DEMOCRATIC AND POPULAR REPUBLIC OF ALGERIA

MINISTRY OF HIGHER EDUCATION AND SCIENTIFIC RESEARCH

Ecole Nationale Polytechnique

Electronic Department

Master thesis in electronics

A comparison study on ECG signal compression methods

BOUGUEZINE Insaf

Supervised by:

PhD. Mourad ADNANE   Prof. Adel BELOUCHRANI

Presented in public on October 8th, 2017

Jury members

President	Mr. Cherif Larbes	Prof.	ENP
Examiner	Mr. Mohamed Salah Ait Cheikh	Prof.	ENP
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	Mr. Adel Belouchrani	Prof.	ENP

ENP 2017

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Dedication

To my father, who taught me that the best kind of knowledge is that which is learned for its own sake;

To my mother, who has always been there for me, doing best what a mother could have ever done;

To my brother Mohamed and sister Yasmine

To my friends for being the most loving, inspiring and supportive friends one could ever ask for.

To Mei, Safa, Houda and to Majd , I would not have made it without you guys.

Acknowledgement

My deepest appreciation goes to all those who have ever been there for me and helped me with the smallest but kindest gesture at any point throughout a long journey.

First and foremost I would like to thank my supervisors Mr.ADNANE and Mr.BELOUCHERANI, the jury members as well as all those whose generous encouragement and support helped me get to this point in my life.

I would like to thank my friends for their unconditional love, loyalty and support in my I cannot cite your names here but you would know be cause I am on your mind and you are on mine.

Last but not least, I would like to thank those whom I can never thank enough. My parents. I am so much of what I have learned from you and for that I am eternally grateful.

ملخص :

الهدف من هذا العمل هو إجراء دراسة مقارنة لمجموعة من أساليب الضغط بما في ذلك دكت، دوت، خوارزميات .DFT , DWT,DCT وطرق تكميم ناقلات مثل خوارزمية LGB. كما يتم عرض استعراض التقنيات المختلفة المستخدمة لتخطيط القلب، والكلام والصورة والفيديو ضغط.

يتم تنفيذ هذه التقنيات على MATLAB r2013a وتقييمها على أساس نسب ضغط كل منها ونسبة الفرق متوسط مربع. وتبين الدراسة أن ضغط DCT هو الأكثر كفاءة بين تقنيات ضغط التحويل الأخرى، وأن تكميم النواقل هو في حالة إشارة تخطيط القلب استراتيجية جيدة لنسبة ضغط أكثر كفاءة

الكلمات الدالة تخطيط القلب ,DCT, DWT, DFT , ضغط.

Résume :

L' objectif dans ce travail est d'étudier un ensemble de méthodes de compression, y compris la DCT, DWT, DFT ainsi que les méthodes de quantification vectorielle comme l'algorithme LGB. Une revue des différentes techniques utilisées pour l'ECG, la parole et l'image est également présentée. Ces techniques sont évaluées en fonction de leurs rapports de compression respectifs et du pourcentage de différence carrée moyenne. L'étude montre que la compression à l'aide de la dct est la plus efficace parmi les autres techniques de compression étudiées et que la quantification vectorielle est dans le cas d'un signal ECG une bonne stratégie pour un ratio de compression plus efficace.

mots clé: ECG, compression, DCT, DWT, DFT, quantification vectorielle, LBG, K-means.

Abstract :

The aim of this work is to carry out a comparison study for a set of compression methods including DCT , DWT,DFT algorithms , and vector quantization methods like the LGB algorithm. A review of the different techniques used for ECG , speech and image and video compression is also presented. These techniques are implemented on MATLAB r2013a and evaluated based on their respective compression ratios and percentage of mean square difference. The study shows that the dct compression is the most efficient amongst other transform compression techniques and that vector quantization is in the case of an ECG signal a good strategy for a more efficient compresson ratio.

Key words : ECG, compressin , DCT , DWT , vector quantization, LBG, K-means algorithms.

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List of Abbreviations

ECG : Elctrocardiogram

DWT: Discret Wavelet Transform

DFT: Discrete Fourier Transform

DCT: Discrete Cosine Transform

PRD: Percentage Root-squared Distortion

CR: compression ratio

Introduction

Electrocardiographic signals may be recorded for hours , even for days and that is for the purpose of detecting occurring abnormalities in the heart's activity that can even be life threatening . Consequently, the produced ECG recording amounts to huge data sizes that not only quickly fill up available storage space but also present a problem when transmitting data.

For these cases, data compression is an essential operation and another key elements in signal processing, which has, in turn, contributed significantly to a new understanding of the ECG and its dynamic properties as expressed by changes in rhythm and beat morphology. Thus, data compression and other signal processing techniques go hand in hand and provide us with a variety of methods that aims at once for the reliability of the received data and its size.

This project aims for a study of these methods. The first chapter give a brief glance at the electrical activity of the heart and the actual significance of the ECG signals. The second chapter gives a review of a multitude of compression algorithms and their different classifications. The third chapter includes a comparison study between different compression methods including the DCT compression, the DWT compression and the DFT compression. It also highlights the efficiency of the vector quantization by quantizing the ECG signal using the K-means algorithm.

CHAPTER 01

Chapter 1

Introduction to ECG compression

This chapter contains background generalities concerning some of the basic properties of the ECG signal. This will help us understand the type of information we are dealing with and set aspects to consider in the compression process as not to alter the signal in an undesirable manner.

1.1 About ECG

An electrocardiogram (ECG) is a simple, noninvasive test where electrodes are placed on the skin of the chest and connected in a specific order to a machine that, when turned on, measures the electrical activity of the heart to detect possible abnormalities.

An electrocardiograph is a machine that is used to perform electrocardiography, and produces the electrocardiogram. The fundamental component to electrocardiograph is the Instrumentation amplifier, which is responsible for taking the voltage difference for each lead and amplifying the signal. ECG voltages measured across the body are on the order of hundreds of microvolts up to 1 millivolt (the small square on a standard ECG is 100 microvolts). This low voltage necessitates a low noise circuit and instrumentation amplifiers are key.

1.1.1 How the heart works

The heart is a muscle about the size of a fist which works like a pump: The right side (see Figure 1) pumps blood into the lungs to pick up oxygen. The left side of the heart receives the oxygen-rich blood from the lungs and pumps it into the body.

The heart has four chambers and four valves and is connected to various blood vessels. Valves are like doors that open and close. They open to allow blood to flow through to the next chamber or to one of the arteries. Then they shut to keep blood from flowing backward, producing a "lub dub sound". A single heart beat (cardio cycle) is a succession of two phases:

- Systole is when the ventricles contract, or squeeze, and pump blood out of the heart:

  From the left side to the rest of the body and from the right side into the lungs to pick up oxygen. The end of this phase is marked by the closing of the pulmonary and aortic valves (the dub sound).

▪ Diastole is when the ventricles relax and fill with blood pumped into them by the atria. [**1**]

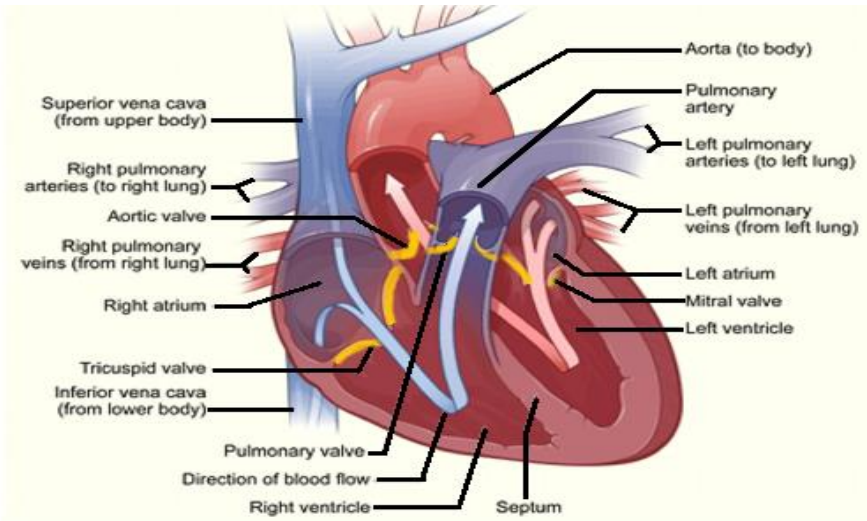


Figure 1 Healthy heart corss-section

1.1.2 Electrical activity of the heart

As any other kind of muscular activity, the contraction and relaxation of the heart are mere consequences of a particular periodic electrical activity: An electrical impulse is generated by a node called the "***Sinoatrial (SA) node"*** in the right atrium and travels to the left atrium and then to the ventricles through pathways that are specialized for rapid transmission of electrical impulses. These electrical signals end up spreading throughout the muscular cells.

The electrical signals are ions (Na^{+}, K^{+}, Ca^{+2}) and their movements trace what is called ion channels. Ion channels help maintain ionic concentration gradients and charge differentials between the inside and outside of the cells. The ions are driven across cell membranes by two main forces:

- Chemical potential: an ion will move down its concentration gradient.
- Electrical potential: an ion will move away from ions/molecules of like charge

The resting potential in a cardiomyocyte (a cardiac muscle cell) is **–90 mV**. When there is a net movement of positive ions into a cell, **the TMP** i.e. the electrical potential difference (i.e., voltage) between the inside and the outside of a cell becomes more positive.

