REPUBLIQUE ALGERIENNE DEMOCRATIQUE ET POPULAIRE

Ministère de l'Enseignement Supérieur et de la Recherche Scientifique

Ecole Nationale Polytechnique

Electronics Department

Laboratoire des Dispositifs de Communication et de
Convertion Photovoltaique

Master thesis on Electronics

Theme

Theoretical study on Selective Harmonics Elemination SHE PWM

Presented by

OUADRIA Anes Abderrahim

Presented publicly on June, 19th 2017

Jury members

SADOUN Rabah	MCA	ENP	President
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ENP 2017

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ACKNOWLEDGEMENTS

I would like to thank My mentor, Amar Guellal for guiding and supporting me over the graduate period. He has set an example of excellence as a researcher, advisor and instructor.

I would like also to thank my professor Cherif LARBES for all of his guidance through this process; his discussion, ideas, and feedback have been absolutely invaluable.

I would like to thank my fellow graduate students who contributed to this research. I am very grateful to all of you.

Finally, we would especially like to thank my family for the love, support, and constant encouragement I got over the years. I undoubtedly could not have done this without them.

الملخص:

على مدى السنوات الماضية، كان هناك اهتمام كبير حول التحكم في سرعة المحرك. و لذلك تم إجراء العديد من الأبحاث لتطوير التقنيات والخوارزميات من أجل مراقبة أفضل للسرعة. تم الإثبات أن العاكس هو أفضل أداة للتحكم في هذه الآلات. فهو يولد إشارة PWM والتي هي عبارة عن نبضات متتالية ذات عرض مختلف يتعلق بالطيف الترددي المراد تحقيقه. إن تقنية تحوير عرض النبض المحسوبة مع القضاء الانتقائي المتناغم على الترددات غير المرغوبة والتحكم في الجهد (SHE PWM) هي بديل جذاب للسيطرة على سرعة المحرك اللامتزامن. وتهدف هذه التقنية لحساب زوايا التبديل، وبالتالي السيطرة على عرض نبض إشارة PWM لتحسين كفاءة المحرك و التقليل من الخسائر.

كلمات مفتاحية: سرعة المحرك, العاكس, PWM, القضاء الانتقائي على الترددات غير المرغوبة, المحرك اللامتزامن.

Résumé:

Au cours des dernières années, de nombreux intérêts ont été accordés au contrôle de la vitesse du moteur à induction, de nombreuses recherches ont donc été faites pour développer une technique et un algorithme pour un meilleur contrôle. Les onduleurs ont été prouvés comme le meilleur contrôleur de ces machines. Ils fournissent un signal PWM qui est une succession d'impulsions, dont la largeur est variable et dépend du spectre fréquentiel désiré. La technique de modulation de largeur d'impulsion calculée avec élimination harmonique sélective et contrôle de tension (SHE PWM) est une alternative attrayante pour la régulation de vitesse d'un moteur à induction. Cette technique vise à calculer les angles de commutation, afin de contrôler la largeur d'impulsion du signal PWM pour améliorer l'efficacité du moteur et réduire les pertes.

Mots clés : vitesse du moteur, onduleur, PWM, élimination harmonique sélective (EHS), moteur asynchrone

Abstract:

Over the last years, many interests were given to the speed control of induction motor, thus so many researches has been made to develop technique and algorithm for a better control. The inverters were proved the best controller of these machines. They provide a PWM signal which is a succession of pulses, whose width is variable, and depends on the desired frequency spectrum. The calculated Pulse Width Modulation technique with Selective Harmonic Elimination and Voltage Control (SHE PWM) is an attractive alternative for speed control of an induction motor. This technique aim to calculate the switching angles, thus control the pulse width of the PWM signal to improve the efficiency of the motor and reduces the losses.

Keywords: Motor speed, inverter, PWM, Selective Harmonic Elimination (SHE), induction motor

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INTRODUCTION

Nowadays, industrial companies use essentially for different application the control of inverters, which are static converters of the DC-AC type, based on pulse-width modulation (PWM) for powering alternating current. Several techniques exist for the PWM control, depending on how the switching times of the switches are defined. Among these are the programmed PWM control, which is the most commonly used for inverters based on asynchronous machines because it has several advantages such as improving harmonic performance by eliminating unwanted harmonics. To eliminate these undesirable an algorithm has been introduced by Turnbull in 1964 and developed later by Patel and Hoftel in 1973. The objective of this algorithm is providing a control signal with a spectrum that have only desired harmonics. In this thesis the objective is to introduce the SHE PWM (Selective Harmonics Elimination Pulse-Width Modulation) and its use in the control systems.

We present in the first chapter the power inverters, describing their structure and topology. Then we presented in details the part of the control of these converters, since it is the part that is related directly to the other chapters in the thesis.

In the second chapter we get through the heart of the control by presenting the PWM which is the maker of the control signal.

The last chapter is devoted for one of the type of the PWM presented on chapter two. In chapter three the algorithm of Patel and Hoft based on harmonics elimination is proposed. The resolution of this algorithm using numerical methods is also described in details.

CHAPTER 1

CHAPTER 1 : POWER INVERTER

1.1 Introduction

Power electronics converters are a family of electrical circuits which convert electrical energy from one level of voltage/current/frequency to another, using semiconductor-based electronic switches. Nowadays, they have been used in several industrial fields for different use such as power supplies, heating systems, audio amplifiers …etc.

