MODIFIED RESPONSE SPECTRUM MODEL FOR THE DESIGN OF STRUCTURES SUBJECTED TO SPATIALLY VARYING SEISMIC EXCITATIONS

by

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Submitted to the Department of Civil Engineering
on May 18th, 1989, in partial fulfillment of the
requirements for the Degree of Doctor of Philosophy

Abstract

An important aspect of earthquake loads exerted on extended structures, or structures founded on several foundations, is the spatial variability of the seismic motion. Hence, a rigorous earthquake resistant design of lifeline structures should account for the spatial character of the seismic input, at least in an approximate way.

A procedure for the modification of the design response spectrum is proposed. It enables addressing the problem of multiply-supported structures subjected to imperfectly correlated seismic excitations by means of an extension to the response spectrum method. A modified response spectrum model is developed for the design of extended facilities subjected to single and multicomponent ground motion, and a modal combination rule is proposed for each case. The modification procedure is based on adjusting each spectral value of the given design response spectrum by means of a correction factor which depends on the structural properties and on the characteristics of the wave propagation phenomenon. Finally, the theoretical model is validated through digital simulation of seismic ground motion, whereby model predictions are found to be in satisfactory agreement with exact results.

Thesis Supervisor: Dr. Eduardo Kausel
Title: Associate Professor of Civil Engineering

To my loving parents

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Chapter 1

Introduction

1.1 Objectives and Scope

The safety incorporated in the design of large or extended structures, such as nuclear power plants, industrial buildings, bridges, or lifelines can significantly be enhanced by improvements in the understanding and the representation of the earthquake loads exerted on these facilities. An important aspect of the earthquake loads for extended structures, or structures founded on several foundations, is the spatial variability of the seismic motion, whose study is now possible as a result of the deployment of strong motion instrument arrays, started at the end of last decade. Therefore, a rigorous seismic analysis of lifeline structures should account for the spatial variability of the ground motion, at least in an approximate way. This situation has motivated the present work, whose main objective is to provide a practical means for the seismic analysis and design of spatially extended facilities accounting for partially correlated seismic excitations. In order to succeed in including such an aspect into seismic design practice, it is essential to provide the engineering community with an attractive means that takes the form of an extension to a commonly used tool. In other words, while a full Random Fields analysis to address this problem would be fastidious and not practical, an attractive alternative for engineering design situations would consist in modifying the design response spectrum, so as to account for the spatial extent of

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the structure to be analyzed (or designed) and for the spatial character of the input motion. Therefore, a procedure for the modification of the design response spectrum is proposed in this thesis, which enables addressing the problem of multiply-supported structures subjected to imperfectly correlated seismic excitations. The technique is an extension to the mode superposition method combined with the response spectrum method, thereby allowing the inclusion of this potentially important problem into seismic design practice.

1.2 Organization

In Chapter 2, a review of past work on the subject of strong motion arrays and spatial variability of ground motion is made, and the relevance of the present thesis work is put into evidence.

In Chapter 3, the general derivation of the modified response spectrum model is presented. It addresses the cases of discrete systems subjected to both single and multicomponent ground motion, and the case of bridges subjected to a single ground motion component.

In Chapter 4, modal combination rules are developed for the cases of single and multicomponent ground motion, accounting for the spatial variability of the seismic excitation.

In Chapter 5, the theoretical response spectrum model is validated through digital simulation of seismic ground motion.

Finally, Chapter 6 summarizes the results and provides suggestions for further

research.

Chapter 2

Background

2.1 Introduction

Earthquakes are complex and heavily damaging phenomena that must be accounted for in estimating the safety of structures. Earthquake ground motions exhibit a high variability in both time and space, and may be probabilistically modeled based upon the theory of Random Fields. While the temporal variability has been the object of extensive research and sophisticated modeling because of its relatively easy quantification from seismograms, the spatial variability, whose investigation requires the deployment of dense networks of strong motion accelerographs, is not yet well understood.

During an earthquake, a structure is subjected to forces not only caused by the inertial loading, but also by the spatially varying nature of the ground motion. This effect, usually referred to as wave passage, is the result of a number of factors, such as site geology and stratigraphy, wave content, dispersion, scattering, etc. The problem of wave passage is of essential importance in the seismic analysis and design of of extended facilities such as dams, nuclear power plants, and bridges, in which differential ground motion can induce substantial additional stresses. Therefore, for a seismic analysis of lifeline structures to be rigorous, it behooves to take into account the spatial variability of the ground motion, at least approximately.

2.2 Review of Previous Work

Past work on the subject of strong motion arrays and spatial variability of ground motion has mainly focused on four research areas, which will succintly be presented in subsequent paragraphs.

1. Design, site selection, and deployment of strong motion instrument arrays (e.g. Iwan, 1979), which is an area that deals, essentially, with topics such as:

- Favorable array locations.

- Design of arrays for source mechanisms and wave propagation studies.

- Design of arrays for local effects studies.

- Array construction and operation, and implementation.

Thus, the goal aimed at in this research area, is an improvement in the understanding of ground motion nature by providing measurements resulting from actual earthquakes.

2. Interpretation of actual earthquake records for specific array-sites and seismic events (e.g. Loh et al., 1982, Bolt et al., 1982, Harichandran and Vanmarcke, 1984, Abrahamson, 1985, Loh and Yeh, 1988). This research area has flourished after the installation of the SMART-1 (Strong Motion Array in Taiwan), located in the north-east corner of Taiwan (Lotung City), and which was begun in September 1980. Being a dense network of strong motion accelerographs, the SMART-1 provides a good op-

portunity for the study of spatial variations of seismic waves. Based on the SMART-1 data, the following topics have been tackled:

- Generation of Fourier amplitude spectra for different station pairs.

- Moving window analysis in the time and frequency domains for the study of spatial variations of ground motion.

- Identification of wave types and their directions of propagation.

- Description of seismic wave coherency and power spectrum as functions of, wave number, frequency, azimuth of propagation and wave type.

- Space-time correlations.

3. Formulation of analytical tools (both deterministic and statistical) for the processing and interpretation of recordings obtained with large instrument arrays (e.g. Burg, 1964, Aki and Richard, 1980, Vanmarcke, 1983, Harichandran and Vanmarcke, 1984, Loh and Yeh, 1988). This research area is a natural complement to the two aforementioned ones, and it focuses on:

- Development of schemes for seismic array data processing.

- Development of analytical models for the space-time variation of earthquake ground motions.

4. Stochastic characterization of spatial variabilities of ground motion, and analysis of structures and facilities to spatially varying seismic motions. Among the contributors to this research area, we can cite the following:

- Kausel and Pais, 1984, investigated the spatial and temporal variabilities of seismic ground motions that may be expected in soil deposits, by means of simple physical models for stochastic SH waves travelling in a homogeneous medium. The statistical properties of the motions at two different points on the surface and/or within the soil mass were computed and analyzed.

- Harada, 1984, presented a probabilistic description of spatial variation of strong earthquake-generated ground displacement. Maximum values for, the ground displacement, the relative displacement between two points on ground surface, and the ground strain, were expressed in terms of the spatial correlation function, which may be estimated from data analysis.

- Harada and Shinozuka, 1986, in a continuation of the work just described, introduced the notion of ground deformation spectra, which express the relationship between the maximum value of the relative displacement between two points on ground surface and the separating distance. The spatial correlation function was estimated from data analysis, and the theoretical development was based on univariate and spatially unidimensional time-space stochastic processes.

- Hindy and Novak, 1980, investigated theoretically the response of buried pipelines to

partially correlated seismic excitations (an exponential decay of cross-spectrum was assumed with respect to frequency and separating distance) in both lateral and longitudinal directions. It was found that partial correlation of seismic excitations could produce excessive stress in the pipe, which would depend on the degree of correlation of the excitation and its frequency content. Further, axial stresses were found to be higher than bending stress, and to be decreasing with the increase of the pipe radius or the wall thickness. The analytical development was based on random vibration theory.

- Lee and Penzien, 1983, presented a stochastic method for seismic analysis of structures and piping systems subjected to multiple support excitations. In either the time domain or the frequency domain, peak response statistics could be found, including the effects of modal cross-correlation and cross-correlation of multiple support excitations.

- Zerva et al., 1988, developed a stochastic model for the ground excitations. Based on this model, the responses of pipelines and single span simply supported bridges to perfectly and partially correlated seismic excitations were investigated. It was found that for continous pipelines, the spatial character of the seismic input was of importance, and that the differential ground motion was likely to cause damage at the joints. However, it was concluded that the effect of differential ground motion was not significant for single-span simply supported bridges, even though it was found that the bridge response at certain sections would be slightly

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higher for the case of partially correlated excitations than for the case of perfectly correlated excitations.

- Harichandran and Wang, 1988, investigated the response of simple beams to spatially varying earthquake excitations, using random vibration theory. It was concluded that the assumption of identical support excitations leads to conservative estimates of the maximum beam response, while the assumption of time-delayed support excitations can lead to unconservative estimates. Incidentally, it is to be noted that these conclusions concern maximum beam response only, and do not apply to arbitrary sections along the beam. Therefore, for design purposes, the situation still needs further investigation.

Having briefly exposed the work done on the subject of spatial variability of ground motion, it behooves to mention that further investigation is still needed with regard to both the seismological aspect and the engineering aspect of the problem. Indeed, from a seismological viewpoint, further exploration of effects of source mechanisms, geology, stratigraphy, local site, etc., ought to be done so as to improve the available ground motion models. Regarding the engineering aspect, on the other hand, it is essential to provide practical means, as an alternative to random vibration theory, for the seismic analysis of elongated structures, incorporating the spatial variability of the ground motion, at least approximately. Also, as an additional note to the engineering aspect, it is important to shed some light on the as yet unresolved issue of the response of structures to incoherent seismic motions, as opposed to the importance

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of wave travelling effects that was recognized about two decades ago in independent pioneering studies by Newmark [33] and Scanlan [40]. As was described earlier, the present thesis work aims at contributing to the engineering aspect of the problem of spatial variability of ground motion.

2.3 Relevance of Proposed Work

From an earthquake engineering viewpoint, the most prominent parameter characterizing ground motions is the maximum ground acceleration, and seismic design criteria are most often expressed in terms of maximum acceleration . A design value for the maximum acceleration may be obtained from an attenuation relationship as a function of a specified magnitude and distance, from correlations with a specified Modified Mercalli Intensity, or directly from a regional risk map which corresponds to an appropriate mean return period.

The maximum acceleration per se is by no means a sufficient representation of earthquake ground motions for seismic analysis and design purposes. It serves, however, as an anchor for more sophisticated and detailed ground motion descriptions. Typically, the seismic input takes the form of time histories, spectral density functions, or smooth response spectra, all of which can be conveniently scaled with respect to the maximum ground acceleration .

Time history analyses performed on different accelerograms - all scaled to the same maximum acceleration - can provide valuable information as to the variability of the seismic response. However, such a task requires extensive calculations and is expensive

for practical purposes.

Spectral density functions along with duration of strong ground motion constitute a second representation of seismic loading. A fundamental property of the spectral density function lies in the fact that the square root of its integral over all frequencies is equal to the r.m.s. ground acceleration. Since the r.m.s. ground acceleration is proportional to the peak ground acceleration, it is possible to scale the seismic input described by a spectral density function by adjusting the area under the spectrum. There are more sophisticated models of spectral density functions, based on the concept of *evolutionary spectra*, which can handle non stationary processes, whose statistics are time dependent. But a spectral density function, as a description mode for a seismic input, is not commonly used in seismic design practice, because it requires involved stochastic analyses.