However these channels are only permeable to a single type of ions. Their permeability is restricted to specific TMPs intervals. Outside these intervals, these channels are closed. Some channels (importantly, fast Na+ channels) are configured to close a fraction of a second after opening; they cannot be opened again until the TMP is back to resting levels, thereby preventing further excessive influx. This results in a very specific variation in the TMP. The latter is measured using different electrodes **.** [**2**] These electrodes are configured as to observe the heart activity from different angles, and that is by placing the electrodes in particular spots of the patient's body. Each pair of these spots is called a **"lead".** One of the most used ECG measurement procedures is the 12-Leads electrocardiography. The image below illustrates the electrodes placements for this method. A single ECG sheet includes recordings for each of the 12 leads. [**3**]

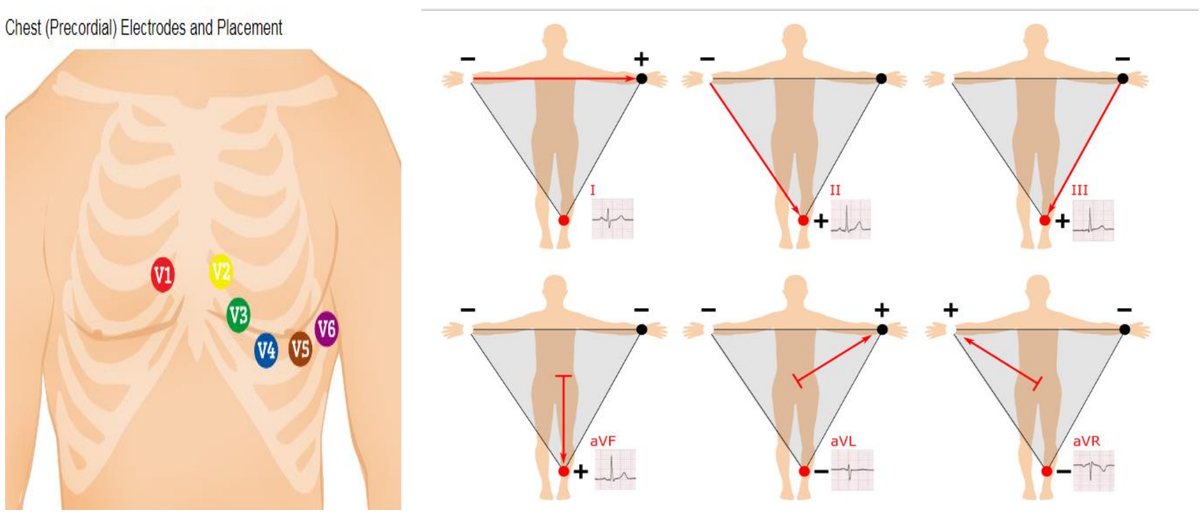


Figure 2 12-lead ECG

1.1.3 Typical ECG waves

The electrical activity of the heart along with the emplacements of the electrodes result in a typical ECG signal. Figure 3 illustrates a typical cardiocylcle recording. It can be seen that normal rhythm produces four waves :a P wave, a QRS complex, a T wave, and a U wave. Each of these waves has a fairly unique pattern.

- The P wave represents atrial depolarization.
- The QRS complex represents ventricular depolarization.
- The T wave represents ventricular repolarization.
- The U wave represents papillary muscle repolarization.

However, the U wave is not typically seen and its absence is generally ignored. Changes in the structure of the heart and its surroundings (including blood composition) change the patterns of these four entities. [**4**]

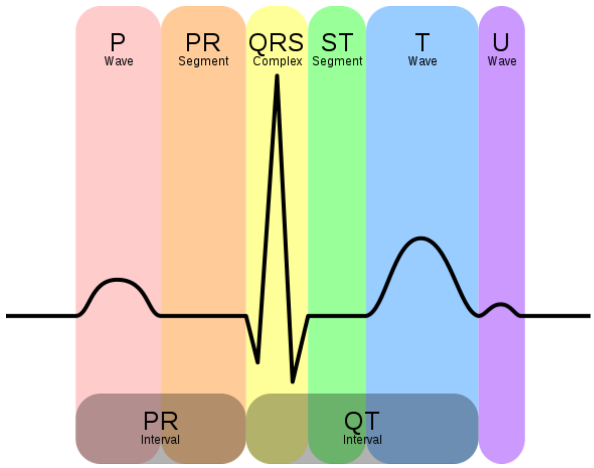


Figure 3 ECG cyle waves

1.2 Why ECG compression is important

Output usually appears on a long scroll of paper that displays a printed graph of activity on a computer screen. ECG can be performed on-site i.e. in a doctor's office or at a hospital, or by the mean of remote ECG monitoring systems. These systems are commonplace medical devices for remote and long term physiological monitoring, especially for that of the elderly and frail patients. They consist of three major components:

- A mobile gateway, deployed on the patient's mobile device, that receives 12-lead ECG signals from any ECG sensor.
- A remote server component that hosts algorithms for accurate annotation and analysis of the ECG signal
- A point-of-care device for the doctor to receive a diagnostic report from the server based on the analysis of the ECG signals [**5**]

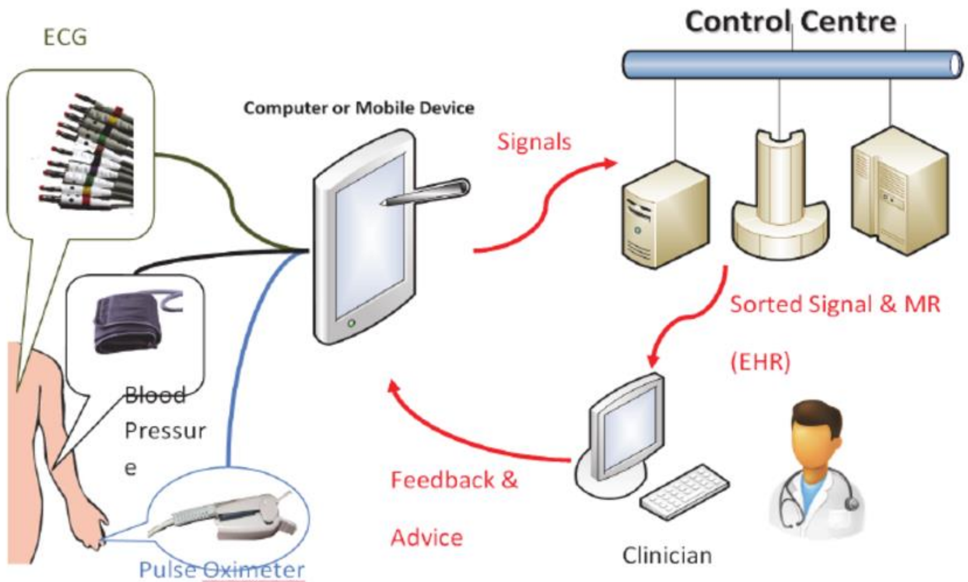


Figure 4 Remote ECG system

A healthy heart beats at an average rate of 1.2 Hz. And all the recordings are to be stored either at the remote server component or at the point-of-care device. Consequently, huge memory is required. Thus, **the limitation in storage** has made **ECG data compression** an important issue of research in biomedical signal processing. In addition to these, there are many advantages of ECG compression such **as transmission speed** of real-time ECG signal is enhanced and is **also economical.**

Conclusion:

In this chapter we introduced some basics about the ECG signal, we acquired a brief but sufficient -to some extent- background on the phenomena behind its generation. These aspects are quiet important when compressing the signal as they allow us to know more about how far we can go in compressing it. This holds when implementing a compression algorithm that allows us to reconstruct the signal based on some key parameters, which is also knows as parametric compression.

CHAPTER 02

Chapter 2

Quantization,compression and encoding techniques

Generally, The aim is to eliminate redundancies as to represent the information using a minimum number of bits (rate) and that is with respect to the quality of the compression. This later is generally expressed in terms of reliability or distortion. Another aspect that is to be considered is the real-time processing: An algorithm can only be used if the available resources allow a real-time execution of this algorithm.

It can easily be noticed that speech signal are similar to ECG signal in terms of quasi-periodicity and frequency band. This latter can also be found in images. Consequently, the idea is to apply some of the techniques applied in speech and image compression, to ECG signals. Considerable efforts have been put into this. Compression methods vary between lossless and lossy methods, scalar and vector quantization based methods , parametric and waveform methods .They can also be divided into direct methods and transform methods.

This chapter presents a review of most of these techniques with their classification. The reader must be familiar with some of the basics of information theory including self-information, entropy, Shannon theorem, distortion, rate and the relationship between the two of them. Nonetheless, some of these preliminaries will be briefly explained. Books like one proposed at [**6**] can be used for further explanations.

2.1 Background on information theory and data compression

2.1.1 Rate-distortion theory

In his 1948 paper, ``A Mathematical Theory of Communication,'' Claude E. Shannon [**7**] formulated the theory of data compression in which he introduced two key elements

- ✓ **Rate**: It is the as the number of bits per data sample to be stored or transmitted
- ✓ **Distortion:** It expression the sparseness of the input sample and the corresponding reconstruction level. It is in most cases defined as the expected value of the square of the difference between input and output signal i.e**. the mean squared error.** There are other forms of distortion like the hamming distortion. However, It is sometimes modeled on **human perception** aesthetics and that is mostly in the case where data is to be perceived by human consumers. For instance: listening to music or watching pictures and videos.