The inverter is a power electronics device for generating alternating voltages and currents from a source of electrical energy of different voltage or frequency. This is the inverse function of a rectifier. They are designed to produce a variable output voltage. Thus, they are often used within motor speed controllers.

In this chapter we present generalities about the inverters. We start with a definition then the voltage inverter types, since it is way famous and used compared to the current inverter. Then we state some application areas of the inverters. And finally, we present the inverters used in control system where we introduce the Pulse Width Modulation (PWM).

1.2 Definition

Voltage inverters are static converters used mainly to supply alternating loads at fixed or variable frequency. The aim is to obtain, for each output voltage, a waveform best approximating the sinusoid.

There are two types of inverters: Voltage inverters and current inverters, depending on the DC input source: voltage source or current source. Voltage inverter technology is the most controlled and is found in most industrial systems and in all power ranges.

1.3 Voltage inverter types

1.3.1 Single Phase Inverters

This type of inverter is intended to supply single phase alternating loads; they are used for low power applications. There are two basic configurations: half-bridge or full bridge (Figure 1-1 and Figure 1-2)[**1**].

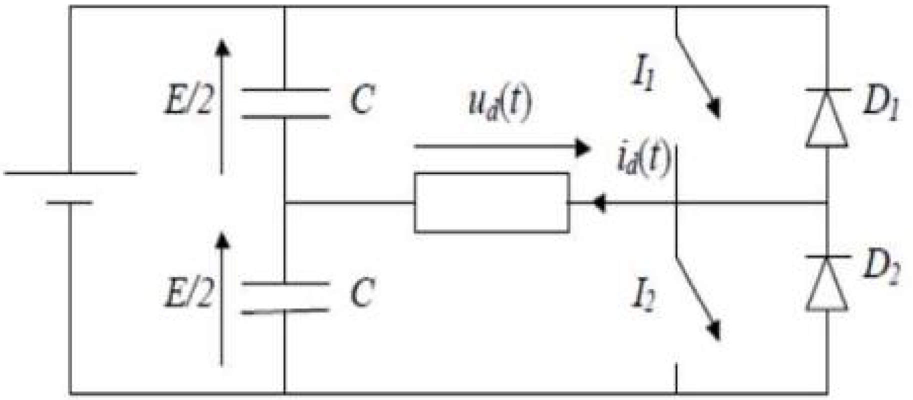


Figure 1-1: single-phase inverter - half-bridge

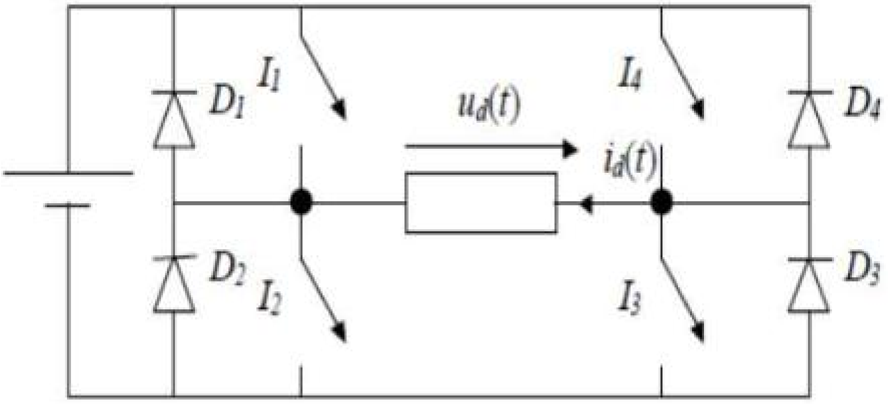


Figure 1-2: single-phase inverter full-bridge

1.3.2 Three Phase Inverters

Single-phase inverters are used for low-power applications, while three-phase inverters cover the medium and high power range. The objective of this topology is to provide a three-phase voltage source whose amplitude, phase and frequency are controllable.

Figure 1-3 shows a three-phase voltage inverter made up of three basic cells. A three-phase load is connected to the middle points a, b and c. Input voltage is usually supplied by a rectifier [**2**].

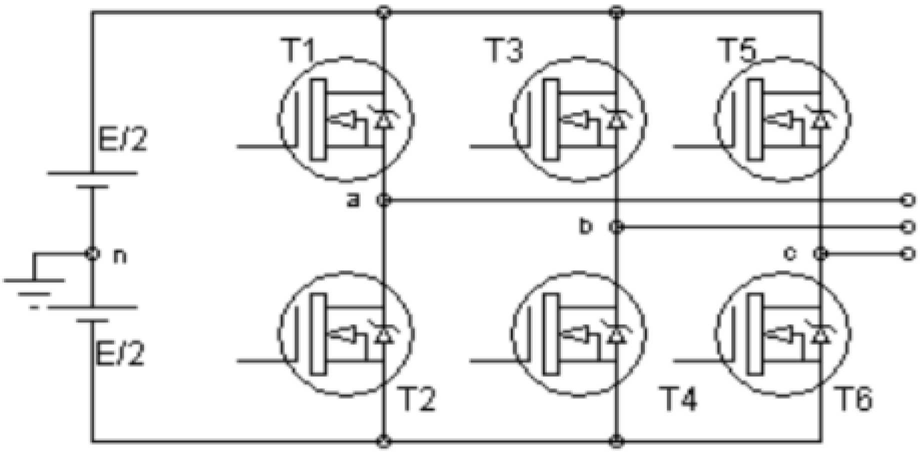


Figure 1-3: three-phase voltage inverter

If the switches of the three cells are controlled with a phase shift of one third of the fundamental period, the three simple voltages V_{an}, V_{bn} and V_{cn} (With respect to the fictive midpoint n at the input) are also phase shifted by one thirds of the fundamental period related to each others.