The last and most common representation of earthquake ground motions for seismic analysis and design is by means of the response spectrum. The combination of this seismic loading representation with the modal superposition method is widely used in earthquake resistant designs of conventional above-ground structures for the approximate computation of the structural seismic response. The fundamental requirement for the applicability of this method is that the structure at hand, simply or multiply supported, be subjected to a uniform translational seismic excitation. Such an assumption seems to be realistic for non elongated structures, which extend over short distances relative to earthquake vibration wavelengths. However, it is not the case for elongated

structures, such as bridges or dams, which are generally subjected to changing motions along their length. This twofold situation has provided the essential impetus for the work contemplated in this thesis, whose objective is the provision of a practical means for the seismic analysis and design of an extended facility accounting, at least in an approximate way, for partially correlated seismic excitations (imperfectly coherent seismic motions combined with wave travelling effects). An attractive alternative for the practical means sought after, would be to develop a procedure that modifies the design response spectrum - assumed to be spatially homogeneous - by means of simple models for the cross-correlation functions. In other words, it will be assumed that the *response spectra* do not change in horizontal planes, even though the *motions* may not be spatially uniform. The cross-correlation characteristics will be obtained from models that account for the physical characteristics of the wave-propagation phenomenon. The advantage of such an alternative, over the random vibration approach for instance, is the possible inclusion of the potentially important problem of spatial variability of earthquake ground motions into seismic design practice.

Chapter 3

General Derivation of the

Modified Response Spectrum Model

3.1 Introduction

In earthquake resistant design, it is common to assume that the entire base of a structure is subjected to a uniform ground motion. In other words, the same seismic motion is assumed to be acting, simultaneously, at all points attaching the structure to the ground. This hypothesis in the treatment of earthquake excitations is undoubtedly advantageous, because it substantially facilitates the dynamic analysis, which can be performed by means of the well established and widely used response spectrum method. However, such a hypothesis inherently implies that the ground motion is a result of spatially uniform, vertically propagating shear waves, or, that the base dimensions of the structure at hand, are small relative to the seismic vibration wavelengths. As a consequence, it is clear that the assumption of uniform ground motion is inappropriate for extended facilities, and hence, that the use of the response spectrum method is invalidated for such structures. Therefore, it behooves to develop a procedure that, not only addresses the problem of multiple support excitation, but also is practical enough, in order to gain acceptance within the engineering design community. As was mentioned in earlier parts of this dissertation, a promising alternative would consist in modifying the design response spectrum, so as to account for the spatial character

of the seismic input. Such a development is the object of the present chapter.

In brief, the design response spectrum, as it is classically understood, is considered to be given at a site of interest. This design response spectrum will be modified, so as to account for the spatial character of the seismic input exerted on an extended facility to be implemented in that site. The modification procedure will be based on adjusting each spectral value of the given design response spectrum. Such a task will be achieved by examining each modal equation, and expressing the mean value of the maximum modal response for the case of partially correlated excitations, in terms of the homologous modal response had the seismic input been uniform. By so doing, a *modified design response spectrum* will be derived, and it will be expressed in terms of the given design response spectrum, by means of a relation that takes the spatially varying nature of the ground motion into account. It is worth mentioning at this point, that relating the mean values of the maximum modal responses, as was stated earlier, is the end result of a multi-step process, which starts by relating the spectral density functions of these very responses. While the spectrum modification procedure will be developed by means of a random vibration analysis, such an analysis will be transparent to the prospective user of the model.

Prior to getting into the development per se, it is important to state the assumptions and hypotheses which the present work is based upon:

- Earthquake ground motions are probabilistically modeled by means of a homogenous space-time random field, which is considered as a collection of temporal random

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processes at each of the spatial locations. Both seismic input and structural output are assumed to be zero-mean stationary Gaussian Processes. In fact, the stationarity of the structural response would be justified even if the seismic input were considered to be transient stationary, so long as its duration would be several times larger than the fundamental period of the structure.

- The Design Response Spectrum (mean value R and standard deviation σ_R) is assumed to be given. It is also assumed to be spatially homogeneous within the site of interest, even though the support motions may be different from one location to another.

- Auto-spectra of the ground motion acceleration are considered to be the same at every location, to be consistent with the given design response spectrum, and to follow a model whose mathematical expression is given.

- Cross-spectra are related to auto-spectra by means of the coherency function, whose mathematical expression is assumed to be given.

- Soil-structure interaction effects are neglected, and support motions are considered to be equal to free-field motions.

- Structures are assumed to be classically damped.

It is important to mention at this stage, that the proposed procedure is independent of the mathematical expressions of both the auto-spectra and the coherency function

of the ground motion. All it is based on, is that these two functions are given, as opposed to being unknown to be determined.

3.2 Discrete Systems

In this section, the derivation of the modified response spectrum model will be presented for extended lumped mass systems, such as industrial buildings. The formulation will, at first, account for one ground motion component, and will, in a second phase, be extended to multicomponent ground motion.

3.2.1 Case of a Single Ground Motion Component

The coupled equations of motion of a linear, lumped mass, multi-degree of freedom, multiply-supported structural system subjected to uniform undimensional translational seismic excitations can be written in matrix form as follows:

$$
\begin{bmatrix} M_s & 0 \\ 0 & M_b \end{bmatrix} \begin{bmatrix} \ddot{U}_s \\ \ddot{U}_b \end{bmatrix} + \begin{bmatrix} C_s & C_{sb} \\ C_{bs} & C_b \end{bmatrix} \begin{bmatrix} \dot{U}_s \\ \dot{U}_b \end{bmatrix} + \begin{bmatrix} K_s & K_{sb} \\ K_{bs} & K_b \end{bmatrix} \begin{bmatrix} U_s \\ U_b \end{bmatrix} = \begin{bmatrix} 0 \\ P_b \end{bmatrix}
\tag{3.2.1.1}
$$

where:

s and b are subscripts referring to structure and base (foundation) respectively.

sb and bs are pairs of subscripts referring to coupling between structure and base.

M refers to mass matrices.

C refers to damping matrices.

K refers to stiffness matrices.

$U, \dot{U}$, and $\ddot{U}$ refer to vectors of, absolute displacement, absolute velocity, and absolute acceleration, respectively.

P_b is a vector of reaction forces at the base (support points).

Note that there is no mass coupling between the structure and the base because masses are lumped.

Let's now decompose the displacements into a rigid body component and a non-rigid body component.

$$\begin{bmatrix} U_s \\ U_b \end{bmatrix} = \begin{bmatrix} T_s U_0 \\ T_b U_0 \end{bmatrix} + \begin{bmatrix} V_s \\ V_b \end{bmatrix} \qquad (3.2.1.2)$$

where:

$T_s U_0$ is the rigid body component of the structural displacement.

V_s is the non rigid body component of the structural displacement (relative displacement).

$T_b U_0$ is the rigid body component of the base displacement..

V_b is the non rigid body component of the base displacement.

U_0 is a scalar in this case, since a single ground motion component is under consideration. $U_0 = u_0$.

T_s and T_b are transformation matrices (influence vectors in this case), for the structure
and the base respectively, which satisfy the rigid body condition:

$$\begin{bmatrix} K_s & K_{sb} \\ K_{bs} & K_b \end{bmatrix} \begin{bmatrix} T_s \\ T_b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad (3.2.1.3)$$

$$\implies K_s T_s + K_{sb} T_b = 0 \qquad (3.2.1.4)$$

Note that likewise, there are no damping forces induced by a rigid body velocity.

Note that $V_b = 0$ since the ground motion is uniform.

Let's combine equations (3.2.1.2) and (3.2.1.1), and make use of the rigid body
condition (3.2.1.3), to write the equations of motion for the structural part:

$$M_s \ddot{V}_s + C_s \dot{V}_s + K_s V_s = -M_s T_s \ddot{u}_0 \qquad (3.2.1.5)$$

Assuming that the structure is classically damped, and using modal coordinates,
one gets:

$$\begin{cases} (\Phi^T M_s \Phi) \ddot{Y} + (\Phi^T C_s \Phi) \dot{Y} + (\Phi^T K_s \Phi) Y = -\Phi^T M_s T_s \ddot{u}_0 \\ \\ V_s = \Phi Y \end{cases} \qquad (3.2.1.6)$$

where:

Y is a vector of modal coordinates.

Φ is a matrix containing the mode shapes of the structure.

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From (3.2.1.6), the kth modal equation can be written as follows:

$$\ddot{y}_k + 2\beta_k\omega_k\dot{y}_k + \omega_k^2 y_k = -\gamma_k\ddot{u}_0 \qquad (3.2.1.7)$$

where:

y_k is the kth modal coordinate

β_k is the viscous damping ratio for mode k, such that:

$$2\beta_k\omega_k = \frac{\phi_k^T C_s \phi_k}{\phi_k^T M_s \phi_k}$$

ω_k is the natural frequency for mode k, such that:

$$\omega_k^2 = \frac{\phi_k^T K_s \phi_k}{\phi_k^T M_s \phi_k}$$

γ_k is the participation factor for mode k, such that:

$$\gamma_k = \frac{\phi_k^T M_s T_s}{\phi_k^T M_s \phi_k}$$

The modal equation (3.2.1.7) could be solved in a number of ways. For design purposes, however, the response spectrum method is the most widely used means to compute the maximum value (in the absolute sense) of the modal response. Namely, if $R(\omega, \beta)$ is the design response spectrum at the site of interest then:

$$\left. y_k \right|_{max} = |\gamma_k| \, R(\omega_k, \beta_k) \qquad (3.2.1.8)$$

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It is important to note that, in fact, $\left|y_k\right|_{\text{max}}$ is the mean value of the maximum modal response.

Alternatively, y_k could be characterized by its spectral density function given by:

$$S_{y_k}(\omega) = \gamma_k^2 \mid H_k(\omega) \mid^2 S_{\ddot{u}_0}(\omega) \qquad (3.2.1.9)$$

where:

$S_{y_k}(\omega)$ is the spectral density function of y_k.

$H_k(\omega) = \dfrac{1}{\omega_k^2 - \omega^2 + 2i\beta_k\omega_k\omega}$ is the complex frequency function (transfer function) for

mode k.

$S_{\ddot{u}_0}(\omega)$ is the spectral density function of the ground acceleration at the site of interest.

Since stationary Gaussian Processes are assumed, a description through spectral density functions is a complete characterization in the probabilistic sense.

Having presented the case of uniform seismic excitations, where any physical response can be computed by means of the response spectrum method combined with the mode superposition method, let's examine the less trivial case of non-uniform seismic input.

The coupled equations of motion of a linear, lumped mass, multi-degree of freedom, multiply-supported structural system subjected to non uniform unidimensional translational seismic excitations are also given, in matrix form, by the general system of equations (3.2.1.1). However, for the case of non uniform seismic input, support

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motions are different from one location to another. For this reason, the structural response will not be decomposed into a rigid body component and a non rigid body component, as for the case of uniform seismic input; because such a decomposition would make the computation of the non rigid body component of the response very complicated. Indeed, this response component would be affected by both the rigid body part of the loading (inertia forces due to uniform seismic input) and the non rigid part of it. This situation would induce a mixed input problem (acceleration and displacement), which is highly undesireable.