Rate-distortion theory says that for a given source and a given distortion measure, there exists a function, R(D), called the rate-distortion function that gives the best possible compression rate. A typical shape of R(D) looks like this

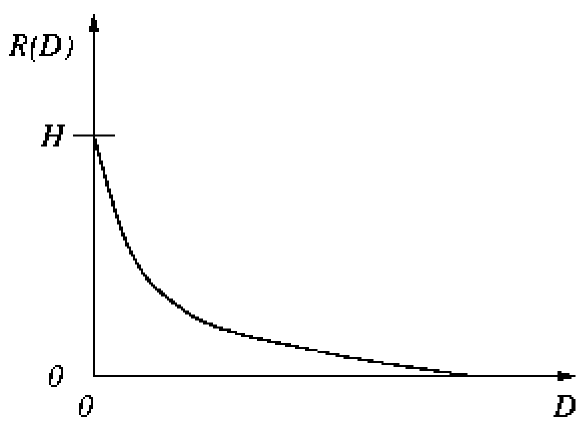


Figure 5 Rate-distorsion diagram

Where H is the entropy of the sequence of data. If the source samples are independent of one another, the rate-distortion function can be obtained by solving the constrained minimization problem which mainly consists of minimizing **the mutual information** of the input and output of the source encoder. [**8**]

2.1.2 Lossy vs. lossless compression

Lossless, as the name implies, involve no distortion (D=0) ,in other words, no loss of information. Consequently, the original data can be recovered exactly from the compressed data. It is generally used for applications that cannot tolerate any difference between the original and reconstructed data like text compression. The minimum possible rate for this type of compression represents the entropy of the signal.

There are also a number of situations in which the decompressed data need not be exactly the same as the original data, that is where lossy compression comes. However, in return for accepting this distortion in the reconstruction, we can generally obtain much higher compression ratios than is possible with lossless compression. Some applications where lossy compression techniques are used include speech compression, image compression, video compression where the fact that the reconstruction is different from the original is generally not important as long as the differences do not result in annoying artifacts that perceptibleto the human's senses .

2.1.3 Quantization

The process of mapping input values from a large set (often a continuous set) to output values in a (countable) smaller set. The output values are referred to as reconstruction level. For each reconstruction level there exists a subspace such that the input elements belonging to this subspace are to be mapped to the same corresponding reconstruction level. A discrete signal can either be quantized one sample at the time, this is known as scalar quantization or divided into blocks of samples such that each block will be entirely mapped to a reconstruction level that is naturally a vector, this is known as vector quantization.

2.1.4 Granular noise and overload noise

Quantization error is composed of two kinds of error, overload error and granular error. The overload error is determined by the location of the quantization regions furthest from the origin, or the boundary. Granular noise concerns the lowest values that are mapped to higher undesirable values.

2.2 Evaluation of performance

A compression algorithm can be evaluated in a number of different ways and that includes: the relative complexity of the algorithm, the memory required to implement the algorithm, how fast the algorithm performs on a given machine, the amount of compression, and how closely the reconstruction resembles the original. For this thesis, we will only consider the last two aspects. The amount of compression is measured through what is called a "compression ratio" which is given by the following formula:

$$\mathrm{Cr} = \frac{\text{Number of bit per uncompressed sample}}{\text{number of bits per compressed sample}}$$

The amount of the introduced distortion is represented by the percentage root-mean-square difference given by the three following different formulas. The choice of the formula depends on the context

$$\mathrm{PRD1} = 100 * \sqrt{\frac{\sum_{i=1}^{N}(xi - yi)^2}{\sum_{i=1}^{N}(xi - moy)^2}}$$

$$\mathrm{PRD2} = 100 * \sqrt{\frac{\sum_{i=1}^{N}(xi - yi)^2}{\sum_{i=1}^{N}(xi)^2}}$$

$$\mathrm{PRD3} = 100 * \sqrt{\frac{\sum_{i=1}^{N}(xi - yi)^2}{\sum_{i=1}^{N}(xi + p)^2}}$$

Where *N* is the number of sample in a given signal, *xi* is the ith sample*, yi* is the corresponding reconstruction level, *moy* is the mean value of the signal, *p* is a *reference value* ,added when recording the ECG signal and is generally pointed out by info files attached to the data file that contains the signals 'representations.

The ECG signal compression can also be evaluated visually by a cardiologist. However this method remains subjective and can only be used to detect obvious alterations on the signal's morphology.

2.3 Compression techniques :

Whether the compression scheme is to be lossy or lossless, depends on the reconstruction requirements. However, there are a large number of techniques that fall into each of these two categories. The exact compression scheme we use will depend on a number of different factors. Some of the most important factors are the characteristics of the data that need to be compressed such as redundancy, periodicity, frequency range, etc

This section presents a review of different algorithm techniques amongst which we chose two compare.

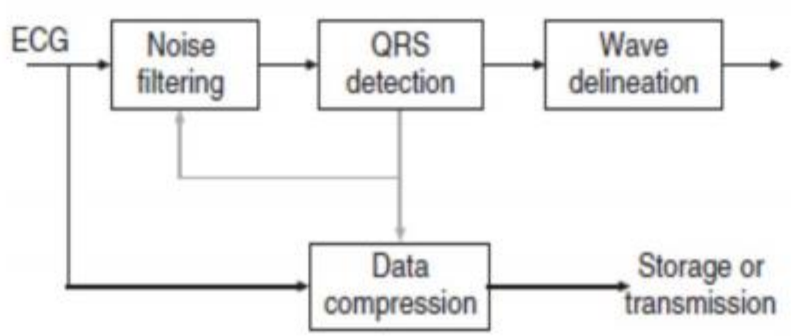


Figure 6 Algorithms for basic ECG signal processing

2.3.1 Lossless algorithms:

As their name indicates ,these algorithms do not introduce any distortion and so allow the original data to be perfectly reconstructed from the compressed data. They include:

- ✓ The Huffman algorithm
- ✓ The run-length algorithm
- ✓ The Shannon algorithm
- ✓ Lempel-Ziv Welch algorithm

2.3.2 Lossy algorithms

The following diagram summarizes the most common compression schemes for ECG and speech signal. Speech signal was considered for its wide resemblance to the ECG.

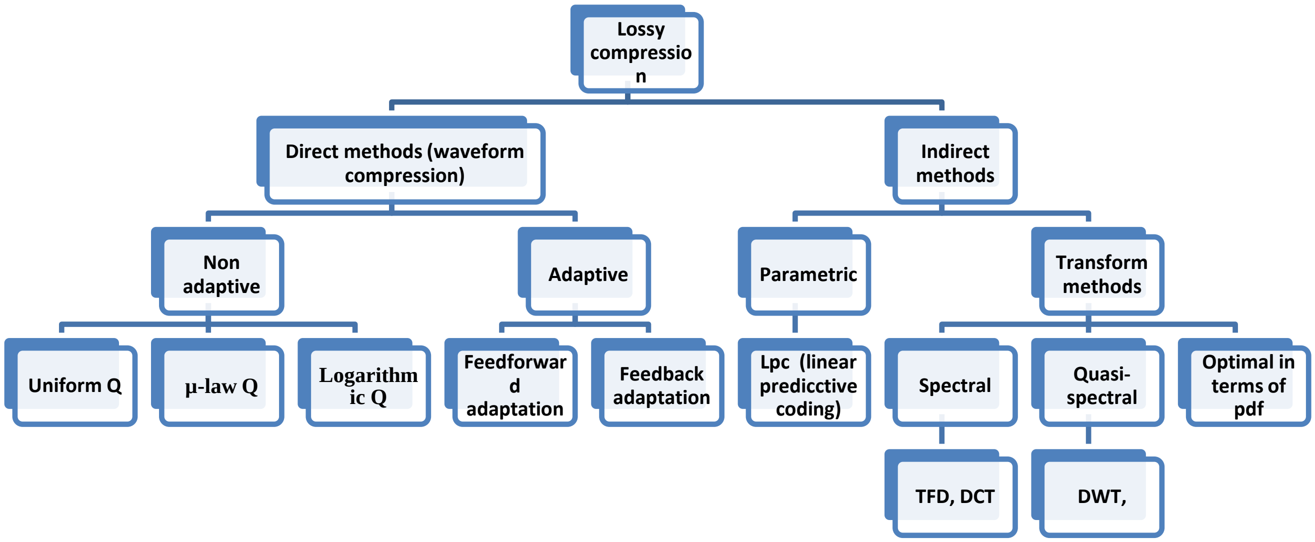


Figure 7 Diagram summarizing a multitude of data compression schemes

2.3.2.1 Waveform non-adaptive coding techniques

These methods rely on direct compression of either

a) *The sample itself:* The following figure represents a typical bloc diagram of the transmission line associated with such techniques.

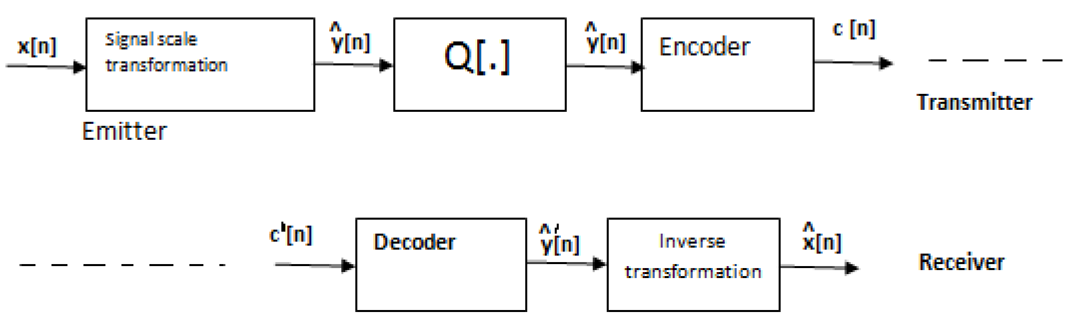


Figure 8 Pulse Code Modulation typical transmission line

The encoder is generally an entropy encoder: The binary word associated with each reconstruction level will depend on the probability distribution of the input signal. In our case, the heart electrical activity doesn't differ majorly from one person to another. Naturally, establishing a statistical model for the input is possible. Thus, it is possible to obtain very efficient encoding rates.