1.3.3 Multi-level inverters

By definition, the multi-level voltage inverter has three or more levels. In the literature we find several multilevel topologies, the most widely used are the looping diode topology, the floating capacitor topology and the cascade topology.

1.4 Application areas

The two main fields of application of conventional voltage inverters are back-up power supplies and control of alternative motors. They are also a part of two main families, respectively fixed frequency systems and variable frequency systems.

1.5 The inverter control:

The role of the control function is to determine the switching times and the logic control commands of the switches in order to obtain a switching sequence of the latter. The choice of a modulation strategy can be made according to the performance desired by the user. All strategies have advantages and disadvantages and can be realized by software or hardware programming.

Several strategies for controlling inverters have been developed in the literature, their principles consist either in

- Generating the control signals of the power switches by servo-controlling the output voltage of the inverter at a sinusoidal voltage reference. This is the so-called implicit command.
- The determination of the switching times of the power components forming the inverter through the Fourier series development of desired output waveforms meeting

well-defined criteria (harmonic ratio, fundamental term value …etc). This is the explicit command.

These control strategies can be distinguished as:

A. Full-wave control (known as 180 °).
B. Shifted control.
C. Pulse width modulation (PWM) control.

The voltage generated by the strategies (a, b) has a rectangular shape, its Fourier series decomposition has shown that this waveform is rich in harmonics. It is useless to attenuate these harmonics by a filter, indeed the frequency is variable, in addition, the first harmonic to be eliminated (the harmonic 5) at a frequency very close to that of the fundamental [**3**].

In a pulse width modulated inverter, instead of forming each alternation of an output voltage with a single rectangular waveform, it is formed from several well-defined waveforms to eliminate harmonics at frequencies close to fundamental. Pulse width modulation (PWM) is the most appropriate technique for controlling the inverter while having good neutralization of the output wave.

1.6 Conclusion

Motor speed control needs are numerous and include things like: industrial motor driven equipment, electric vehicles, rail transport systems, and power tools. The inverters are the heart of motor speed control, what make them very invaluable. In this chapter we have presented the power inverters and their control function. The control strategies have been stated. The PWM has proved being the best control technique. In the next chapter, the PWM is explained in details.

CHAPTER 2

CHAPTER 2 : PWM

2.1 Introduction

At the end of the last century, one of the results of the development of power electronics is the Pulse Width Modulation technique. It is the heart of the control of static converters. The objective of the PWM technique in controlling a voltage inverter is to have a fast response and high performance.

In this chapter the PWM is presented the PWM, starting with a definition explain its principle. Then we present some of the areas where this technique has been used. At the end we pass to the type of the PWM, stating its most used two types, the generated and the calculated PWM. This last is based on a technique of elimination of unwanted harmonics called SHE PWM (Selective Harmonics Elimination Pulse-Width Modulation).

2.2 Definition

Pulse Width Modulation (PWM) is a technique commonly used to synthesize continuous signals using circuits with "all-or-nothing" operation, or more generally discrete state circuits. The general principle is that by applying a succession of discrete states for well-chosen durations, we can obtain any intermediate value on average over certain duration.

If we consider for example a pulse waveform $f(t)$ with a period T, and a duty cycle D and f_{max}, f_{min}as the maximum and the minimum of this function.

The average value of the pulse waveform is given by:

$$\bar{f} = \frac{1}{T}\int_{0}^{T} f(t)dt \tag{2-1}$$

Knowing that :

$$\begin{cases} f(t) = f_{max} & when \quad 0 < t < D.T \\ f(t) = f_{min} & when \quad D.T < t < T \end{cases} \tag{2-2}$$

From the (2-1) and (2-2) we obtain:

$$\bar{f} = \frac{1}{T}\left(\int_{0}^{DT} f_{max}\, dt + \int_{DT}^{T} f_{min}\, dt\right) \tag{2-3}$$

$$\bar{f} = \frac{1}{T}(D.T.f_{max} + T(1-D)f_{min}) \tag{2-4}$$

$$\bar{f} = D.f_{max} + (1-D)f_{min} \tag{2-5}$$

As we can see from the (2-5**)**, the average value depend on the duty cycle, thus it depend of switching times.

2.3 Application Areas

The most frequent uses are:

- Digital-to-analog conversion
- Class D amplifiers, in audio
- Switching power supplies, variable speed drives, and more generally all power electronics devices using MOSFET, IGBT, GTO components.

It is also possible to transmit data by this method. It is used mainly in the power electronics converters control such as inverters control, as it is in our study.

2.4 The types of PWM

The choice of the technique depends on the type of the machine to be controlled, the type of the semiconductors, the power involved and the simplicity or the complexity of the algorithms to be implanted.

Several techniques for obtaining PWM waves are used to generate the control signal necessary for the control and blocking of the semiconductor elements[**2**].

2.4.1 The generated PWM

The generated PWM (triangulo-sinusoidal) control consists in comparing a reference voltage value of frequency f_r, the image of the desired signal with the output called modulator, with a triangular carrier or with a sawtooth of frequency f_c. The points of intersection between the modulator and the carrier generate an on/off switching, thus constituting a pulse of variable duration and the set of these pulses thus reconstitutes the fundamental of the reference sinusoid.