The strategy to be adopted for the problem of multiple support excitation, is a decomposition of the structural response into a pseudo-static component and a dynamic component [9]. Hence, let us perform the following decomposition:

$$
\begin{bmatrix} U_s \\ U_b \end{bmatrix} = \begin{bmatrix} U_s^s \\ U_b \end{bmatrix} + \begin{bmatrix} V_s \\ 0 \end{bmatrix}
\tag{3.2.1.10}
$$

where:

U_s^s is the pseudo-static component of structural displacement.

V_s is the dynamic component of structural displacement.

For the static (pseudo-static) case, (3.2.1.1) yields:

$$
\begin{bmatrix} K_s & K_{sb} \\ K_{bs} & K_b \end{bmatrix} \begin{bmatrix} U_s^s \\ U_b \end{bmatrix} = \begin{bmatrix} 0 \\ P_{bs} \end{bmatrix}
\tag{3.2.1.11}
$$

where:

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P_{bs} is a vector of reaction forces at the support points due to static displacements:

U_s^s.

$$(3.2.1.11) \implies K_s U_s^s + K_{sb} U_b = 0 \tag{3.2.1.12}$$

$$\implies U_s^s = -K_s^{-1} K_{sb} U_b \tag{3.2.1.13}$$

From (3.2.1.11), the equations of motion for the structural part read as follows:

$$M_s \ddot{V}_s + C_s \dot{V}_s + K_s V_s = - M_s \ddot{U}_s^s - C_{sb} \dot{U}_b$$
$$- (K_s U_s^s + K_{sb} U_b) \tag{3.2.1.14}$$

By virtue of (3.2.1.12), and by neglecting the support damping contributions and the structural damping term, one gets:

$$M_s \ddot{V}_s + C_s \dot{V}_s + K_s V_s = -M_s \ddot{U}_s^s \tag{3.2.1.15}$$

Which can be rewritten as:

$$M_s \ddot{V}_s + C_s \dot{V}_s + K_s V_s = M_s K_s^{-1} K_{sb} \ddot{U}_b \tag{3.2.1.16}$$

by making use of (3.2.13).

Note that it is possible to retrieve the equations of motion for the case of uniform seismic input. Indeed, $U_b = T_b U_0 \implies U_s^s = -K_s^{-1} K_{sb} T_b U_0$ from (3.2.1.13)

$$\implies U_s^s = -K_s^{-1}(-K_s T_s)U_0 \quad \text{from (3.2.1.4)}.$$

$$\implies U_s^s = T_s U_0 \tag{3.2.1.17}$$

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By substituting (3.2.1.17) into (3.2.1.15), one gets the equations of motion for the case of uniform seismic motion given by (3.2.1.5). Hence, if seismic motions are uniform, then the static component of the structural response is equal to the rigid body component of it.

After this brief remark, let's proceed with the case of non uniform seismic input. Assuming classical damping, and using modal coordinates, one can transform (3.2.1.16) into:

$$
\begin{cases}
(\Phi^T M_s \Phi)\ddot{Y} + (\Phi^T C_s \Phi)\dot{Y} + (\Phi^T K_s \Phi)Y = \Phi^T M_s K_s^{-1} K_{sb}\ddot{U}_b \\
V_s = \Phi Y
\end{cases}
\tag{3.2.1.18}
$$

$\ddot{U}_b$ is a vector containing support accelerations.

$\ddot{U}_b = [\ddot{u}_i]^T, \quad i = 1, 2, ..., n$

i indicates the support number (or the support point).

n is the total number of supports.

From (3.2.1.18), the kth modal equation can be written as follows:

$$
\ddot{y}_k + 2\beta_k\omega_k\dot{y}_k + \omega_k^2 y_k = \sum_{i=1}^{n} A_{ki}\ddot{u}_i
\tag{3.2.1.19}
$$

i indicates the support point

k indicates the mode.

Because of the nature of the seismic loading, (3.2.1.19) cannot be solved by means of the design response spectrum, which was used to solve (3.2.1.7). However, y_k can be

characterized in a stochastic sense by the following:

$$S_{y_k}^p(\omega) = |H_k(\omega)|^2 \, S_{\ddot{s}_k}(\omega) \qquad (3.2.1.20)$$

where:

$S_{y_k}^p(\omega)$ is the spectral density function of y_k for the case of partially correlated excitations (non uniform seismic input).

The superscript p stands for partially correlated excitations.

$$\ddot{s}_k(t) = \sum_{i=1}^{n} A_{ki} \ddot{u}_i(t) \qquad (3.2.1.21)$$

$S_{\ddot{s}_k}(\omega)$ is the spectral density function of $\ddot{s}_k$.

Let's express $S_{\ddot{s}_k}(\omega)$ in terms of auto-spectra and cross-spectra of the ground acceleration at the different supports. Prior to doing so, let's do the same work for the autocorrelation function $R_{\ddot{s}_k}(\tau)$.

$$
\begin{aligned}
R_{\ddot{s}_k}(\tau) &= E[\ddot{s}_k(t)\ddot{s}_k(t+\tau)] \\
&= E\left[(\sum_{i=1}^{n} A_{ki}\ddot{u}_i(t))(\sum_{i=1}^{n} A_{ki}\ddot{u}_i(t+\tau))\right] \\
&= E\left[\sum_{i=1}^{n}\sum_{j=1}^{n} A_{ki} A_{kj}\ddot{u}_i(t)\ddot{u}_j(t+\tau)\right]
\end{aligned}
$$

By changing order of operation between expectation and summation, one gets:

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$$R_{\ddot{s}_k}(\tau) = \sum_{i=1}^{n}\sum_{j=1}^{n} A_{ki} A_{kj} E\left[\ddot{u}_i(t)\ddot{u}_j(r+\tau)\right]$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{n} A_{ki} A_{kj} R_{\ddot{u}_i\ddot{u}_j}(\tau)$$

Hence,

$$R_{\ddot{s}_k}(\tau) = \sum_{i=1}^{n}\sum_{j=1}^{n} A_{ki} A_{kj} R_{\ddot{u}_i\ddot{u}_j}(\tau) \qquad (3.2.1.22)$$

By taking Fourier transforms in (3.2.1.22), one gets:

$$S_{\ddot{s}_k}(\omega) = \sum_{i=1}^{n}\sum_{j=1}^{n} A_{ki} A_{kj} S_{\ddot{u}_i\ddot{u}_j}(\omega) \qquad (3.2.1.23)$$

Let's now make use of the following equations:

$$S_{\ddot{u}_i\ddot{u}_j}(\omega) = S_{\ddot{u}_j\ddot{u}_i}^{*}(\omega) \qquad (3.2.1.24)$$

$$S_{\ddot{u}_i\ddot{u}_j}(\omega) = r_{ij}(\omega, d_{ij}) S_{\ddot{u}_0}(\omega) \qquad (3.2.1.25)$$

Where:

* indicates complex conjugate.

$r_{ij}(\omega, d_{ij})$ is the coherency function of the seismic motion, which is taken to be an exponentially decaying function of both frequency, ω, and distance separating two locations, d_{ij}.

$S_{\ddot{u}_0}(\omega)$ is the autospectrum of the ground acceleration common to all locations.

By virtue of (3.2.1.24) and (3.2.1.25), (3.2.1.23) can be rewritten as:

$$
\begin{aligned}
S_{\ddot{s}_k}(\omega) &= \sum_{i=1}^{n} A_{ki}^2 S_{\ddot{u}_0}(\omega) + \sum_{i=1}^{n}\sum_{\substack{j=1\\j\neq i}}^{n} A_{ki} A_{kj} S_{\ddot{u}_i \ddot{u}_j}(\omega)\\
&= \sum_{i=1}^{n} A_{ki}^2 S_{\ddot{u}_0}(\omega) + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} A_{ki} A_{kj}\, \text{Real}\,(S_{\ddot{u}_i \ddot{u}_j}(\omega))\\
&= \left[\sum_{i=1}^{n} A_{ki}^2 + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} A_{ki} A_{kj}\rho_{ij}\right] S_{\ddot{u}_0}(\omega)
\end{aligned}
$$

where ρ_{ij} is the real part of the coherency function. It is the frequency dependent spatial correlation coefficient.

Hence,

$$
S_{\ddot{s}_k}(\omega) = \left[\sum_{i=1}^{n} A_{ki}^2 + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} A_{ki} A_{kj}\rho_{ij}\right] S_{\ddot{u}_0}(\omega) \tag{3.2.1.26}
$$

Therefore, $S_{y_k}^p(\omega)$ can be written as follows:

$$
\left.\begin{array}{c}
(3.2.1.20)\\[2mm]
(3.2.1.26)
\end{array}\right\} \implies
$$

$$
S_{y_k}^p(\omega) = |H_k(\omega)|^2 \left[\sum_{i=1}^{n} A_{ki}^2 + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} A_{ki} A_{kj}\rho_{ij}\right] S_{\ddot{u}_0}(\omega) \tag{3.2.1.27}
$$

Combining (3.2.1.9) and (3.2.1.27), it is possible to express the spectral density function of y_k for the case of partially correlated excitations, $S_{y_k}^p(\omega)$, in terms of the spectral density function of y_k for the case of fully correlated excitations, $S_{y_k}(\omega)$:

$$
S_{y_k}^p(\omega) = \frac{1}{\gamma_k^2} \left[\sum_{i=1}^{n} A_{ki}^2 + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} A_{ki} A_{kj}\rho_{ij}\right] S_{y_k}(\omega) \tag{3.2.1.28}
$$

By considering all $\ddot{u}_i$'s identical (uniform seismic input) in the kth modal equation for

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the case of non uniform seismic input (3.2.1.14), one gets:

$$\ddot{y}_k + 2\beta_k \omega_k \dot{y}_k + \omega_k^2 y_k = \ddot{u}_0(\sum_{i=1}^{n} A_{ki}) \qquad (3.2.1.29)$$

where $\ddot{u}_0(t)$ is the ground acceleration acting at all support points if the ground motion is assumed to be uniform (no loss of coherence and no travelling wave effects). Comparing (3.2.1.29) and (3.2.1.7), and identifying participation factors, one gets:

$$\gamma_k = -\sum_{i=1}^{n} A_{ki} \qquad (3.2.1.30)$$

Before getting any further in the development, let us examine two extreme cases:

1. Case of fully correlated support motions: ρ_{ij}'s $= 1$ (3.2.1.28) reads as follows:

$$
\begin{aligned}
S_{y_k}^p(\omega) &= \frac{1}{\gamma_k^2}\left[\sum_{i=1}^{n} A_{ki}^2 + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} A_{ki} A_{kj}\right] S_{y_k}(\omega) \\
&= \frac{1}{\gamma_k^2}\left(\sum_{i=1}^{n} A_{ki}\right)^2 S_{y_k}(\omega)
\end{aligned}
$$

Using (3.2.1.30), one gets:

$$S_{y_k}^p(\omega) = S_{y_k}(\omega)$$

Which is an expected result because one should be able to get to the case of uniform seismic input starting from the case of non uniform seismic input and considering all support motions to be identical.