The quantizer : is generally uniform. The step is **pre-fixed** according to the input signal as to minimize the granular and overload noises as much as possible. Consequently, the reconstructed signal is the closest to the input signal when this latter is in his typical shape but not in the case of a severe alteration of the signal which reflects an anomaly that could be life-threatening. Hence, the necessity of an adaptive quantizer.

The transformation is used to cope with the dynamic range of the signal. The quantized value in this case is not that of the sample but that of its transform. Different transforms can be used such as the µ- law transform which not only reduces the dynamicity problem but also enhances the SNR. [**9**]

b) The sample itself - (minus) a predicted value of the sample using previous sample which is knows as differential coding.

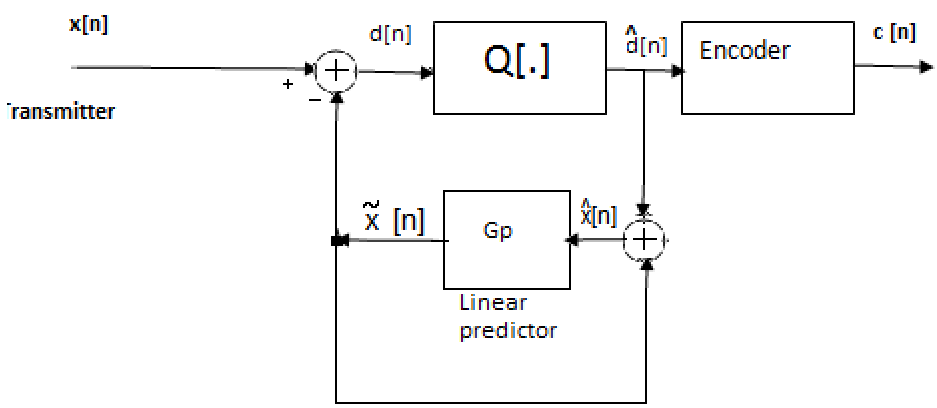


Figure 9 Differential coding transmission line - transmitter

This kind of coding techniques takes advantages of the redundancy in the samples and presents a better SNR then simple coding.

$$SNR = \frac{\sigma x^2}{\sigma e^2} = \frac{\sigma x^2}{\sigma d^2} * \frac{\sigma d^2}{\sigma e^2} = Gp * SNR_Q \ldots \ (3.2.1.1.1)$$

Where σx^2 is the energy of the input signal, σe^2 is the energy of the error signal e[n] ,

Where σd^2 is the energy of the differential signal.

2.3.2.2 *Adaptive coding*

The quantizers used adaptive coding are adaptive quantizers which "adapt themselves" to the statistics of the input. This allows us to deal with the mismatch problem i.e., the fact that several things might change in the input with respect to the assumed statistics, including the mean, the variance and the pdf. The encoder has to be designed such that the parameters of the quantizer are to change to fit the input. for example: the step size of the quantizer is not pre-determined but is calculated as to minimize the square error of the quantizer.

There are two ways to adapt the quantizer parameters : Forward adaptive quantization

a. Forward adaptive quantization

The source output is divided into blocks of data. Each block is analyzed before quantization, and the quantizer parameters are set accordingly. The settings of the quantizer are then transmitted to the receiver as side information. This allows for a noticeable rise in the SNR which reduces the overall rate.

The size of one block has to be small enough as to capture the statistical changes in the input but large enough as to minimize the side information flow.

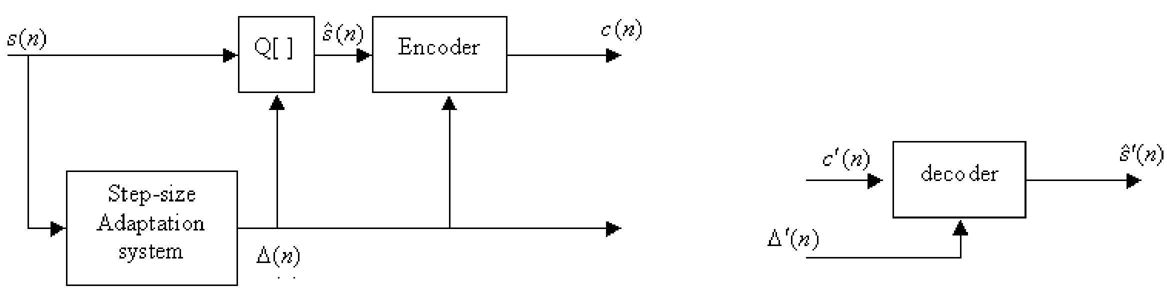


Figure 10 Adaptive quantization encoder +decoder [10]

One of the widely used forward adaptive techniques is the adaptive prediction. Which is similar to a DPCM excluding the fact that the predictor coefficients are adapted to the input.

b. Backward Adaptive Quantization

The quantizer's parameters are set according the values of the c[n] which is known to both the encoder and the decoder and that is not to include the side information in the sent package. Only the past samples are available for use so this method induces more distortion compared to the forward method but allows for a better rate.

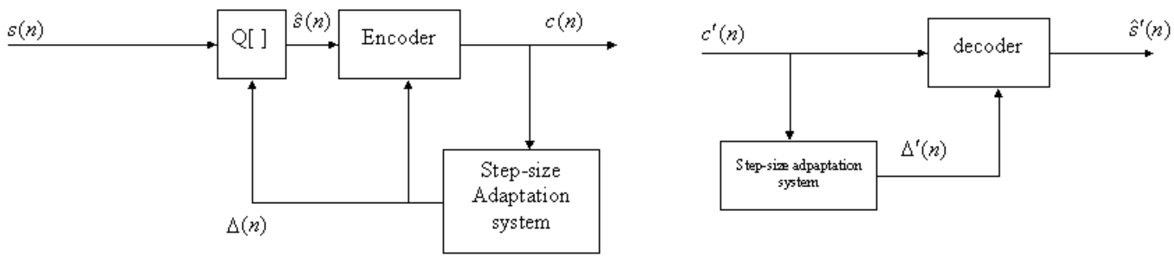


Figure 11 Backward adaptive quantization

The mismatch in this case can be observed from the output. If the step size is well matched to the input, the probability that an input of the quantizer would land in a particular interval would be consistent with the pdf. Otherwise, If Δ is smaller than what it should be, the input will fall in the outer levels of the quantizer an excessive number of times. On the other hand, if Δ is larger than it should be for a particular source, the input will fall in the inner levels an excessive number of times. One approach to proceed is to observe the output for a long period of time. Another approach which was proposed by Nuggehally S. Jayant consisted of adjusting the step size based only on the previous samples as shown in [**6**] **.**

There are many backward adaptive quantization techniques amongst which we enumerate: the Adaptive binary delta modulation in which the quantizer of the difference signal has two possible outputs +Δ (if the x[n]> x and - Δ if x[n]< x[n]) the value of Δ is t be adjusted based on the past one or two samples.

Another aspect of the coders with linear predictors that should be mentioned is that the order of prediction generally doesn't exceed p=2, because the SNR (p) takes a quasi-constant value from p=2 on. [**11**]

2.3.2.3 Parametric coding

This kind of techniques requires the studying of the reproduction of the signal in order to know how to re-synthesize it based on a number of essential information. The mission of the coder in this case is simply to extract these information from the signal and encode them. This logic is used for what we call "Linear Predictive Coding" (of speech). Where the vocal tract is studied and modeled. Speech is regenerated at the decoder in a way that is very similar to how it is produced. This kind of compression works for signals with typical shape and parameters.

Comparing the ECG signal and the speech signal, it can be seen that the ECG signal is similar to the speech signal for the fact that it is quasi-periodic and it presents key elements that can be detected. Hypothetically, if we detect the waves mentioned in section 1.1.3, the period (the heart rate) and the amplitude of these waves, we can regenerate the signal.

2.3.2.4 Transform coding techniques

Generally, signal is divided into blocks of data, then a transform is applied to each of these blocks. The idea is to find a reversible transformation that removes the redundancy by decorrelating the data, then a signal can be stored more efficiently. The basis vectors of the new space define the linear transformation of the data.

2.3.2.4.1 Karhunen Loeve transform (Optimal coding)

The basis vectors of the KLT are the eigenvectors of the signal covariance matrix. For each block of data, the covariance matrix is computed and diagonalized. This eliminates the correlation of neighboring samples, and the resulting set of coefficients can be encoded with fewer bits for a given distortion than the raw data.

This transform is said to be optimal in terms of minimizing the bit rate. [**12**]

2.3.2.4.2 spectral transforms (non optimal coding)

This type of transforms allows us to take advantage of the frequency distribution of the signal. This lies in the fact that the energy that corresponds to the coefficients of the transform decreases rapidly with the range i.e. , for a signal with a frequency distribution that's concentrated around a given frequency f. The coefficients of this transform are high around this frequency and decrease remarkably as we move to further frequencies. Consequently, given a tolerable distortion D, it is possible to discard coefficients with the least coefficients by setting a threshold.