The study of this type of PWM is based on two parameters that characterize the control:

The modulation index im which is the result of the ratio of the frequencies of the carrier f_c to the reference f_r. If im is integer the modulation is synchronous. It is asynchronous otherwise.

The rate of the modulation rm which is the result of the ratio of the amplitudes of the voltages of the reference V_{rm} to that of the carrier V_{cm}.

$$im = \frac{fc}{fr}$$

$$rm = \frac{V_{rm}}{V_{cm}}$$

To eliminate even-order harmonics and third-order harmonics, modulation index im must be odd and multiple of 3.

The most suitable carrier for two-level converters is the bipolar triangular. The Figure 2-1 shows the principle of generating the control pulses of a two-stage voltage inverter[**2**].

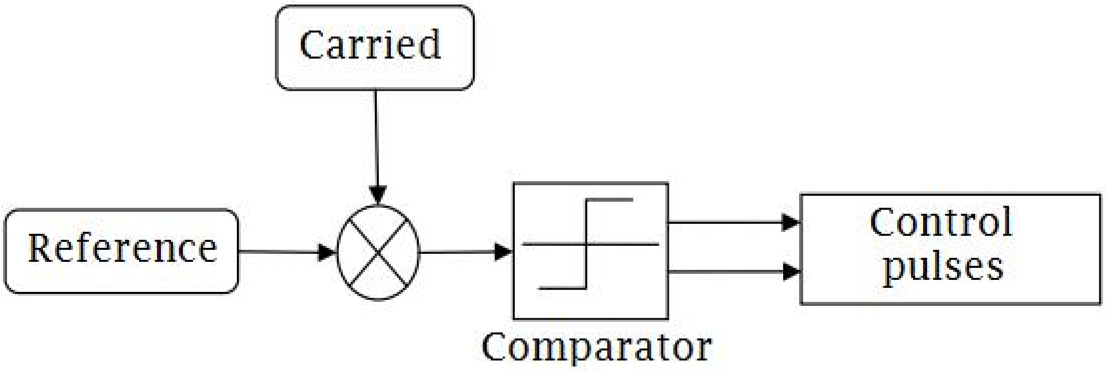


Figure 2-1: Principle of the PWM Sinus- Triangle for single-phase inverter

For a single-phase inverter of the Figure 1-2 , the application of this control strategy gives the results cited in the Figure 2-2 [**2**].

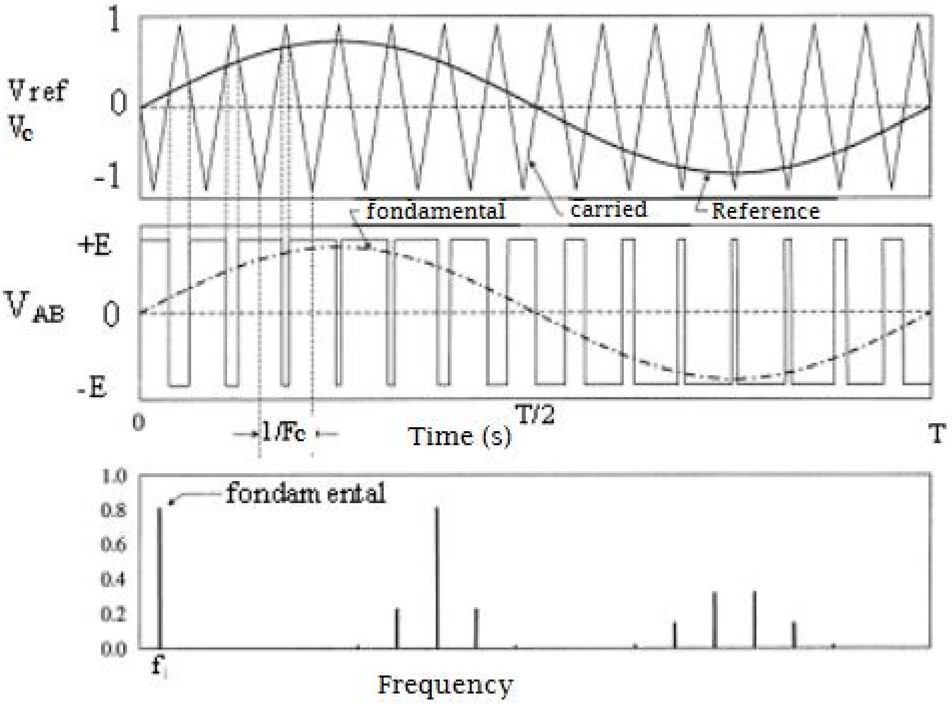


Figure 2-2: Form of control signals, output voltage and its spectrum

2.4.2 The calculated PWM (programmed)

This technique consists in calculating the switching times of the switches so as to meet certain criteria relating to the frequency spectrum of the wave delivered by the inverter. These operating sequences are then stored and returned cyclically in order to control the switches. The criteria usually adopted are the elimination of specified harmonics or the elimination of harmonics in a specified frequency band.

The programmed PWM technique is based on the Patel and Hoft algorithm. In this technique, it is possible to control the fundamental of the voltage PWM and to cancel the amplitudes of the (m-1) first harmonics[**3**].