2. Case of uncorrelated support motions ρ_{ij}'s $= 0$ (3.2.1.28) reads as follows:

$$S_{y_k}^p(\omega) = \frac{1}{\gamma_k^2}\left[\sum_{i=1}^{n} A_{ki}^2\right] S_{y_k}(\omega)$$

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$$\implies S^p_{y_k}(\omega) = \frac{\sum\limits_{i=1}^{n} A^2_{ki}}{\left(\sum\limits_{i=1}^{n} A_{ki}\right)^2} S_{y_k}(\omega)$$

Closing this parenthesis, and pursuing the procedure development, it has been noted that (3.2.1.28) expresses the relationship between spectral density functions for the cases of partially and fully (perfectly) correlated seismic excitations. Let us now relate the corresponding response spectra. In other words, let us express the response spectrum (mean value of the maximum response) for the case of partially correlated seismic excitations in terms of the "classical" response spectrum (mean value of the maximum response for the case of uniform seismic input).

Finding the mean value of the maximum of a stochastic variable given its spectral density function is a complex problem, and exact solutions are not available. However, under reasonable assumptions, satisfactory results can be reached. For the present case, and as was stated in the introduction to this chapter, all stochastic processes are assumed to be zero-mean, stationary, and Gaussian. Such simplifying assumptions, but not unjustified for seismic motions, render the task of seeking an expected maximum value feasible. To briefly justify the aforecited assumptions, it is worth mentioning that while earthquake induced ground motions are inherently non-stationary, the strong phase of such motions, during which the peak response generally occurs, is ususally considered to be nearly stationary. The Gaussian character of the seismic motions is acceptable on the basis of the central limit theorem, since the earthquake motions

result from the accumulation of a large number of randomly arriving pulses.

If $x(t)$ is a random process satisfying the precedent assumptions, then a prescribed duration τ must be imposed in order to find a realistic maximum absolute value [35]. For the present case, τ is the duration of the strong motion phase. Let the maximum value be x_τ such that:

$$x_\tau = \overset{max}{{}^\tau} \mid x(t) \mid \tag{3.2.1.31}$$

Because of the random nature of $x(t)$, x_τ is also random. But in practice, the interest is in the mean value (and the standard deviation) $\bar{x}_\tau$ and σ_{x_τ}, of x_τ.

Assuming the excursions above a certain level (barrier) of the process $x(t)$ to be mutually independent and to have a negligible duration (Poisson process), Davenport [11] has shown that the mean value $\bar{x}_\tau$ and the standard deviation σ_{x_τ} are approximately given by:

$$\bar{x}_\tau = \left[\sqrt{2 ln\nu\tau} + \frac{\gamma}{\sqrt{2 ln\nu\tau}} \right] \sigma_x \tag{3.2.1.32}$$

$$\sigma_{x_\tau} = \frac{\pi}{\sqrt{6}} \frac{1}{\sqrt{2 ln\nu\tau}} \sigma_x \tag{3.2.1.33}$$

where:

σ_x is the root mean square of the process $x(t)$ given by:

$$\sigma_x^2 = \int_{-\infty}^{\infty} S_x(\omega) d\omega$$

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$S_x(\omega)$ is the spectral density function of $x(t)$.

τ is the prescribed duration.

γ is Euler's constant $= 0.577216$.

ν is the mean zero-crossing rate of the process $x(t)$

Prior to further elaborating on ν, let us define some necessary parameters:

$G_x(\omega)$, the one-sided spectral density function of the process $x(t)$ defined by:

$$\sigma_x^2 = \int_0^\infty G_x(\omega)\,d\omega$$

or

$$G_x(\omega) = \begin{cases} 2S_x(\omega) & \omega \geq 0 \\ \\ 0 & \omega < 0 \end{cases}$$

$S_x(\omega)$ being an even function of ω.

λ_i, i^{th} spectral moment of the process $x(t)$ given by:

$$\lambda_i = \int_0^\infty \omega^i G_x(\omega)\,d\omega$$

Having defined these parameters,

$$\nu = \frac{1}{\pi}\frac{\sigma_{\dot{x}}}{\sigma_x} = \frac{1}{\pi}\sqrt{\frac{\lambda_2}{\lambda_0}}$$

where $\sigma_{\dot{x}}$ is the root mean square of the process $\dot{x}(t)$, given by:

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$$\sigma_{\dot{x}}^2 = \int_0^\infty G_{\dot{x}}(\omega) d\omega = \int_0^\infty \omega^2 G_x(\omega) d\omega = \lambda_2$$

Note that $\sigma_x^2 = \lambda_0$.

For the sake of completeness, one ought to mention that more involved work has been done on the distribution of the extreme values of the process $x(t)$. Namely, Vanmarcke [47] derived a distribution for the first-crossing time of a given level by $| \, x \, |$, which takes into account the statistical dependence among crossings. Based on Vanmarcke's work, Der Kiureghian [13] has given empirical expressions for $\bar{x}_r$ and σ_{x_r}. The difference between Der Kiureghian's expressions and Davenport's expressions increases with decreasing bandwidth of the process. This difference results from the fact that for narrow-band processes, statistical dependence should not be disregarded. However, in many applications, including earthquake engineering, the description of the dynamic loads contains considerable inaccuracies [36]. Therefore, for the purposes of the present work, the practical and satisfactory model proposed by Davenport will be retained.

Starting from (3.2.1.28) relating power spectra, let us try to relate response spectra (mean value of maximum response) by means of Davenport's expressions:

$$(3.2.1.28) \implies$$

$$G_{y_k}^p(\omega) = \frac{1}{\gamma_k^2} \left[\sum_{i=1}^n A_{ki}^2 + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^n A_{ki} A_{kj} \rho_{ij} \right] G_{y_k}(\omega) \qquad (3.2.1.33)$$

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The above equation relates the one-sided spectral density functions of y_k for the cases of partially correlated seismic excitations and fully correlated seismic excitations. Having related the one-sided spectral density functions, one can relate the spectral moments: Let λ_m and λ_m^p be the mth spectral moments of the modal response y_k for the cases of fully correlated and partially correlated seismic excitations, respectively.

Hence,

$$\lambda_m = \int_0^\infty \omega^m G_{y_k}(\omega)\,d\omega \qquad (3.2.1.34)$$

$$\lambda_m^p = \int_0^\infty \omega^m G_{y_k}^p(\omega)\,d\omega \qquad (3.2.1.35)$$

Let's express λ_m^p in terms of λ_m.

$$
\begin{aligned}
\lambda_m^p &= \int_0^\infty \omega^m G_{y_k}^p(\omega)\,d\omega \\
&= \int_0^\infty \omega^m \frac{1}{\gamma_k^2} \left[\sum_{i=1}^n A_{ki}^2 + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^n A_{ki}A_{kj}\rho_{ij} \right] G_{y_k}(\omega)\,d\omega \\
&= \frac{1}{\gamma_k^2} \left[\sum_{i=1}^n A_{ki}^2 \int_0^\infty \omega^m G_{y_k}(\omega)\,d\omega + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^n A_{ki}A_{kj} \int_0^\infty \rho_{ij}\omega^m G_{y_k}(\omega)\,d\omega \right]
\end{aligned}
$$

Define:

$$\rho_{ijmk} = \frac{\int_0^\infty \rho_{ij}\omega^m G_{y_k}(\omega)\,d\omega}{\int_0^\infty \omega^m G_{y_k}(\omega)\,d\omega} \qquad (3.2.1.36)$$

Using (3.2.1.34) and (3.2.1.36), λ_m^p can be rewritten as:

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$$\lambda_m^p = \frac{1}{\gamma_k^2} \left[\lambda_m \sum_{i=1}^{n} A_{ki}^2 + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} A_{ki} A_{k_j} \rho_{ijm_k} \lambda_m \right]$$

Hence,

$$\lambda_m^p = \frac{1}{\gamma_k^2} \left[\sum_{i=1}^{n} A_{ki}^2 + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} A_{ki} A_{kj} \rho_{ijm_k} \right] \lambda_m \qquad (3.2.1.37)$$

(3.2.1.37) relates the mth spectral moments. Of interest for the evaluation of the peak response statistics are the zeroth and second spectral moments. Incidentally, ρ_{ij0k} represents the spatial correlation (cross-correlation) coefficient between the relative responses of two identical oscillators (ω_k, β_k) respectively subjected to $\ddot{u}_i(t)$ and $\ddot{u}_j(t)$. While ρ_{ij2k} represents the spatial correlation (cross-correlation) coefficient between the relative velocities of the two identical oscillators subjected to the same respective seismic loading.

Having related spectral moments, let's relate necessary parameters for the evaluation of the peak response statistics:

1. Root Mean Square of the Processes:

Let σ_{y_k} and $\sigma_{y_k}^p$ be the root mean square of the modal response y_k for the cases of uniform and non uniform seismic input, respectively:

$$\sigma_{y_k} = \sqrt{\lambda_0} \qquad (3.2.1.38)$$

$$\sigma_{y_k}^p = \sqrt{\lambda_0^p} \qquad (3.2.1.39)$$

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Let's express $\sigma_{y_k}^p$ in terms of σ_{y_k}:

$$\sigma_{y_k}^p = \sqrt{\lambda_0^p} = \left[\frac{1}{\gamma_k^2} \left[\sum_{i=1}^{n} A_{ki}^2 + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} A_{ki} A_{kj} \rho_{ij0k} \right] \lambda_0 \right]^{1/2}$$

Hence,

$$\sigma_{y_k}^p = \left[\frac{1}{\gamma_k^2} \left[\sum_{i=1}^{n} A_{ki}^2 + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} A_{ki} A_{kj} \rho_{ij0k} \right] \right]^{1/2} \sigma_{y_k} \qquad (3.2.1.40)$$

2. Mean Zero-Crossing Rate:

Let ν and ν^p be the mean zero-crossing rates of the modal response y_k for the cases of fully correlated and partially correlated seismic excitations:

$$\nu = \frac{1}{\pi} \left(\frac{\lambda_2}{\lambda_0} \right)^{1/2} \qquad (3.2.1.41)$$

$$\nu^p = \frac{1}{\pi} \left(\frac{\lambda_2^p}{\lambda_0^p} \right)^{1/2} \qquad (3.2.1.42)$$

Let's express ν^p in terms of ν:

$$\nu^p = \frac{1}{\pi} \left(\frac{\lambda_2^p}{\lambda_0^p} \right)^{1/2} = \frac{1}{\pi} \left[\frac{1}{\gamma_k^2} \frac{\left[\sum_{i=1}^{n} A_{ki}^2 + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} A_{ki} A_{kj} \rho_{ij2k} \right] \lambda_2}{\frac{1}{\gamma_k^2} \left[\sum_{i=1}^{n} A_{ki}^2 + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} A_{ki} A_{kj} \rho_{ij0k} \right] \lambda_0} \right]^{1/2}$$

$$= \left[\frac{\sum_{i=1}^{n} A_{ki}^2 + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} A_{ki} A_{kj} \rho_{ij2k}}{\sum_{i=1}^{n} A_{ki}^2 + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} A_{ki} A_{kj} \rho_{ij0k}} \right]^{1/2} \frac{1}{\pi} \left(\frac{\lambda_2}{\lambda_0} \right)^{1/2}$$

Using (3.2.1.41), ν^p can be rewritten as:

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$$\nu^p = \left[\frac{\sum_{i=1}^{n} A_{ki}^2 + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} A_{ki}A_{kj}\rho_{ij2k}}{\sum_{i=1}^{n} A_{ki}^2 + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} A_{ki}A_{kj}\rho_{ij0k}} \right]^{1/2} \nu \qquad (3.2.1.43)$$