All the transforms that are used in signal compression are generally linear transformations, that is we can get the sequence $\{y_n\}_{0\leq n<N}$ from the sequence $\{x_n\}_{0\leq n<N}$ as :

$$y_n = \sum_{i=0}^{N-1} x_i a_{n,i}$$

2.3.2.4.2.1 Discrete Fourier transform

The DFT represents the input signal as a linear combination of weighted basis functions that are related to its frequency components. The forward transform is defined by

$$y(n) = \sum_{k=0}^{N-1} x(k) e^{-j2\pi kn/N}$$

The inverse transform is given by :

$$x(k) = \frac{1}{N} \sum_{n=0}^{N-1} y(n) e^{j2\pi kn/N}$$

For ECG signals it is the high frequency coefficients that are rejected. However, this methods presents some drawbacks that include:

- The coefficients are complex
- It does not give a time- frequency of representation of the signal. This problem can be solved by the use of time window to selected a limited number of sample and evaluate the frequency distribution of the signal within this time frame. However the precision in the frequency domain is limited by the Heisenberg incertitude theorem.

2.3.2.4.2.2 Discrete cosine transform

DCT decomposes the signal into a sum of cosine functions oscillating at different frequencies.

DCT performs better than the DFT, the output is a real number. However the lack of time-frequency information is still there.

The forward discrete cosine transform is given by several formulas amongst which we can mention :

$$y(k) = \frac{1}{2}\left(x0 + (-1)^k x(N-1)\right) + \sum_{n=1}^{N-2} x(n)\ \cos\left[\frac{\pi}{N-1} nk\right] \quad \text{Equation 0-1}$$

The DCT is used in JPEG image compression, MJPEG, MPEG, DV, Daala, and Theora video compression. An imagine is typically divided into 8x8 blocs. There, the two-dimensional (DCT-II) blocks are computed ,i.e., the DCT-II formula is applied to each row and column of the block. The result is an 8 × 8 transform coefficient array in which the (0,0) element (top-left) is the DC (zero-frequency) component and entries with increasing vertical and horizontal index values represent higher vertical and horizontal spatial frequencies. A mask is applied to discard some of the coefficients that are less than a prefixed threshold, and the results are quantized and entropy coded. [**13**]

2.3.2.4.3Quasi-spectral transforms (non optimal coding)

Similarly to spectral transforms, these transforms are mere decomposition of the targeted time domain discrete function (i.e. our signal) into weighted combination of orthogonal base functions, such as Walsh functions , Haar wavelet and many other types of wavelets.

These methods do not guaranty a concentration of energy on a limited number of coefficients, however they are easier to compute

2.3.2.4.3.1 Discrete wavelet transform (DWT)

This transform captures both frequency and location information (location in time) which gives it an advantage over Fourier transforms

The idea behind DWT is that an efficient decorrelation can be achieved by splitting the data into two half-rate subsequences, carrying information respectively on the approximation and detail of the original signal, or equivalently on the low- and high-frequency half-bands of its spectrum. Since most of the signal energy of the ECG signal is typically concentrated in the lowpass frequencies, this process splits the signal in a very significant and a little significant part, leading to good energy compaction. The procedure can be iterated on the low pass subsequence by means of the filter bank configuration as shown in Figure 12.

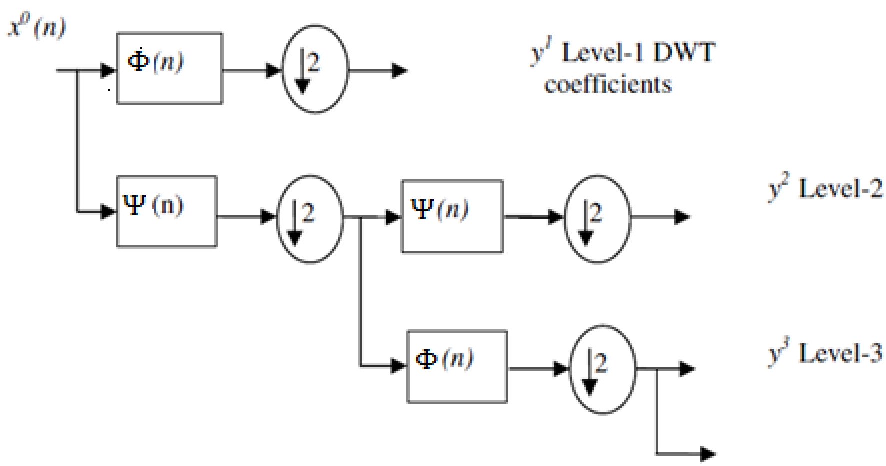


Figure 12 Filter bank representation of DWT decomposition [14]

This transform is an implementation of the wavelet transform using a discrete set of wavelet scales and translations obeying some defined rules. [**15**] $W\Psi(j,k)$ and $W\varphi(j0,k)$ are given by the forward DWT formula :

$$W\varphi(j0,k) = \frac{1}{N}\left[\sum_{k} s(n)\ \Phi\ j0,k\ \ (n)\ \right]$$

$$W\Psi(j,k) = \frac{1}{N}\left[\sum_{k} s(n)\ \Psi\ j0,k\ \ (n)\ \right]$$

Where the $\Phi\ j0,k\ (n)$ and $\Psi\ j0,k\ (n)$ represent the scaling function and the wavelet function respectively.

There exists a variety of scaling functions which can be used such that from each scaling function a corresponding wavelet function can be derived, resulting a typical type of wavelet transforms

The inverse transform formula represent the method to extract the signal original and is given by:

$$x[n] = \frac{1}{N}\left[\sum_{k} W\varphi(j0,k)\ \Phi\ j0,k\ (n)\ + \sum_{j=j0}^{\infty}\sum_{k} W\Psi(j,k)\Psi j,k\ (n)\right]$$

2.4 Vector quantization:

The set of the quantizer's output points is called the codebook of the quantizer, and the process of placing these output points is often referred to as codebook design.

By dividing the source outputs into groups and encoding each group as a single block, we can obtain efficient lossy as well as lossless compression algorithms in several ways. By “efficient” we mean a lower distortion for a given rate, or a lower rate for a given distortion.

First, it can easily be seen how the structure in the form of correlation between source can make it more efficient to look at sequences of source outputs rather than looking at each sample separately : For 2 samples that are correlated the total number of possible combinations of the two bits as a whole is smaller than (number of possibilities for the first one multiplied number of possibilities for the second one) and that is simply because some 2-sample combinations don’t even occur and we can simply not represent them at all. This is only possible in case of vector quantization.

Second, we can move some points to the origin as shown in Figure 13. These points are combinations of multiple samples that are of the least probability. This can augment the SNR to a small extent. Whether this improvement is small or not depends on the application.

As we block the input into larger and larger blocks or vectors, these higher dimensions provide even greater flexibility and the promise of further gains to be made. [**6**]

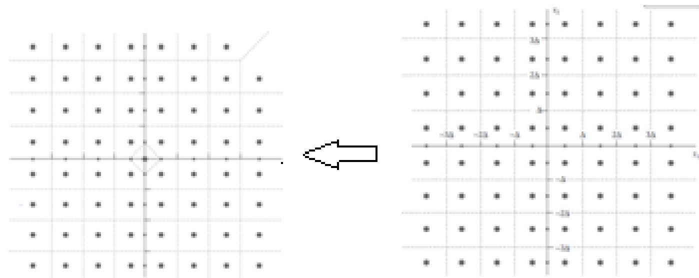


Figure 13 Vector quantization

2.4.1 Linde-Buzo-Gray Algorithm

The idea is to place the quantizer's output points where the source output vectors are most likely to assemble. This kind of algorithms are purely "statistical". They provide the most optimal performance terms of distortion. However they are intricate to implement and are generally used for correcting other static quantizers or testing their optimality.

The original Linde-Buzo-gray algorithms uses the probability distribution function to calculate the assembling points which are simply the centroids of each updated cluster.

The pdf is not always available. Consequently another algorithm is used. This algorithm is known by the " k-means algorithm" and is described as follows:

1. Start with an initial set of reconstruction values $\left\{Y_i^{(0)}\right\}_{i=1}^{M}$ and a set of training vectors $\{X_n\}_{n=1}^{N}$ which are simply the input chain of samples cut into vectors. Set k=0, D =0. Select a threshold Ɛ
2. Clustering: assign each of the training vectors to a quantization region. The quantization regions $\left\{V_i^{(k)}\right\}_{i=1}^{M}$ given by

   $V_i^{(k)} = \{Xn: d(xn, Yi) < d(Xn, Yj) \forall j \neq i\}$ i=1,2,...,M

   We assume that none of the quantization regions are empty. For the case where $V_i^{(k)}$ is empty , we generally take an output point from another cluster that is the farthest from its corresponding centroid.
3. Compute the average distortion D $^{(k)}$ between the training vectors and the representative reconstruction value.
4. If $\frac{(D^{(k)} - D^{(k-1)})}{D^{(k)}} <$ Ɛ , stop; otherwise, continue.
5. k = k+1. find new reconstruction values $\left\{Y_i^{(0)}\right\}_{i=1}^{M}$ that are th average value of the elements of each of the quantization regions $V_i^{(k-1)}$. Go to step 2.

2.4.2 Structured quantizers

These quantizers are organized in such a way that it is easy to pick which part contains the desired output vectors. This type of quantizer includes :Tree- structure quantizers , Fischer quantizers , Polar and spherical vector quantizers, etc

2.4.3 TCQ quantizer

Its entropy-constrained variants provide some of the best performance when encoding random sources. This quantizer can be viewed as a vector quantizer with very large dimension, but a restricted set of values for the components of the vectors.

It is not necessarily a vector quantizer, although it can be used to quantize vectors at the time. However, for it to quantizer a single elements, it relies on the minimization of the distortion error of an entire sequence of samples.

The codebook with R-dimensional codewords (vectors or scalars) include $2^{(R+1)}$ reconstruction levels that will be divided into two sets of 2^R elements. Each chosen element determines the set that is to use for the proceeding element and so on , and each element is chosen as to minimize distortion. However, it may be advantageous, at times, to accept poor quantization for several samples so that several samples down the line, the quantization can result in less distortion.