The programmed PWM voltage is defined as a function of the exact switching angles $\alpha_1,\dots \alpha_m$ which correspond to the switching times of the programmed PWM voltage from a positive value +E/2 to a negative value -E/2 or inversely (in single-phase inverters). A digital circuit then generates the programmed PWM voltage as a function of time.

It can be said that the programmed PWM technique has many advantages:

- Controlling the voltage V of the fundamental.
- Variation of the frequency f of the fundamental by using the relation of conversion of an angular value in time value: $\alpha = 2\pi ft$.
- Elimination of the (m-1) first harmonics.

These advantages make it possible to replace the ideal sinusoidal power supply with a convenient power supply having a harmonic ratio which can be reduced at will.

2.5 Conclusion

In this chapter we have showed the way how the control signal has been generated for adjustable frequency loads, and the concept of regulating the switched state of power electronic devices, especially power inverters. Unfortunately, voltage control can only be accompanied by undesirable harmonics as a result of the inherent switched nature of modem power electronic equipment. Thus a SHE PWM (Selective Harmonics Elimination Pulse-Width Modulation) has been introduced. The next chapter is devoted for this technique.

CHAPTER 3

CHAPTER 3 : SHE PWM

3.1 Introduction

In control systems, the unwanted harmonics may produce vibrations and undulations of torque and many undesirable consequences. A technique that consists in forming the output wave of a succession of slots of variable and controllable widths has been introduced. The switching angles are determined so as to eliminate certain disturbing harmonics in the output wave and improve the efficiency of the inverter-machine system by reducing torque ripples, as well as current peaks and losses in the machine. The calculation of these angles with this method is based on the nonlinear and transcendental equations. This has forced researchers to use numerical methods such as Newton-Raphson.

In this chapter we mainly introduce that technique. Then we present the algorithm of Patel and Hoft, made for the calculation of these angles.

3.2 Patel and Hoft PWM algorithm [4]

The PWM signals describing the three output voltages of the converter must have properties which help to guide their characteristics towards those of a sine wave. In order to approach them as much as possible, we may in some cases attribute to them the same properties of symmetry as a sinusoidal wave. The aim of this technique is to eliminate a certain number of low-order harmonics and to control the fundamental wave. The output voltage of the inverter is defined as a function of the exact switching angles $\alpha_1,\ldots, \alpha_m$ corresponding to the switching times of the voltage from a positive to negative value or vice versa. The index m is the number of switching angles of the output voltage of the inverter per quarter wave. The output voltage of the inverter is constructed to have half-wave symmetry (odd function with respect to the angle π). This symmetry makes it possible to eliminate certain types of harmonics, which simplifies the Fourier series development of this voltage and reduces the harmonic ratio. Then, the amplitude of the fundamental is fixed to the value im and the amplitudes of the (m-1) first harmonics are canceled.

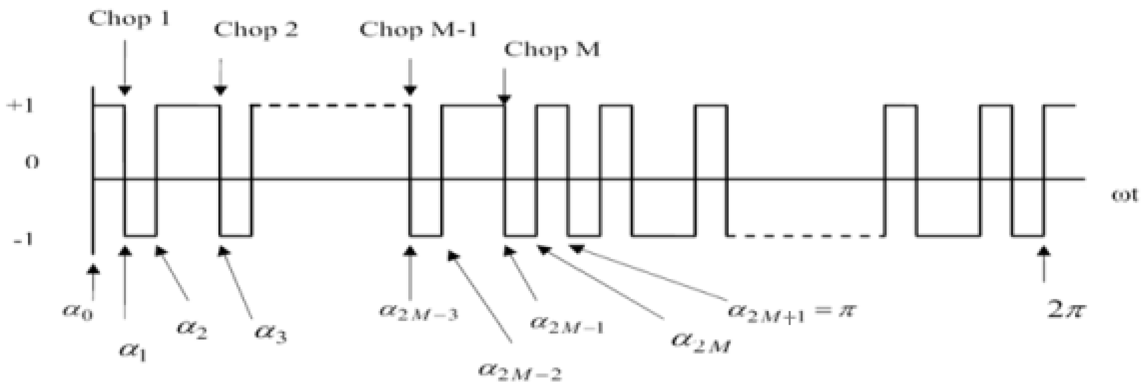


Figure 3-1: The normalized voltage output of half-bridge inverter

It is assumed that the output voltage is periodic and of unit amplitude. Let f be the function representing the PWM signal as a function of α (α = ωt). We can write therefore:

$$f(\alpha) = -f(\alpha + \pi) \tag{3-1}$$

The function f can be decomposed into Fourier series:

$$f(\alpha) = a_0 + \sum_{n=0}^{\infty} (a_n \sin(n\alpha) + b_n \cos(n\alpha)) \tag{3-2}$$

Where:

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} f(\alpha) d\alpha \tag{3-3}$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(\alpha)\sin(n\alpha) d\alpha \tag{3-4}$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(\alpha)\cos(n\alpha) d\alpha \tag{3-5}$$

Since $f(\alpha)$ has a half-wave symmetry $f(\alpha) = -f(\alpha + \pi)$, the mean value a_0 equal to zero and only the odd harmonics exist. Consequently, the index n takes the odd values 1, 3, 5, 7, 9, ...