3. Peak Factors:

By virtue of (3.2.1.32) and (3.2.1.33) (Davenport's expressions), the peak response statistics for the case of uniform and non uniform seismic input can be written as follows:

a) Uniform seismic input

$$\begin{cases} \text{Mean}(\max(y_k)_\tau) & = \quad p_\tau \cdot \sigma_{y_k} \qquad (3.2.1.44) \\[2mm] \sigma \ (\max(y_k)_\tau) & = \quad q_\tau \cdot \sigma_{y_k} \qquad (3.2.1.45) \end{cases}$$

where p_τ and q_τ are peak factors given by:

$$p_\tau = \sqrt{2ln\nu\tau} + \frac{\gamma}{\sqrt{2ln\nu\tau}} \qquad (3.2.1.46)$$

$$q_\tau = \frac{\pi}{\sqrt{6}}\frac{1}{\sqrt{2ln\nu\tau}} \qquad (3.2.1.47)$$

b) Non uniform seismic input:

$$\begin{cases} \overset{p}{\text{Mean}}(\max(y_k)_\tau) & = \quad p_\tau^p \cdot \sigma_{y_k}^p \qquad (3.2.1.48) \\[2mm] \sigma^p(\max(y_k)_\tau) & = \quad q_\tau^p \cdot \sigma_{y_k}^p \qquad (3.2.1.49) \end{cases}$$

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where p_τ^p and q_τ^p are peak factors given by:

$$p_\tau^p = \sqrt{2ln\nu^p\tau} + \frac{\gamma}{\sqrt{2ln\nu^p\tau}} \qquad (3.2.1.50)$$

$$q_\tau^p = \frac{\pi}{\sqrt{6}} \frac{1}{\sqrt{2ln\nu^p\tau}} \qquad (3.2.1.51)$$

Let's express p_τ^p in terms of p_r, and q_τ^p in terms of q_r.

$$p_\tau^p = \sqrt{2ln\nu^p\tau} + \frac{\gamma}{\sqrt{2ln\nu^p\tau}} = \sqrt{2ln\nu\tau_1} + \frac{\gamma}{\sqrt{2ln\nu\tau_1}}$$

$$q_\tau^p = \frac{\pi}{\sqrt{6}} \frac{1}{\sqrt{2ln\nu^p\tau}} = \frac{\pi}{\sqrt{6}} \frac{1}{\sqrt{2ln\nu\tau_1}}$$

where τ_1 is such that:

$$\nu^p\tau = \nu\tau_1 \implies \tau_1 = \frac{\nu^p}{\nu}\tau$$

$$\implies \tau_1 = \left[\frac{\sum_{i=1}^{n}A_{ki}^2 + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}A_{ki}A_{kj}\rho_{ij2k}}{\sum_{i=1}^{n}A_{ki}^2 + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}A_{ki}A_{kj}\rho_{ij0k}}\right]^{1/2}\tau \qquad (3.2.1.52)$$

Hence,

$$p_\tau^p = \sqrt{2ln\nu\tau_1} + \frac{\gamma}{\sqrt{2ln\nu\tau_1}}$$

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$$q_\tau^p = \frac{\pi}{\sqrt{6}} \frac{1}{\sqrt{2ln\nu\tau_1}}$$

And finally:

$$p_\tau^p = p_{\tau_1} \qquad\qquad (3.2.1.53)$$

$$q_\tau^p = q_{\tau_1} \qquad\qquad (3.2.1.54)$$

The last two equations express that the peak factors for the case of partially correlated seismic excitations, over a duration τ, are equal to the peak factors for the case of fully correlated seismic excitations, over a duration τ_1. τ_1 and τ are related by (3.2.1.52).

At present, the final step in the process is reached, and it is to relate the peak response statistics in each one of the two cases of seismic excitations.

$$(3.2.1.48) \implies \overset{p}{M}ean(\max(y_k)_\tau) = p_\tau^p \sigma_{y_k}^p$$

$$(3.2.1.40)\ (3.2.1.53) \implies \overset{p}{M}ean(\max(y_k)_\tau) = p_{\tau 1}\sigma_{y_k}\frac{1}{\mid\gamma_k\mid}\left[\sum_{i=1}^{n}A_{ki}^2 + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}A_{ki}A_{kj}\rho_{ij0k}\right]^{1/2}$$

$$\implies \overset{p}{M}ean(\max(y_k)_\tau) = Mean(\max(y_k)_{\tau_1})\frac{1}{\mid\gamma_k\mid}\left[\sum_{i=1}^{n}A_{ki}^2 + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}A_{ki}A_{kj}\rho_{ij0k}\right]^{1/2}$$

$$(3.2.1.55)$$

Hence, (3.2.1.55) expresses the mean value of the maximum k^{th} modal response for the case of partially correlated seismic excitations, in terms of the maximum k^{th} modal response for the case of fully correlated seismic excitations. Let's define a modified response spectrum, which accounts for the spatial character of the seismic input as follows:

$$Mean^p(\max(y_k)_r) = \mid \gamma_k \mid R_r^p(\omega_k, \beta_k)$$
(3.2.1.56)

where $R_r^p(\omega, \beta)$ is the modified response spectrum, and γ_k is the participation factor for mode k.

Also,

$$Mean(\max(y_k)_{r_1}) = \mid \gamma_k \mid R_{r_1}(\omega_k, \beta_k)$$
(3.2.1.57)

where $R_{r_1}(\omega, \beta)$ is the response spectrum which assumes identical support motions.

Hence, (3.2.1.55), (3.2.1.56), and (3.2.1.57) $\Longrightarrow$

$$R_r^p(\omega_k, \beta_k) = R_{r_1}(\omega_k, \beta_k) \left[\sum_{i=1}^{n} A_{ki}^2 + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} A_{ki} A_{kj} \rho_{ij0k} \right]^{1/2} \frac{1}{\mid \gamma_k \mid}$$
(3.2.1.58)

$$\rho_{ij0k} = \frac{\int_0^\infty \rho_{ij} G_{y_k}(\omega) d\omega}{\int_0^\infty G_{y_k}(\omega) d\omega} = \frac{\int_0^\infty \rho_{ij} \mid H_k(\omega) \mid^2 G_{\ddot{u}_0}(\omega) d\omega}{\int_0^\infty \mid H_k(\omega) \mid^2 G_{\ddot{u}_0}(\omega) d\omega}$$
(3.2.1.36)

Note that the pair (ω_k, β_k) is used in a generic (dummy) sense in (3.2.1.58). Alternatively, (3.2.1.58) enables any modal response to be modified to account for partially

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correlated seismic excitations. However, the modification factor (or correction factor) will vary from mode to mode.

For the sake of completeness, let us also mention that an analogous equation to (3.2.1.55) could be arrived at for the standard deviation of the maximum modal response, namely,

$$(3.2.1.49) \implies \sigma^p(\max(y_k)_r) = q_r^p \sigma_{y_k}^p$$

$$(3.2.1.40) \implies \sigma^p(\max(y_k)_r) = q_{r_1}\sigma_{y_k}\frac{1}{|\gamma_k|}\left[\sum_{i=1}^{n}A_{ki}^2 + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}A_{ki}A_{kj}\rho_{ij0k}\right]^{1/2}$$

$$(3.2.1.54)$$

$$(3.2.1.45) \implies \sigma^p(\max(y_k)_r) = \sigma(\max(y_k)_{r_1})\frac{1}{|\gamma_k|}\left[\sum_{i=1}^{n}A_{ki}^2 + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}A_{ki}A_{kj}\rho_{ij0k}\right]^{1/2}$$

$$(3.2.1.59)$$

Hence, (3.2.1.59) expresses the standard deviation of the maximum modal response for the case of non uniform seismic input, in terms of the standard deviation of the maximum modal response for the case of uniform seismic input.

An analogous equation to (3.2.1.59) could be written for the standard deviation to be accounted for, in the modified response spectrum:

$$\sigma(R_r^p(\omega_k,\beta_k)) = \sigma(R_{r_1}(\omega_k,\beta_k))\frac{1}{|\gamma_k|}\left[\sum_{i=1}^{n}A_{ki}^2 + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}A_{ki}A_{kj}\rho_{ij0k}\right]^{1/2} \quad (3.2.1.60)$$

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where:

$\sigma(R_{\tau_1}(\omega, \beta))$ is the standard deviation relative to the "classical" response spectrum, which assumes identical support motions.

$\sigma(R_\tau^p(\omega, \beta))$ is the modified standard deviation to account for the spatial character of the seismic input. It is relative to the modified response spectrum.

For practical purposes τ and τ_1 will be considered equal. There is a number of reasons favoring such a simplifying consideration. Firstly, the sensitivity of peak response statistics to the prescribed duration (strong phase duration) is not very pronounced because of the logarithmic function in the expression of the peak factors. Secondly, the measurement of the strong phase duration has some inherent inaccuracy. Thirdly, equation (3.2.1.52) suggests that from a practical viewpoint, the ratio of τ_1 to τ may be assumed to be close to unity. Therefore, if τ_1 and τ are taken to be equal, then they both can be omitted in the final equations where they appear as parameters of the response spectra. Such an omission is by no means detrimental to the formulation presented herein, since the response spectrum, as a means of representing earthquake ground motion, does not include the strong phase duration.

By virtue of the above, (3.2.1.58) could be rewritten as:

$$R^p(\omega_k, \beta_k) = R(\omega_k, \beta_k) \left[\sum_{i=1}^{n} A_{ki}^2 + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} A_{ki} A_{kj} \rho_{ijk} \right]^{1/2} \frac{1}{|\gamma_k|} \qquad (3.2.1.61)$$

Likewise, (3.2.1.60) could be rewritten as:

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$$\sigma(R^p(\omega_k, \beta_k)) = \sigma(R(\omega_k, \beta_k)) \left[\sum_{i=1}^{n} A_{ki}^2 + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} A_{ki} A_{kj} \rho_{ijk} \right]^{1/2} \frac{1}{|\gamma_k|} \qquad (3.2.1.62)$$

Where in both equations ρ_{ij0k} has been written as ρ_{ijk} for the sake of ease of notation.