This results in $2^{R.L}$ possible paths for a sequence of L samples which can present a real computational challenge. Fortunately, there is a technique that can be used to keep this explosive growth of choices under control. The technique, called the *Viterbi* algorithm ,is based on the elimination of the long path as we quantize the sample as explained in [**14**].

CHAPTER 3

Chapter 3

Introduction

This chapter includes a comparison study of the three transform compression methods: the discrete cosine transofrm, the discrete fourier transform and the wavelet transform , as well as a comparison study of the scalar and vector quantization. These methods are evaluated based on the evaluation methods presented in section 2.2

These methods are applied on data from the MIT-BIH, availabe on [**16**] [**17**] data base and implemented using Matlab R2013a.

3.1 Signal denoising

Ecg signals are exposed to several noise sources and that includes : Power line interference, muscle contractions, electrode contact noise, noise generated by electronic devices used in signal processing, breath, etc Several technique can be used to reduce noise.

For this application, we use wavelets, mainly for their ability to localize features in our data to different scales. The wavelet transform leads to a sparse representation for the ecg signal: it concentrates signal features in a few large-magnitude wavelet coefficients. Wavelet coefficients which are small in value are typically noise and can thereby be reduced or utterly removed without affecting the signal or image quality. The wavelets used in this case are debauchies1 wavelets.

Ps: the signal is 10 seconds and contains 3600 samples. For this application we only consider the first 1000 samples.

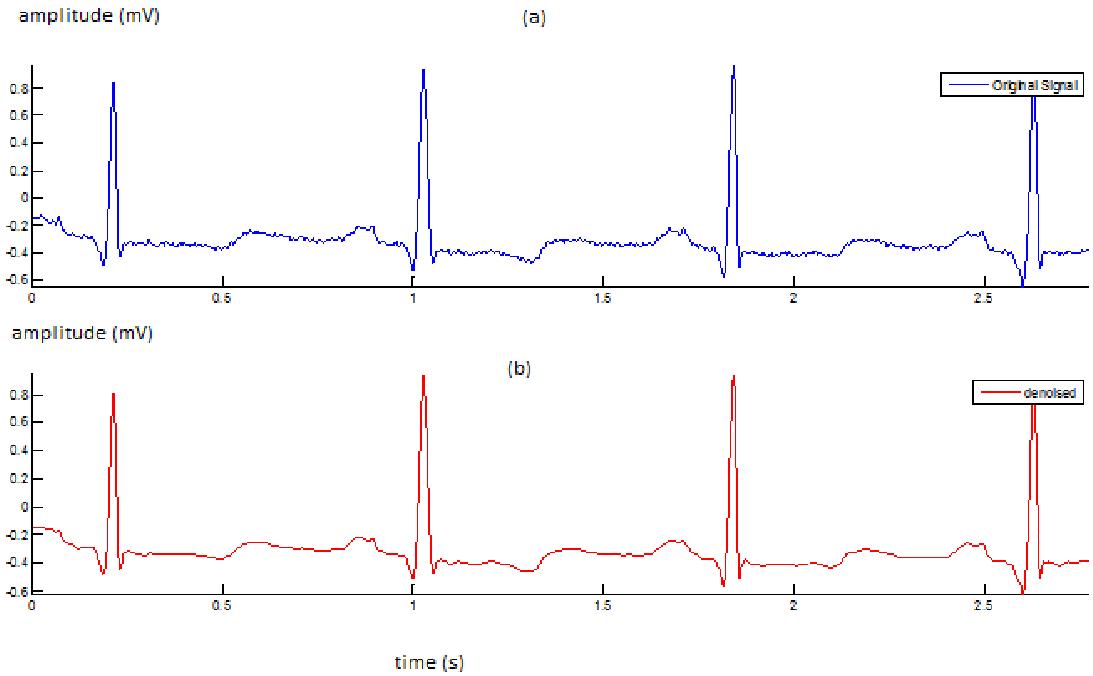


Figure 14 Signal denoising using wavelets

3.2 Compression

3.2.1 Spectral transform compression techniques

These methods are characterized by algorithms that can be generally described in the following steps.

- ✓ The spectral transform of the signal is computed over the 1000 samples
- ✓ It can be seen that energy of the signal is concentrated on the low frequencies. Only these coefficients are to be sent. The number of these coefficient can be reduced further using a threshold and that is in exchange of higher PRD. Thresholding is set according to typical values of the transform.
- ✓ Quantization of the transform coefficients
- ✓ Lossless compression of the quantizer outputs: Methods like Huffman coding are applied. For this study, we leave aside this step and assume that the number of bits assigned to one codeword depends only on the range of the quantizer and the value of the step.
- ✓ We suppose that the transmission channel is lossless.
- ✓ Inverse lossless compression
- ✓ Reconstruction of the signal by inverse transform.

When it comes to some methods, more steps can be included such as : decimation / interpolation, truncating the transform vector due to symmetrical properties, etc.

3.2.1.1 DFT compression

The DFT is computed. Both the imaginary and the real part are submitted to the procedure mentioned in the introduction of this section. Note that the DFT for a real signal is to be symmetrical. Consequently, only half of the transform is sufficient to represent the signal. This allows us to keep a maximum coefficient number of 1000.

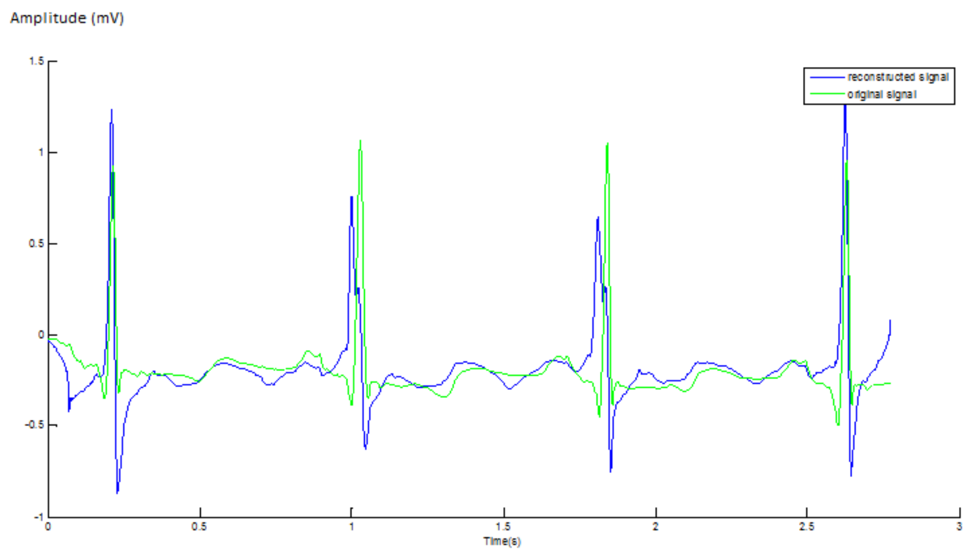


Figure 15 DFT compression :original and reconstructed signals

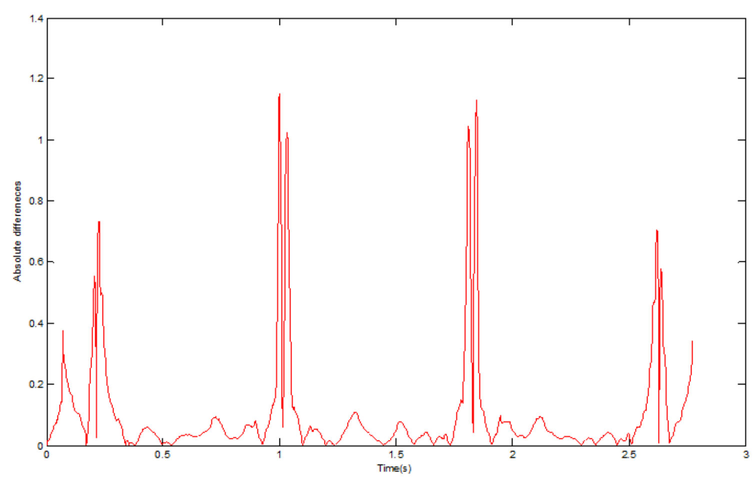


Figure 16 Absolute difference

PRD = 109.6 % CR = 9.09 % Codeword size = 12

Comment: The reconstructed signal is complex. Only the real part is plotted. It can clearly be seen that signal is drowned in the quantization noise.