Thus, the factors a_n and b_n are given by:

$$\begin{gathered} a_0 = 0 \\ a_n = \frac{2}{\pi}\int_0^{\pi} f(\alpha)\sin(n\alpha) d\alpha \\ b_n = \frac{2}{\pi}\int_0^{\pi} f(\alpha)\cos(n\alpha) d\alpha \end{gathered} \tag{3-6}$$

Replacing $f(\alpha)$ in the equation (3-6)

$$a_n = \frac{2}{\pi}\left[\int_{a_0}^{a_1} (-1)^0 \sin(n\alpha) d\alpha\right] + \ldots\ldots + \frac{2}{\pi}\left[\int_{a_{2m}}^{a_{(2m+1)}} (-1)^{2M} \sin(n\alpha) d\alpha\right] \tag{3-7}$$

Where

$$a_n = \frac{2}{\pi}\left[\sum_{k=0}^{2m}\int_{a_k}^{a_{(k+1)}}(-1)^k \sin(n\alpha)d\alpha\right]$$

$$a_n = \frac{2}{\pi}\left[\sum_{k=0}^{2m}(-1)^k(\cos(n\alpha_k) - \cos(n\alpha_{k+1}))\right]$$

With $\alpha_{2m+1} = \pi$ and $\alpha_0 = 0$ and $\alpha_1 < \alpha_2 < \alpha_3 < \cdots < \alpha_{2M+1}$

Thus, $\cos(n\alpha_0) = 0$ and $cos(n\alpha_{2m+1}) = (-1)^n$. Moreover, since n is odd, then:

$$a_n = \frac{4}{\pi n}\left[1 + \sum_{k=1}^{2m}(-1)^k \cos(n\alpha_k)\right] \tag{3-8}$$

With the same calculation we find

$$b_n = \frac{4}{\pi n}\left[-\sum_{k=1}^{2m}(-1)^k \sin(n\alpha_k)\right] \tag{3-9}$$

Since $f(\alpha)$ has half-wave symmetry, and according to the Figure 3-1 we have:

$\alpha_k = \pi - \alpha_{2m-k+1}$

Thus

$\sin(n\alpha_k) = \sin(\mathrm{n}(\pi - \alpha_{2m-k+1})) = \sin(n\pi)\cos(n\alpha_{2m-k+1}) - \cos(n\pi)\sin(n\alpha_{2m-k+1})$

For n odd: $\sin(n\pi) = 0$ and $\cos(n\pi)$=-1

Thus,
$$\sin(n\alpha_k) = \sin(n\alpha_{2m-k+1}) \tag{3-10}$$

Replacing (3-10) in (3-9) we find:

$$b_n = \frac{4}{\pi n}\sum_{k=1}^{m}(\sin(n\alpha_k) - \sin(n\alpha_{2m-k+1})) = 0$$

In the other hand we have

$$\cos(n\alpha_k) = \cos(\mathrm{n}(\pi - \alpha_{2m-k+1})) = \cos(n\pi)\cos(n\alpha_{2m-k+1}) - \sin(n\pi)\sin(n\alpha_{2m-k+1})$$

Thus,

$$\cos(n\alpha_k) = -\cos(n\alpha_{2m-k+1}) \tag{3-11}$$

Replacing (3-11) in (3-8)

We find

$$a_n = \frac{4}{\pi n}\left[1 + 2\sum_{k=1}^{m}(-1)^k \cos(n\alpha_k)\right] \tag{3-12}$$

Each equation (3-12) has m unknown variables $\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_m$. The problem is to calculate the values of these switching angles which make it possible to cancel the amplitudes a_n of the first (m-1) harmonics f_n and to assign an im value to the amplitude a_1 of the fundamental f_1.

On the other hand, two voltage harmonics must be eliminated in order to eliminate a current harmonic. The first value of m is set to 3 so that the amplitude of the fundamental is fixed to a given value (m being the number of quarter-wave switching per half wave). Consequently, when m is increased successively by 2, the number of current harmonics that will be eliminated is increased by 1.

It should be noted that the value of the modulation index im assigned to the fundamental is a dimensionless index varying from 0 to 1. To obtain the corresponding value in volt, multiply by E/2 for the mono-phase bridge and by E for the three-phase inverter.

The equations (3-13) form a system of m nonlinear equations with m unknown.

$$\begin{cases} f_1(\alpha_1,\alpha_2,\alpha_3,\ldots\alpha_4) = \frac{4}{\pi}\left[1 + 2\sum_{k=1}^{m}(-1)^k\cos(\alpha_k)\right] + im = 0 \\ f_2(\alpha_1,\alpha_2,\alpha_3,\ldots\alpha_4) = \frac{4}{5\pi}\left[1 + 2\sum_{k=1}^{m}(-1)^k\cos(5\alpha_k)\right] = 0 \\ f_3(\alpha_1,\alpha_2,\alpha_3,\ldots\alpha_4) = \frac{4}{7\pi}\left[1 + 2\sum_{k=1}^{m}(-1)^k\cos(7\alpha_k)\right] = 0 \\ \quad\vdots \\ f_m(\alpha_1,\alpha_2,\alpha_3,\ldots\alpha_4) = \frac{4}{n\pi}\left[1 + 2\sum_{k=1}^{m}(-1)^k\cos(n\alpha_k)\right] = 0 \end{cases} \tag{3-13}$$

3.3 The exact switching angles values

The system equations (3-13) are nonlinear and transcendent; it has been shown that they can be solved using numerical methods [**4**].