A last aspect, which will be examined in the context of the present formulation, is the situation where a given mode does not contribute to the structural response under uniform seismic excitations. In other words, the case where $\gamma_k = 0$ is under consideration. It is important to point out that this situation implies, by no means, that this very mode will have no contribution to the structural response under non uniform seismic excitations. This fact can be observed from the following:

$$\gamma_k = 0 \Longrightarrow S_{y_k}(\omega) = \gamma_k^2 \mid H_k(\omega) \mid^2 S_{\ddot{u}_0}(\omega) = 0 \qquad \text{from} (3.2.1.9)$$

Also,

$$\gamma_k = 0 \Longrightarrow \sum_{i=1}^{n} A_{ki} = 0, \text{ from} (3.2.1.30)$$

However,

$$\gamma_k = 0 \nLongrightarrow S_{y_k}^p(\omega) = \mid H_k(\omega) \mid^2 \left[\sum_{i=1}^{n} A_{ki}^2 + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} A_{ki} A_{kj} \rho_{ij} \right] S_{\ddot{u}_0}(\omega) = 0, \text{ from} (3.2.1.27)$$

Hence, while the spectral density function of y_k for the case of uniform seismic input vanishes, that for the case of non uniform seismic input does not. The way to proceed in this case would be as follows:

$$\text{Let} \quad S_k(\omega) = | H_k(\omega) |^2 S_{\ddot{u}_0}(\omega) \tag{3.2.1.63}$$

which represents the spectral density function of y_k for the case of uniform seismic input, and for a participation factor γ_k equal to unity. Alternatively, y_k would be such that:

$$\ddot{y}_k + 2\beta_k \omega_k \dot{y}_k + \omega_k^2 y_k = -\ddot{u}_0 \tag{3.2.1.64}$$

Hence,

$$S_{y_k}^p(\omega) = \left[\sum_{i=1}^{n} A_{ki}^2 + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} A_{ki} A_{kj} \rho_{ij} \right] S_k(\omega) \tag{3.2.1.65}$$

Following the procedure presented earlier, one gets:

$$\overset{p}{Mean}(\max(y_k)) = \text{Mean}(\max(y_k)) \left[\sum_{i=1}^{n} A_{ki}^2 + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} A_{ki} A_{kj} \rho_{ijk} \right]^{1/2} \tag{3.2.1.66}$$

$$\sigma^p(\max(y_k)) = \sigma(\max(y_k)) \left[\sum_{i=1}^{n} A_{ki}^2 + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} A_{ki} A_{kj} \rho_{ijk} \right]^{1/2} \tag{3.2.1.67}$$

where:

$$\overset{p}{Mean}(\max(y_k)) = 1 \cdot R^p(\omega_k, \beta_k) \tag{3.2.1.68}$$

$$\text{Mean}(\max(y_k)) = 1 \cdot R(\omega_k, \beta_k) \tag{3.2.1.69}$$

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$$\sigma^p(\max(y_k)) = \sigma(R^p(\omega_k, \beta_k)) \qquad (3.2.1.70)$$

$$\sigma(\max(y_k)) = \sigma(R(\omega_k, \beta_k)) \qquad (3.2.1.71)$$

Therefore, the modified response spectrum model is given by:

$$R^p(\omega_k, \beta_k) = R(\omega_k, \beta_k) \left[\sum_{i=1}^{n} A_{ki}^2 + 2\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} A_{ki} A_{kj} \rho_{ijk} \right]^{1/2} \qquad (3.2.1.72)$$

And the standard deviation to be accounted for is given by:

$$\sigma(R^p(\omega_k, \beta_k)) = \sigma(R(\omega_k, \beta_k)) \left[\sum_{i=1}^{n} A_{ki}^2 + 2\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} A_{ki} A_{kj} \rho_{ijk} \right]^{1/2} \qquad (3.2.1.73)$$

Notice that (3.2.1.63) and (3.2.1.65) could have been used from the beginning of the development, thereby avoiding the issue of mode participation. It has not been done so, however, so as to put into focus the interesting issue of having a mode which does not participate to the structural response under uniform seismic excitations, but which does participate to it under non uniform seismic excitations. This issue will be illustrated in the last part of this chapter.

As a closing statement for this section, it is worth mentioning that the developed procedure enables the use of the mode superposition method in combination with a modified response spectrum model to treat the problem of multiply-supported structures subjected to imperfectly correlated seismic excitations.

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3.2.2 Case of Multicomponent Ground Motion

Having presented the case of a single ground motion component in the previous section, let's extend the formulation to the case of multicomponent ground motion. The assumption of uncorrelated (statistically independent) ground motion components, which is routinely made in practice, will also be followed in the present development. The reason for adopting such an assumption, is that accounting for statistical dependence would not necessarily improve the quality of the formulation. On the contrary, it would require a more involved work, throughout which many unresolved questions would arise, that would necessitate making supplementary assumptions, whose underlying justification would be more mathematical convenience than physical evidence.

From an organizational viewpoint, the present section will be very similar to the previous one. For the sake of brevity, results arrived at in the previous section and needed in the present one will merely be used as such, without redevelopment.

The coupled equations of motion of a linear, three-dimensional, lumped mass, multi-degree of freedom, multiply-supported structural system subjected to uniform (or non uniform), three-dimensional translational seismic excitations can be expressed in matrix form by means of (3.2.1.1):

$$
\begin{bmatrix} M_s & 0 \\ 0 & M_b \end{bmatrix} \begin{bmatrix} \ddot{U}_s \\ \ddot{U}_b \end{bmatrix} + \begin{bmatrix} C_s & C_{sb} \\ C_{bs} & C_b \end{bmatrix} \begin{bmatrix} \dot{U}_s \\ \dot{U}_b \end{bmatrix} + \begin{bmatrix} K_s & K_{sb} \\ K_{bs} & K_b \end{bmatrix} \begin{bmatrix} U_s \\ U_b \end{bmatrix} = \begin{bmatrix} 0 \\ P_b \end{bmatrix} \quad (3.2.1.1)
$$

All parameters have already been defined. The advantage of the above equation is

that it is general enough to encompass different situations (uniform seismic input, non uniform seismic input, unidimensional seismic input, three-dimensional seismic input). Clearly though, the size and the structure of the parameters will accordingly vary from one situation to another.

Let's first examine the case of uniform seismic input (fully correlated seismic excitations). The equations of motion for the structural part are given by (3.2.1.5), in which the parameters are adjusted to the three-dimensional case:

$$M_s \ddot{V}_s + C_s \dot{V}_s + K_s V_s = -M_s T_s \ddot{u}_0 \qquad (3.2.1.5)$$

The intermediate steps between (3.2.1.1) and (3.2.1.5) have been purposely skipped:

$$\ddot{u}_0 = [\ddot{u}_{01} \quad \ddot{u}_{02} \quad \ddot{u}_{03}]^T \qquad (3.2.2.1)$$

where:

$\ddot{u}_{01}$ and $\ddot{u}_{02}$ are the ground accelerations for the two horizontal components.

$\ddot{u}_{03}$ is the ground acceleration for the vertical component.

Assuming that the structure is classically damped, and using modal coordinates, one gets:

$$\begin{cases} (\Phi^T M_s \Phi)\ddot{Y} + (\Phi^T C_s \Phi)\dot{Y} + (\Phi^T K_s \Phi)Y = -\Phi^T M_s T_s \ddot{u}_0 \\ \\ V_s = \Phi Y \end{cases} \qquad (3.2.2.2)$$

where all parameters have previously been defined. From (3.2.2.2), the kth modal equation can be written as follows:

$$\ddot{y}_k + 2\beta_k\omega_k\dot{y}_k + \omega_k^2 y_k = -\gamma_{k_1}\ddot{u}_{01} - \gamma_{k_2}\ddot{u}_{02} - \gamma_{k_3}\ddot{u}_{03} \tag{3.2.2.3}$$

where γ_{kq} is the participation factor for mode k associated with the qth ground motion component.

$k = 1, 2, ...,$ total number of modes.

$q = 1, 2, 3.$

$$\gamma_{kq} = \frac{\phi_k^T M_s T_{sq}}{\phi_k^T M_s \phi_k} \tag{3.2.2.4}$$

where T_{sq} is such that:

$$T_s = [T_{s1} \ \ T_{s2} \ \ T_{s3}] \tag{3.2.2.5}$$

(3.2.2.5) expresses that the influence matrix T_s is the assemblage of three vectors, $T_{sq}(q = 1, 2, 3)$, each one relative to a ground motion component.

Let $R_q(\omega, \beta)$ $(q = 1, 2, 3)$ be the design response spectrum corresponding to the qth ground motion component at the site of interest. If the ground motion components are uncorrelated, then it can be shown that the mean value of the maximum modal response is approximately given by the SRSS (square root of sum of square) combination of the three component contributions:

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$$| y_k |_{max} = \sqrt{\sum_{q=1}^{3} \gamma_{kq}^2 R_q^2(\omega_k, \beta_k)} \qquad (3.2.2.6)$$

$| y_k |_{max}$ is in fact the mean value of the maximum modal response since R_q's give mean values.

The spectral density function of y_k is given by:

$$S_{y_k}(\omega) = | H_k(\omega) |^2 S_{\ddot{s}_k}(\omega) \qquad (3.2.2.7)$$

where:

$$\ddot{s}_k(t) = \sum_{q=1}^{3} \gamma_{kq} \ddot{u}_{0q}(t) \qquad (3.2.2.8)$$

Since the ground motion components are uncorrelated, $S_{\ddot{s}_k}(\omega)$ will be given by:

$$S_{\ddot{s}_k}(\omega) = \sum_{q=1}^{3} \gamma_{kq}^2 S_{\ddot{u}_{0q}}(\omega) \qquad (3.2.2.9)$$

Combining (3.2.2.9) and (3.2.2.7), one gets:

$$S_{y_k}(\omega) = \sum_{q=1}^{3} \gamma_{kq}^2 | H_k(\omega) |^2 S_{\ddot{u}_{0q}}(\omega) \qquad (3.2.2.10)$$

(3.2.2.10) may be used to derive (3.2.2.6). Indeed, let's express the one-sided spectral density function of y_k, $G_{y_k}(\omega)$:

$$G_{y_k}(\omega) = \sum_{q=1}^{3} \gamma_{kq}^2 | H_k(\omega) |^2 G_{\ddot{u}_{0q}}(\omega) \qquad (3.2.2.11)$$

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where:

$G_{\ddot{u}_{0q}}(\omega)$ is the one-sided spectral density function of $\ddot{u}_{0q}(t)$.

Let's relate the zeroth spectral moments:

$$\lambda_{0k} = \sum_{q=1}^{3} \lambda_{0kq} \qquad (3.2.2.12)$$

where:

$$\lambda_{0k} = \int_{0}^{\infty} G_{y_k}(\omega) d\omega \qquad (3.2.2.13)$$

$$\lambda_{0kq} = \int_{0}^{\infty} \gamma_{kq}^2 \mid H_k(\omega) \mid^2 G_{\ddot{u}_{0q}}(\omega) d\omega \qquad (3.2.2.14)$$

Note that:

$$\lambda_{0k} = \left(\frac{\mid y_k \mid_{max}}{p_k} \right)^2 \qquad (3.2.2.15)$$

$$\lambda_{0kq} = \left(\frac{\gamma_{kq} R_q(\omega_k, \beta_k)}{p_{kq}} \right)^2 \qquad (3.2.2.16)$$

where p_k and p_{kq} are peak factors. For practical purposes, as described in [14] and [42], the peak factors will be considered approximately equal. Therefore, by substituting (3.2.1.15) and (3.2.1.16) into (3.2.1.12), one gets:

$$\mid y_k \mid_{max} = \sqrt{\sum_{q=1}^{3} \gamma_{kq}^2 R_q^2(\omega_k, \beta_k)} = \text{Mean}(\max(y_k))$$

which is (3.2.2.6).

Having presented the case of a three-dimensional uniform seismic input, whose three translational components are uncorrelated, let's examine the case of a non uniform seismic input (partially correlated seismic excitations). The assumption of statistically independent ground motion components is, of course, retained.

The equations of motion for the structural part are given by (3.2.1.16), in which the parameters are adjusted to the three-dimensional case:

$$M_s \ddot{V}_s + C_s \dot{V}_s + K_s V_s = M_s K_s^{-1} K_{sb} \ddot{U}_b \qquad (3.2.2.16)$$

$\ddot{U}_b$ is a vector containing support accelerations in three directions.

$$\ddot{U}_b = [\ddot{u}_{qi}]^T \qquad (3.2.2.17)$$

where:

$q = 1, 2, 3$ refers to the ground motion component.