3.2.1.2 DCT compression

The principle of this method is described in section 2.3.2.4.2.2

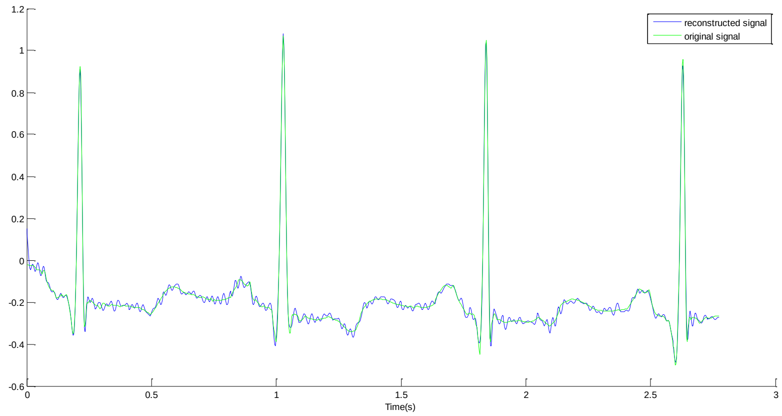


Figure 17 DCT compression: reconstructed and original signals

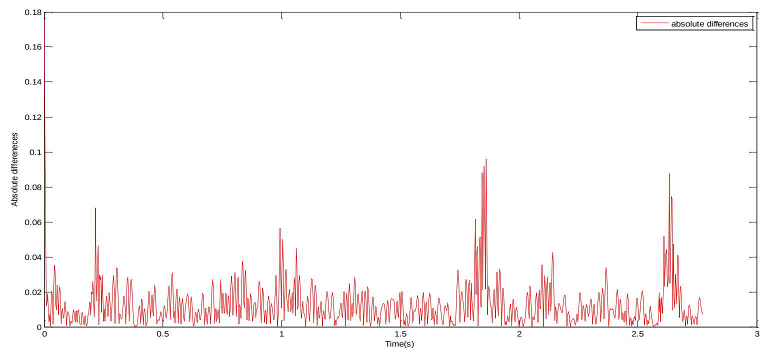
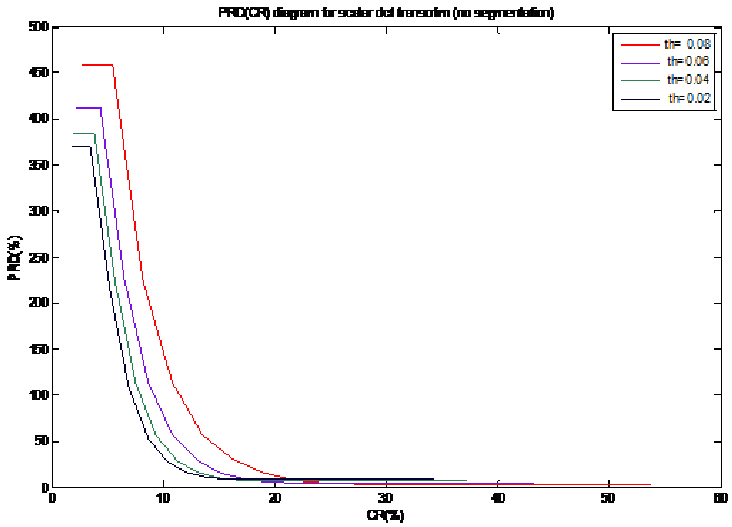
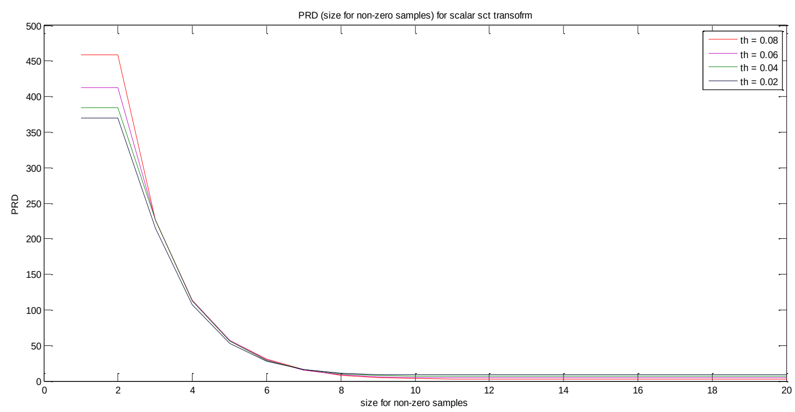


Figure 18 DCT compression : absolute differences

Uniform quantizer 10 bits threshold = 0.08

CR= 17,18% PRD = 8,67 %





3.2.1.3 DCT II compression

This method allows for a time-frequency description of the signal. The signal is divided into a set of blocks. Each block is compressed using the simple cosine method. The total reconstructed signal is formed by concatenating the quantized blocks of samples.

PRD and CR are calculated for each block. PRDm and CRm represent their average value.

PRDm (average) = 9.83% CR (average) = 16.16%

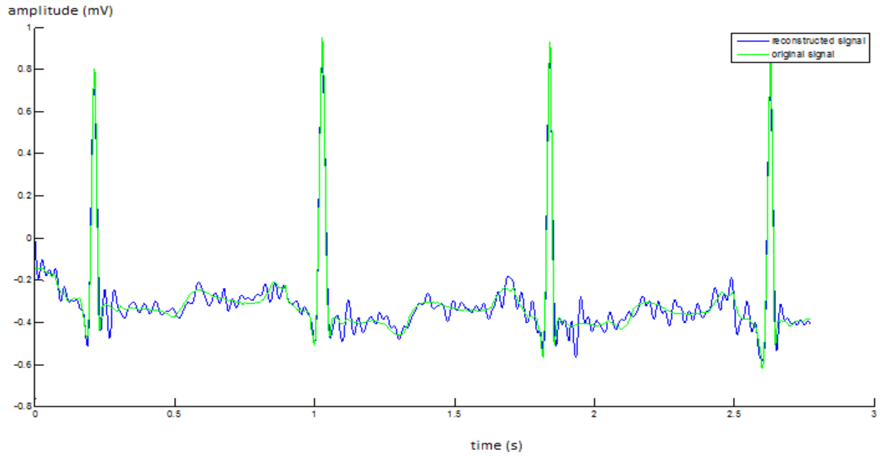


Figure 19 DCT II compression: Reconstruction and original signals

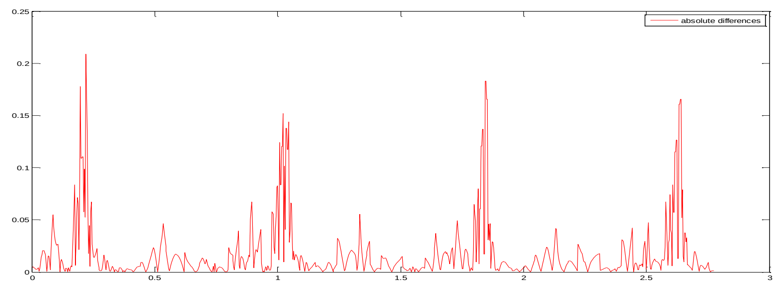


Figure 20 DCT II comression: absolute differences

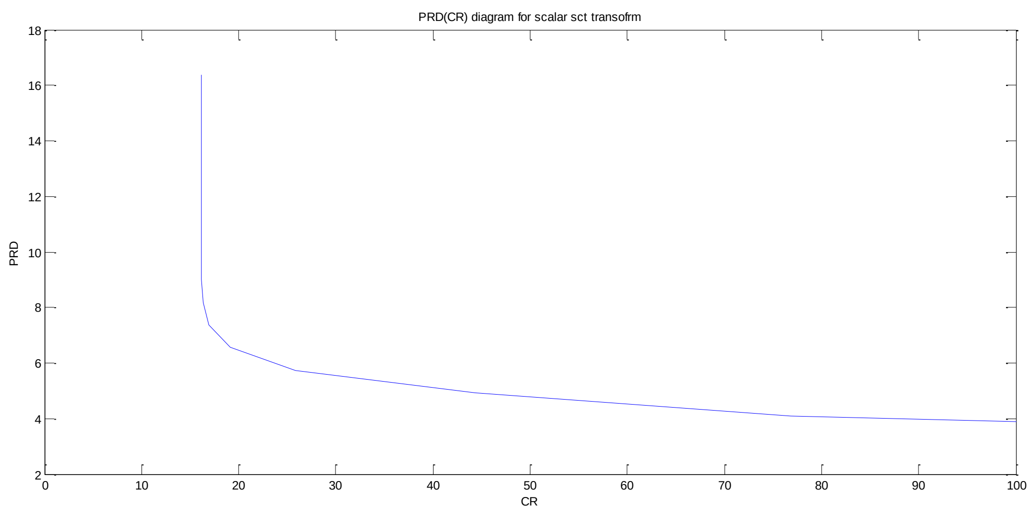


Figure 21 DCT II compression: PRD(CR) diagram

3.2.1.4 *Wavelet transform compression*

Several wavelets families can be used. The following two presented the best efficiency.

a. Debauchies2 :

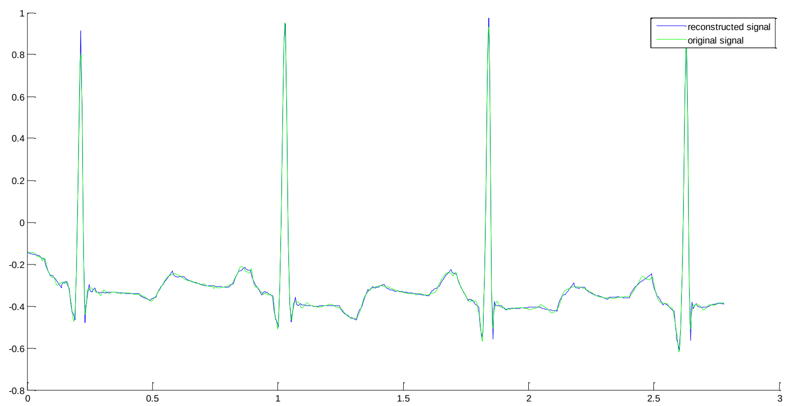


Figure 22 Debauchies2 wavelet transform compression

PRD = 7.62 % CR= 20.15% PFERL2 = 99.62 %

Where PFLR is equal to vector-norm of CXC/vector-norm of C. CxC is the vectors containing the low frequency components. C is the vector containing all the coefficients of the transform. This factor reflects the energy-conservativity of this method.

b. Haar wavelets :