For this method to converge, we assign a negative value (-im) to the fundamental. This corresponds to a phase shift of π of the fundamental. This phase shift has no effect on the controlled machine.

A simple technique to solve these equations is to use Newton-Raphson method. We must have a good initial estimate of the exact solution sought, so that this method accurate solution and good convergence. Alternatively, more complicated gradient search methods can be used to obtain the solutions. Indeed, the Taufik, Mellitt, and Goodman algorithm is used to quickly estimate the initial values of the nonlinear system solution.

3.3.1 TMG algorithm [5]

Some results has shown that for a given odd value for m, the trajectories of the odd switching angles form a parallel lines over the most range of im. Also, the gradients of the trajectories reduce with increasing values of m. the variation of the normalized gradient is given by

$$\Delta_k = \frac{\frac{6\ 0(k+1)}{(m+1)} - \alpha_k}{2\times 6\ 0/(m+1)} \qquad \text{for } k \text{ odd}$$

$$\Delta_k = \frac{\alpha_k - \frac{6\ 0(k+1)}{(m+1)}}{2\times 6\ 0/(m+1)} \qquad \text{for } k \text{ even} \tag{3-14}$$

The switching angles α_k are given by:

For k odd

$$\alpha_k = \frac{60(k+1)}{(m+1)} - \left[\frac{2\times 60}{(m+1)} \times \frac{\Delta_k}{0.8} \times NP1\right] \tag{3-15}$$

For k even

$$\alpha_k = \frac{60k}{(m+1)} - \left[\frac{2\times 60}{(m+1)} \times \frac{\Delta_k}{0.8} \times NP1\right] \tag{3-16}$$

With NP1 is the modulation rate identical to im, it takes values between 0 and 0.8.

3.3.2 The algorithm of switching angles calculation

Let the solution vector of the system of equation (3-13) be:

$$\alpha^* = (\alpha_1^*, \alpha_2^*, \alpha_3^*, \dots, \alpha_m^*)$$

$f_i(\alpha)$ are the equation of the system (3-13) with $i = 1,2,\dots,m$

We have $f_i(\alpha) = 0$

By developing the functions f_i in Taylor series in the vicinity of an estimate $\alpha^{(k)}$ (k^{th} iteration) close to α^*, we obtain:

$$f_i(\alpha) = f_i\left(\alpha^{(k)} + (\alpha^* - \alpha^{(k)})\right)$$

$$f_i(\alpha) = f_i\left(\alpha^{(k)}\right) + \sum_{j=1}^{m} \left[\frac{\partial f_i(\alpha)}{\partial \alpha_j}\right]_{\alpha=\alpha^{(k)}} \left(\alpha_j^* - \alpha_j^{(k)}\right) + \ldots \tag{3-17}$$

$$\ldots + \frac{1}{2}\sum_{j=1}^{m}\sum_{r=1}^{m}(\alpha_j^* - \alpha_j^k)(\alpha_r^* - \alpha_r^k)\left[\frac{\partial^2 f_i(\alpha)}{\partial \alpha_j \partial \alpha_r}\right]_{\alpha\alpha^{(k)}} + \ldots + \cdots = 0$$

Since $\alpha^{(k)}$ is close to α^* we neglect $(\alpha_j^* - \alpha_j^k)^2$ and higher order terms.

The equation (3-17) becomes:

$$f_i\left(\alpha^{(k)}\right) = -\sum_{j=1}^{m}\left[\frac{\partial f_i(\alpha)}{\partial \alpha_j}\right]_{\alpha=\alpha^{(k)}}\left(\alpha_j^* - \alpha_j^{(k)}\right) \tag{3-18}$$

By introducing the matrix of the first derivatives:

$$E^{(k)} = (E_{ij}^{(k)})$$

With $E_{ij}^{(k)} = \left[\frac{\partial f_i(\alpha)}{\partial \alpha_j}\right]_{\alpha=\alpha^{(k)}}$

Thus,

$$E^{(k)} = \frac{8}{\pi}\begin{bmatrix} \sin(\alpha_1) & -\sin(\alpha_2) & \cdots & \sin(\alpha_m) \\ \sin(5\alpha_1) & \sin(5\alpha_2) & \cdots & \sin(5\alpha_m) \\ \vdots & \vdots & \ddots & \vdots \\ \sin(n\alpha_1) & \sin(n\alpha_2) & \ldots & \sin(n\alpha_m) \end{bmatrix}$$

where n and m odd and different from a multiple of 3.

The error vector is defined by

$$\Delta\alpha^{(k)} = (\Delta\alpha_1^{(k)}, \Delta\alpha_2^{(k)}, \Delta\alpha_3^{(k)}, \ldots, \Delta\alpha_m^{(k)})$$

Where $\Delta\alpha_j^{(k)} = \alpha_j^* - \alpha_j^{(k)}$

We define:

$$F^{(k)} = (F_1^{(k)}, F_2^{(k)}, F_3^{(k)}, \ldots, F_m^{(k)})$$

With $F_i^{(k)} = -f_i\left(\alpha^{(k)}\right)$

By replacing in (3-18) we obtain:

$$E^{(k)}\Delta\alpha^{(k)} = F^{(k)}$$

The vector $\Delta\alpha^{(k)}$ is the unknown. The system (3-13) is linear so we can determine $\Delta\alpha^{(k)}$.