$i = 1, 2, ..., n$ refers to the support point.

Assuming classical damping, and using modal coordinates, one gets:

$$\begin{cases} (\Phi^T M_s \Phi)\ddot{Y} + (\Phi^T C_s \Phi)\dot{Y} + (\Phi^T K_s \Phi)Y = \Phi^T M_s K_s^{-1} K_{sb} \ddot{U}_b \\ V_s = \Phi Y \end{cases} \qquad (3.2.2.18)$$

From (3.2.2.18), the kth modal equation can be written as :

$$\ddot{y}_k + 2\beta_k \omega_k \dot{y}_k + \omega_k^2 y_k = \sum_{q=1}^{3}\sum_{i=1}^{n} A_{kqi} \ddot{u}_{qi} \qquad (3.2.2.19)$$

where k indicates the mode, q the ground motion component, and i the support point. Notice that if $\ddot{u}_{qi} = \ddot{u}_{0q}$ (uniform seismic input), then by comparing (3.2.2.3) and (3.2.2.19), one gets:

$$\gamma_{kq} = -\sum_{i=1}^{n} A_{kqi} \qquad (3.2.2.20)$$

Because of the non uniform nature of the seismic loading, (3.2.2.19) cannot be solved by means of the design response spectrum, as was the case for (3.2.2.3) (uniform seismic input). Hence, let's characterize y_k in a stochastic sense by expressing its spectral density function:

$$S_{y_k}^p(\omega) = \mid H_k(\omega) \mid^2 S_{\ddot{s}_k}(\omega) \qquad (3.2.2.21)$$

where:

$$\ddot{s}_k(t) = \sum_{q=1}^{3}\sum_{i=1}^{n} A_{kqi}\ddot{u}_{qi}(t) \qquad (3.2.2.22)$$

Notice that $\ddot{s}_k(t)$ in (3.2.2.8) is different form $\ddot{s}_k(t)$ in (3.2.2.22).

Let's express $S_{\ddot{s}_k}(\omega)$ in terms of auto-spectra and cross-spectra of ground acceleration at the different supports and for the different ground motion components. Let's first do the same work for the auto-correlation function $R_{\ddot{s}_k}(\tau)$.

$$R_{\ddot{s}_k}(\tau) \;=\; E[\ddot{s}_k(t)\ddot{s}_k(t+\tau)]$$

$$= \; E\left[(\sum_{q=1}^{3}\sum_{i=1}^{n}A_{kqi}\ddot{u}_{qi}(t))(\sum_{q=1}^{3}\sum_{i=1}^{n}A_{kqi}\ddot{u}_{qi}(t+\tau))\right]$$

$$= \; E\left[\sum_{q=1}^{3}\sum_{r=1}^{3}\sum_{i=1}^{n}\sum_{j=1}^{n}A_{kqi}A_{krj}\ddot{u}_{qi}(t)\ddot{u}_{rj}(t+\tau)\right]$$

By exchanging expectation and summation, one gets:

$$R_{\ddot{s}_k}(\tau) = \sum_{q=1}^{3}\sum_{r=1}^{3}\sum_{i=1}^{n}\sum_{j=1}^{n}A_{kqi}A_{krj}E\left[\ddot{u}_{qi}(t)\ddot{u}_{rj}(t+\tau)\right]$$

Since the different components are uncorrelated, $R_{\ddot{s}_k}(\tau)$ will reduce

$$R_{\ddot{s}_k}(\tau) = \sum_{q=1}^{3}\sum_{i=1}^{n}\sum_{j=1}^{n}A_{kqi}A_{krj}R_{\ddot{u}_{qi}\ddot{u}_{qj}}(\tau) \qquad (3.2.2.23)$$

By taking Fourier transforms in (3.2.2.23), one gets:

$$S_{\ddot{s}_k}(\omega) = \sum_{q=1}^{3}\sum_{i=1}^{n}\sum_{j=1}^{n}A_{kqi}A_{kqj}S_{\ddot{u}_{qi}\ddot{u}_{qj}}(\omega) \qquad (3.2.2.24)$$

Let's make use of the following equations:

$$S_{\ddot{u}_{qi}\ddot{u}_{qj}}(\omega) = S^{*}_{\ddot{u}_{qj}\ddot{u}_{qi}}(\omega) \qquad (3.2.2.25)$$

$$S_{\ddot{u}_{qi}\ddot{u}_{qj}}(\omega) = r_{qij}S_{\ddot{u}_{0q}}(\omega) \qquad (3.2.2.26)$$

Where:

* indicates complex conjugate.

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r_{qij} is the coherency function of the seismic motion for component q (along axis q) between stations i and j. It will be assumed that for component q, there will be a loss of correlation along axis q only. Hence, depending upon the location of the support points i and j there may or may not be a loss of correlation so far as component q is concerned.

$S_{\ddot{u}_{0q}}(\omega)$ is the autospectrum of the ground acceleration corresponding to component q, and common to all locations.

By virtue of (3.2.2.25) and (3.2.2.26), (3.2.2.24) can be rewritten as:

$$S_{\ddot{s}_k}(\omega) = \sum_{q=1}^{3}\left[\sum_{i=1}^{n}A_{kqi}^2 S_{\ddot{u}_{0q}}(\omega) + \sum_{i=1}^{n}\sum_{\substack{j=1\\j\neq i}}^{n}A_{kqi}A_{kqj}S_{\ddot{u}_{qi}\ddot{u}_{qj}}(\omega)\right]$$

$$= \sum_{q=1}^{3}\left[\sum_{i=1}^{n}A_{kqi}^2 S_{\ddot{u}_{0q}}(\omega) + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}A_{kqi}A_{kqj}\mathrm{Real}(S_{\ddot{u}_{qi}\ddot{u}_{qj}}(\omega))\right]$$

$$= \sum_{q=1}^{3}\left[\sum_{i=1}^{n}A_{kqi}^2 + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}A_{kqi}A_{kqj}\rho_{qij}\right]S_{\ddot{u}_{0q}}(\omega)$$

where ρ_{qij} is the frequency dependent spatial correlation coefficient. It is the real part of the coherency function r_{qij}.

Hence,

$$S_{\ddot{s}_k}(\omega) = \sum_{q=1}^{3}\left[\sum_{i=1}^{n}A_{kqi}^2 + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}A_{kqi}A_{kqj}\rho_{qij}\right]S_{\ddot{u}_{0q}}(\omega) \qquad (3.2.2.27)$$

Therefore, $S_{y_k}^p(\omega)$ can be written as follows:

$$\left.\begin{array}{c}(3.2.2.21)\\[2mm](3.2.2.27)\end{array}\right\} \Longrightarrow$$

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$$S_{y_k}^p(\omega) = \sum_{q=1}^{3} \mid H_k(\omega) \mid^2 \left[\sum_{i=1}^{n} A_{kqi}^2 + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} A_{kqi} A_{kqj} \rho_{qij} \right] S_{\ddot{u}_{0q}}(\omega) \qquad (3.2.2.28)$$

It is appropriate at this point to recall (3.2.2.10), which expresses the spectral density function of y_k for the case of uniform seismic input:

$$S_{y_k}(\omega) = \sum_{q=1}^{3} \gamma_{kq}^2 \mid H_k(\omega) \mid^2 S_{\ddot{u}_{0q}}(\omega) \qquad (3.2.2.10)$$

By comparing (3.2.2.28) and (3.2.2.10), one can observe that both $S_{y_k}^p(\omega)$ and $S_{y_k}(\omega)$ can be written in an analogous way:

$$S_{y_k}^p(\omega) = \sum_{q=1}^{3} S_{y_{kq}}^p(\omega) \qquad (3.2.2.29)$$

$$S_{y_k}(\omega) = \sum_{q=1}^{3} S_{y_{kq}}(\omega) \qquad (3.2.2.30)$$

where:

$$S_{y_{kq}}^p = \mid H_k(\omega) \mid^2 \left[\sum_{i=1}^{n} A_{kqi}^2 + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} A_{kqi} A_{kqj} \rho_{qij} \right] S_{\ddot{u}_{0q}}(\omega) \qquad (3.2.2.31)$$

$$S_{y_{kq}} = \gamma_{kq}^2 \mid H_k(\omega) \mid^2 S_{\ddot{u}_{0q}}(\omega) \qquad (3.2.2.32)$$

Hence, the contribution from component q to the spectral density furnciton of y_k for the case of partially correlated seismic excitations can be expressed in terms of its homologous for the case of fully correlated seismic excitations as follows:

$$S_{y_{kq}}^p(\omega) = \frac{1}{\gamma_{kq}^2}\left[\sum_{i=1}^{n}A_{kqi}^2 + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}A_{kqi}A_{kqj}\rho_{qij}\right]S_{y_{kq}}(\omega) \qquad (3.2.2.33)$$

which is the equation that was arrived at for the case of a single ground motion component (3.2.1.28). Therefore, based on the results reached in the previous section, the mean values of the maximum modal responses due to component q, for the cases of partially and fully correlated seismic excitations are related as follows:

$$\overset{p}{Mean}(\max(y_{kq})) = \text{Mean}(\max(y_{kq}))\frac{1}{\mid \gamma_{kq}\mid}\left[\sum_{i=1}^{n}A_{kqi}^2 + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}A_{kqi}A_{kqj}\rho_{qijk}\right]^{1/2}$$

$$(3.2.2.34)$$

where:

$$\rho_{qijk} = \frac{\int_0^\infty \rho_{qij}G_{y_{kq}}(\omega)d\omega}{\int_0^\infty G_{y_{kq}}(\omega)d\omega} \qquad (3.2.2.35)$$

$$G_{y_{kq}}(\omega) = \gamma_{kq}^2\mid H_k(\omega)\mid^2 G_{\ddot{u}_{0q}}(\omega) \qquad (3.2.2.36)$$

ρ_{qijk} represents the spatial correlation (cross-correlation) coefficient between the relative responses of two identical oscillators (ω_k, β_k) respectively subjected to $\ddot{u}_{qi}(t)$ and $\ddot{u}_{qj}(t)$.

Proceeding in an analogous way to that of the previous section, a modified response spectrum which accounts for spatial correlation of seismic motions for component q can be expressed:

$$R_q^p(\omega_k, \beta_k) = R_q(\omega_k, \beta_k) \left[\sum_{i=1}^{n} A_{kqi}^2 + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} A_{kqi} A_{kqj} \rho_{qijk} \right]^{1/2} \frac{1}{|\gamma_{kq}|} \qquad (3.2.2.37)$$

R_q^p is the mean value of the maximum modal response (for a participation factor equal to unity) due to ground motion component q accounting for non uniform seismic excitations. R_q is the homologous value for the case of uniform seismic excitations. R_q is assumed to be given.

At present, let's express the maximum modal value of the response in terms of the maximum modal values due to each ground motion component.

$$(3.2.2.29) \implies G_{y_k}^p(\omega) = \sum_{q=1}^{3} G_{y_{kq}}^p(\omega) \qquad (3.2.2.38)$$

Where $G_{y_k}^p(\omega)$ and $G_{y_{kq}}^p(\omega)$ are one-sided spectral density functions.