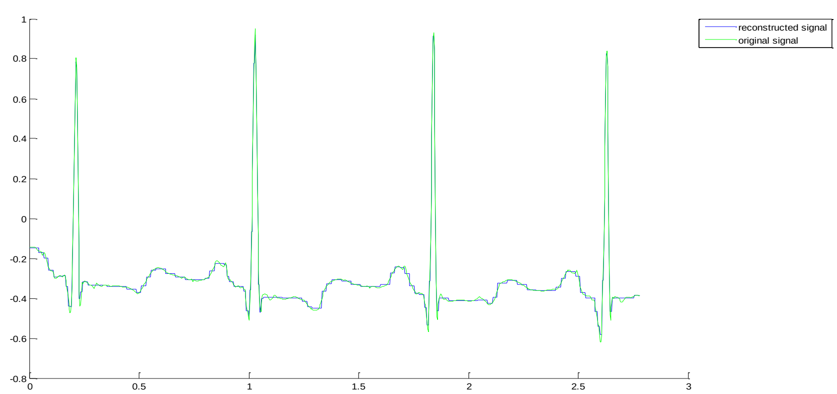


Figure 23 Haar Wavelet transform compression

PRD= 7.67 % CR= 21.32 % PFERL2 = 99.97 %

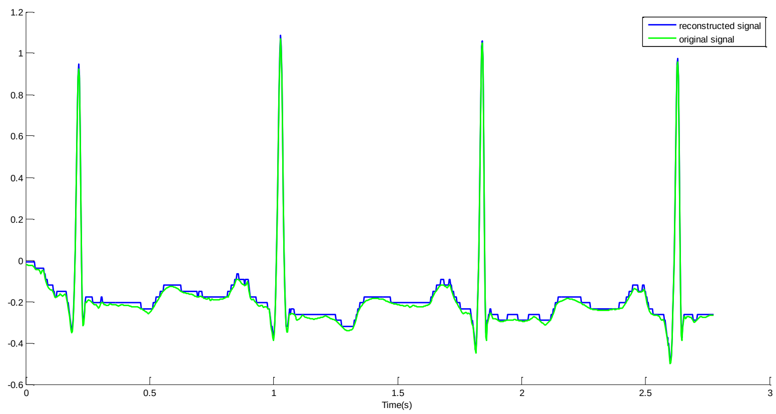
3.2.1.5 Comparison of the transform methods

The DFT decoder outputs a complex signal that is to be treated and converted to the actual real signal. Notes that it is important to conserve both its real and imaginary parts.

On the contrary, the DCT outputs real values.It is much easier to compute and it presents the best efficiency in both segmented and non-segmented signal cases.

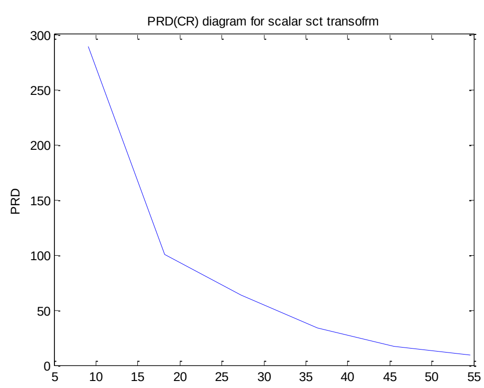
The wavelet transform can also be used, the "Haar" wavelets present the best results and they also produce a real output.

3.2.2 Scalar uniform quantization



PRD = 7.33 % CR = 54% !!!

The compression ratio which, in turn, is related to the PRD is determined by the size of one codeword.



Conclusion We can see how a simple direct quantification is nowhere near our requirements, hence the need for difference methods

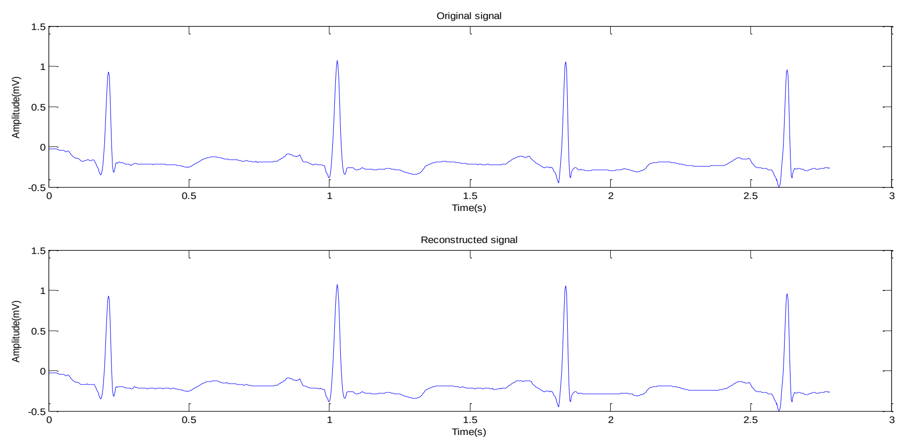
3.2.3 LGB vector quantization

i. First step is to build the quantizer : We use the entire 10s duration signal for better performance on other typical ECG signals. Next, and using the method described in section 2.4.1, we extract a matrix containing the respective representative vectors from each cluster. The vectors are to constitute the codebook.

ii. Second, it is the signal that is divided into blocks of the same length as that of the codewords

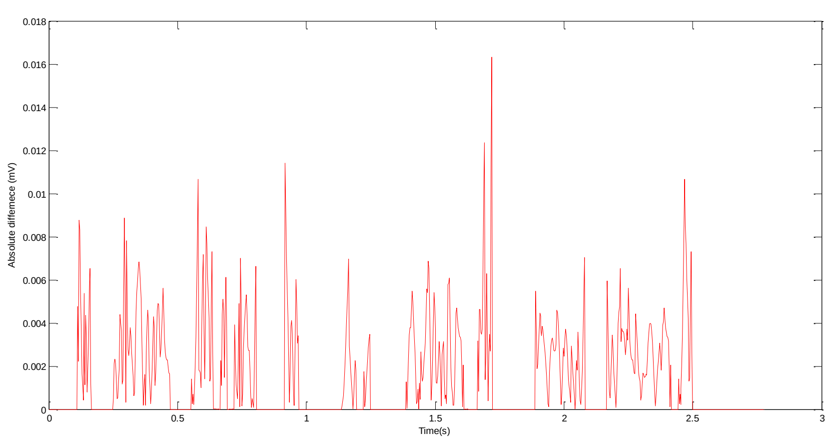
iii. Each vector is quantified to the representative value of the cluster.

iv. The reconstructed signal is rebuilt by concatenating the blocks from step iii



Comment: Visually, the reconstructed signal is practically identical to the original signal. This is due to the fact that the k-means algorithm is optimal in terms of distortion.

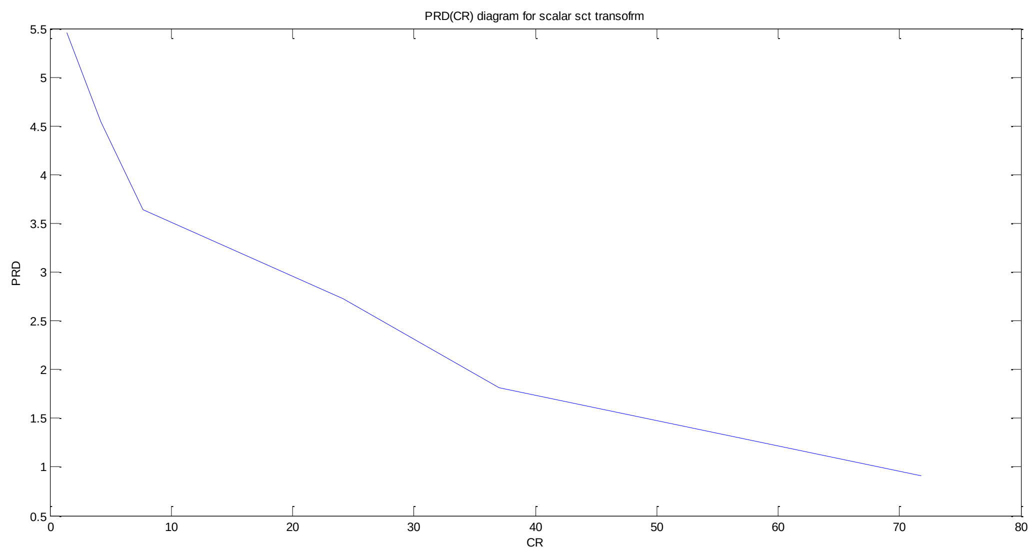
This can be confirmed through the absolute differences diagram.



For CR =   5.45%   PRD = 1,34 %

Comment: This method present excellent values of CR and PRD for a codeword size of 6bits

We vary the codeword size from 1 to 6 (which is the maximum value given the number of blocks) and calculate the corresponding CR and PRD values. This results in the following diagram



Comment : The PRD value does not exceed 5% !!!!

Conclusion :

The k-means algorithm is optimal in terms of CR and PRD. However it relies entirely on the statistics of the data, and much more sophisticated than a simple uniform quantizer.

Vector quantization is very efficient when it comes to ECG signals as it quasi-periodic. A good choice of the division pattern can help us take advantage of this property.

CONCLUSION

Conclusion

The aim of this project was to have a closer look at the compression methods applied on ECG, to understand the importance of it and the different challenges that it presents.

The first chapter gave a brief description of the activity of the heart. The second chapter introduces a set of compression methods and their classification while the third chapter concluded the work with a demonstration of transform methods where we got to compare the methods in terms of PRD and CR and noticed the suitability of DCT for this application. This comparison was followed by another demonstration where we got to observe the optimality of LBG algorithms and the efficiency of vector quantization.

This project is interesting as it presents an "appetizing" introduction of data compression techniques, as more work is to be done at the computational level.

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