Once $\Delta\alpha^{(k)}$ is determined, a better estimate $\alpha^{(k+1)}$ of α^* is obtained by the relation:

$$\alpha^{(k+1)} = \alpha^{(k)} + \Delta\alpha^{(k)}$$

We continue until we get:

$\left|\alpha^{*} - \alpha^{(k)}\right| \rightarrow 0$

In practice, α * being the unknown, the operations are stopped by one of the following tests:

- $k \geq k_{max}$
- $\left|f_i\left(\alpha^{(k)}\right)\right| \leq E_0$

Where E_0 is an upper bound of the error fixed a priori and k_{max} the maximum number of permissible iterations.

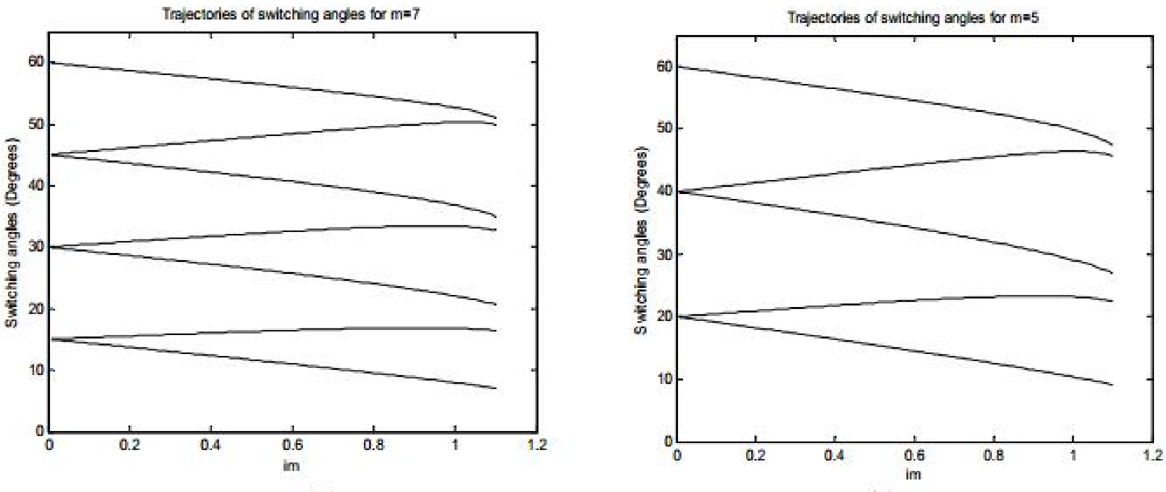
3.4 Results

Indeed, a program has been developed in Matlab, to calculate the exact switching angles whatever the value of the index im [**3**].

The results for, im vary from 0.8 until 0.85 and for m=5 are given in the Table 3-1 and the Figure 3-2 [**3**] represents the trajectories of the switching angles for m= 5 and m=7.

Table 3-1: Exact switching angles for m=5

im	0.80	0.81	0.82	0.83	0.84
α_1	12,5371338	12,4341423	12,3307175	12,226843	12,122501
α_2	23,1789197	23,1989684	23,2176852	23,2349915	23,2508024
α_3	31,9273421	31,8035533	31,6784384	31,551926	31,4239391
α_4	45,5983321	45,6575784	45,7158885	45,773182	45,8293692
α_5	52,5370215	52,4271602	52,3161698	52,2039674	52,090461



(a) (b)

Figure 3-2: trajectories of the exact switching angles: a- m=7 and b- m=5

3.5 Conclusion

To conclude, in this chapter we have seen a technique that are largely used in many control systems in different industrial fields. This technique has showed satisfied results. Although this technique is invaluable, it still has some difficulties of real-time implementation or realization. In this chapter we have focused on Patel and Hoft algorithm, explaining their algorithm to calculate the exact switching angles. Moreover, TMG algorithm has been stated, for its capability to accelerate the calculation by finding an approximate solution that would be helpful for Newton-Raphson method.

GENERAL CONCLUSION

In this thesis a research about the Pulse Width Modulation (PWM) and its contribution in the controls system, has been made. Despite the apparent nature of the PWM, its study is complex and affects several fields of activity: electronics, signal processing, acoustics and automatic. This function, which practically links the drive to the motor, cannot be seen singularly. The study of the PWM results in the study of several subsystems in which the PWM plays the essential role.

Given its advantages, better elimination of unwanted harmonics, lower switching losses and the minimum pulse width limit can be reached easily, SHE PWM control seems to be suitable candidate to control the speed of An Asynchronous machine. However, the calculation of the exact values of the switching angles, when using this technique, requires the resolution of a system of m nonlinear equations with m unknown in real time. For the system resolution we used the iterative method of Newton-Raphson.

Moreover, to ensure convergence, Taufik, Mellitt and Goodman's algorithm were used to estimate the initial values of the solution.

The calculation of the switching angles by this method is precise but requires a very high computing time which prevents an online real-time speed control. Consequently, the SHE PWM technique is an "off-line" technique. In other words, the exact values of the switching angles are first calculated on an off-line computer, and then these values are stored in memory so that they can be used for controlling the speed of the motor. Such a method requires a large memory to store all calculated angles and achieve good accuracy. This disadvantage prompted the researchers to design other techniques allowing the calculation of the switching angles of the SHE PWM on-line and in real time.

In view of this research we plan to develop less complex algorithm or innovate new methods for an easier implementation and realization, so that to be able to use the PWM with more relief, regarding its importance in control system.

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