By going to zeroth spectral moments, one gets:

$$\lambda_{0k}^p = \sum_{q=1}^{3} \lambda_{0kq}^p \qquad (3.2.2.39)$$

where:

$$\lambda_{0k}^p = \int_0^\infty G_{y_k}^p(\omega) d\omega \qquad (3.2.2.40)$$

$$\lambda_{0kq}^p = \int_0^\infty G_{y_{kq}}^p(\omega) d\omega \qquad (3.2.2.41)$$

Note that:

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$$\lambda_{0k}^p = \left(\frac{\overset{p}{M}ean(\max(y_k))}{p_k}\right)^2 \tag{3.2.2.42}$$

$$\lambda_{0kq}^p = \left(\frac{\gamma_{kq} R_q^p(\omega_k, \beta_k)}{p_{kq}}\right)^2 \tag{3.2.2.43}$$

By virtue of a previous argument on peak factors, and by substituting (3.2.2.42) and

(3.2.2.43) into (3.2.2.39), one gets:

$$\overset{p}{M}ean(\max(y_k)) = \sqrt{\sum_{q=1}^{3} \gamma_{kq}^2 (R_q^p(\omega_k, \beta_k))^2} \tag{3.2.2.44}$$

Let's compare (3.2.2.44) with (3.2.2.6):

$$Mean(\max(y_k)) = \sqrt{\sum_{q=1}^{3} \gamma_{kq}^2 R_q^2(\omega_k, \beta_k)} \tag{3.2.2.6}$$

As can be observed form the two previous equations, a procedure has been proposed,

based on a modified response spectrum model, which enables the computation of

the mean value of the maximum modal response for the case of non-uniform seismic

input. The way the mean value of the maximum modal response is expressed is exactly

analogous to the way it would be expressed had the seismic input been uniform. The

only difference is that the "classical" response spectrum is replaced by a modified

response spectrum model which accounts for the spatial character of the seismic input.

It is worth mentioning that, for practical purposes, providing a combination rule for

the case of non uniform seismic motions which is exactly analogous to that for the case

of uniform seismic motions is very important. Analogously to the previous section,

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the standard deviation to be accounted for in the modified response spectrum model, relative to component q, is given by:

$$\sigma(R_q^p(\omega_k,\beta_k)) = \sigma(R_q(\omega_k,\beta_k)) \left[\sum_{i=1}^{n} A_{kqi}^2 + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} A_{kqi} A_{kqj} \rho_{qijk}\right]^{1/2} \frac{1}{\mid \gamma_{kq} \mid} \qquad (3.2.2.45)$$

where $\sigma(R_q^p)$ and $\sigma(R_q)$ are the standard deviations to be taken into account for the cases of non uniform and uniform seismic input, respectively.

Finally, the standard deviation of the maximum modal response y_k will be expressed as:

$$\sigma^p(\max(y_k)) = \sqrt{\sum_{q=1}^{3}\gamma_{kq}^2 \left[\sigma(R_q^p(\omega_k,\beta_k)\right]^2} \qquad (3.2.2.46)$$

As a last remark, let us note that the case of no participation of a given mode under a given component of uniform seismic motion, is handled in exactly the same way, based on the results of the previous section. Namely,

$$R_q^p(\omega_k,\beta_k) = R_q(\omega_k,\beta_k) \left[\sum_{i=1}^{n} A_{kqi}^2 + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} A_{kqi} A_{kqj} \rho_{qijk}\right]^{1/2} \qquad (3.2.2.47)$$

$$\sigma(R_q^p(\omega_k,\beta_k)) = \sigma(R_q(\omega_k,\beta_k)) \left[\sum_{i=1}^{n} A_{kqi}^2 + 2\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} A_{kqi} A_{kqj} \rho_{qijk}\right]^{1/2} \qquad (3.2.2.48)$$

$$\overset{p}{Mean}(\max(y_{kq})) = R_q^p(\omega_k,\beta_k) \qquad (3.2.2.49)$$

$$\sigma^p(\max(y_{kq})) = \sigma(R_q^p(\omega_k, \beta_k)) \qquad (3.2.2.50)$$

The combination of the different component contributions is performed as already presented.

3.2.3 Application to Shear Buildings Subjected to a Single Ground Motion Component

The purpose of the present section is to illustrate the use of the procedure developed earlier, through the relatively easy case of frame structures behaving as shear beams (shear buildings), subjected to a single ground motion component. If the seismic excitations at the support points are fully correlated (i.e. no loss of coherence and no phase delay), then the maximum modal responses can be obtained form the design response spectrum $R(\omega, \beta)$, given at the site of interest. However, if the seismic excitations are imperfectly (partially) correlated, may it be due to a loss of coherence, a phase delay, or both, $R(\omega, \beta)$ is no longer the means by which the maximum modal responses can be obtained. A modified response spectrum model, which accounts for spatial variability of earthquake ground motions, has been derived. It has been expressed in terms of, the given design response spectrum $R(\omega, \beta)$, at the site of interest, and the given coherency characteristics of the seismic motion.

3.2.3.1 Single Bay Shear Buildings

Consider the case of a N-story single bay frame structure behaving as a shear

beam (rigid girders, axial deformations in columns neglected), subjected to a single horizontal component of partially correlated ground motions (fig. 3.1). For the sake of simplicity, column stiffnesses will be assumed equal.
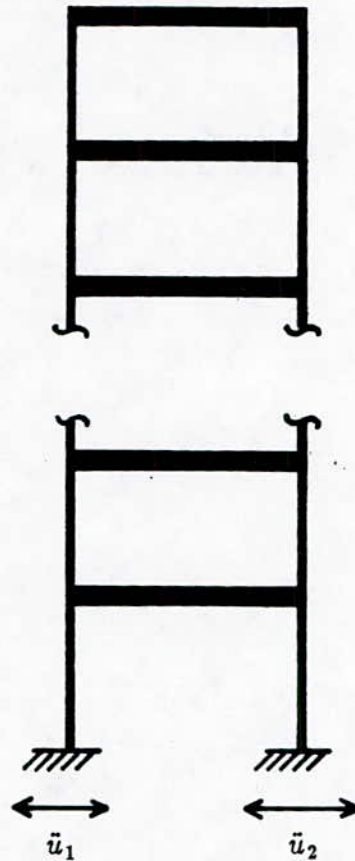
Figure 3.1

The dynamic equations for the N degree-of-freedom structural part can be written based on (3.2.1.16).

Namely,

$$M_s \ddot{V}_s + C_s \dot{V}_s + K_s V_s = M_s K_s^{-1} K_{sb} \ddot{U}_b \qquad (3.2.1.16)$$

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All parameters retain the same definition as in earlier parts. Because of the column stiffnesses, purposely chosen to be equal, (3.2.1.16) reduces for the present case to the following:

$$M_s\ddot{V}_s + C_s\dot{V}_s + K_sV_s = -M_sE\ddot{\bar{s}} \qquad (3.2.3.1.1)$$

where:

$E^T = [11\cdots 1]$ is an influence vector.

$\ddot{\bar{s}}(t) = \dfrac{1}{2}(\ddot{u}_1(t) + \ddot{u}_2(t))$.

Assuming that the system is classically damped, and using modal coordinates, one gets:

$$\begin{cases} (\Phi^T M_s\Phi)\ddot{Y} + (\Phi^T C_s\Phi)\dot{Y} + (\Phi^T K_s\Phi)Y = -\Phi^T M_sE\ddot{\bar{s}} \\ V_s = \Phi Y \end{cases} \qquad (3.2.3.1.2)$$

From which the kth modal equation can be written as:

$$\ddot{y}_k + 2\beta_k\omega_k\dot{y}_k + \omega_k^2y_k = -\gamma_k\ddot{\bar{s}}(t) \qquad (3.2.3.1.3)$$

where γ_k is the participation factor for mode k.

$$\gamma_k = \frac{\phi_k^T M_s E}{\phi_k^T M_s \phi_k}$$

The previous modal equation could be rewritten as:

$$\ddot{y}_k + 2\beta_k\omega_k\dot{y}_k + \omega_k^2y_k = -\frac{\gamma_k}{2}\ddot{u}_1 - \frac{\gamma_k}{2}\ddot{u}_2 \qquad (3.2.3.1.4)$$

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which follows the same pattern as the general modal equation (3.2.1.19):

$$\ddot{y}_k + 2\beta_k\omega_k\dot{y}_k + \omega_k^2 y_k = \sum_{i=1}^{n} A_{ki}\ddot{u}_i \qquad (3.2.1.19)$$

Note that in the present case n (the number of support points) is equal to two. And that:

$$A_{k1} = -\frac{\gamma_k}{2} \qquad (3.2.3.1.5)$$

$$A_{k2} = -\frac{\gamma_k}{2} \qquad (3.2.3.1.6)$$

Note also that (3.2.1.30) is satisfied:

$$\gamma_k = -\sum_{i=1}^{n} A_{ki} \qquad (3.2.1.30)$$

Therefore, by applying (3.2.1.61) to this case, the modified response spectrum model can be expressed as follows:

$$R^p(\omega_k, \beta_k) = R(\omega_k, \beta_k)\left[\frac{1+\rho_{12k}}{2}\right]^{1/2} \qquad (3.2.3.1.7)$$

where:

ρ_{12} is the frequency-dependent spatial correlation between stations 1 and 2.

$$\rho_{12k} = \frac{\int_0^\infty \rho_{12}G_{y_k}(\omega)d\omega}{\int_0^\infty G_{y_k}(\omega)d\omega}$$

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$$G_{y_k}(\omega) = \gamma_k^2 \mid H_k(\omega) \mid^2 G_{\ddot{u}_0}$$

Hence, the maximum modal response (in the mean value sense) for equation (3.2.3.1.4) can be expressed as:

$$\overset{p}{M}ean(\max(y_k)) = \mid \gamma_k \mid R^p(\omega_k, \beta_k) \tag{3.2.3.1.8}$$

It can be observed that the expression of the maximum modal response for the case of partially correlated seismic motions is exactly analogous to that for the case of fully correlated seismic motions. The difference is that $R^p(\omega, \beta)$ has replaced $R(\omega, \beta)$.

Prior to ending the present case, let's examine the extreme situations for (3.2.3.1.7):

1. $\ddot{u}_1(t)$ and $\ddot{u}_2(t)$ are fully correlated (i.e. no loss of coherence and no phase delay, $\ddot{u}_1(t) = \ddot{u}_2(t)$), $\rho_{12} = \rho_{120} = 1$, and the relation between the response spectra reads as follows:

$$R^p(\omega, \beta) = R(\omega, \beta)$$

which is an expected result.

2. $\ddot{u}_1(t)$ and $\ddot{u}_2(t)$ are fully negatively correlated (a particular case of time-delayed excitations: no loss of coherence, and out-of-phase excitations, $\ddot{u}_1(t) = -\ddot{u}_2(t)$), $\rho_{12} = -1$ $(\rho_{120} = -1)$, and the relation between the response spectra shows that the response spectrum for the case of partially correlated excitations is null.

$$R^p(\omega, \beta) = 0$$

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In words, such a loading leads to no response for the given structural system. However, such a result can by no means be generalized to any single bay shear building in which the column stiffnesses would not be equal. Further, it is interesting to note that for the case at hand (single bay shear building with identical columns), the non uniform seismic loading could be decomposed into a symmetrical part (rigid body component) and an antisymmetrical part (non rigid body component) (fig. 3.2). And one would observe that the antisymmetrical part of the loading does not contribute to the structural response, although it does produce stresses in the members.
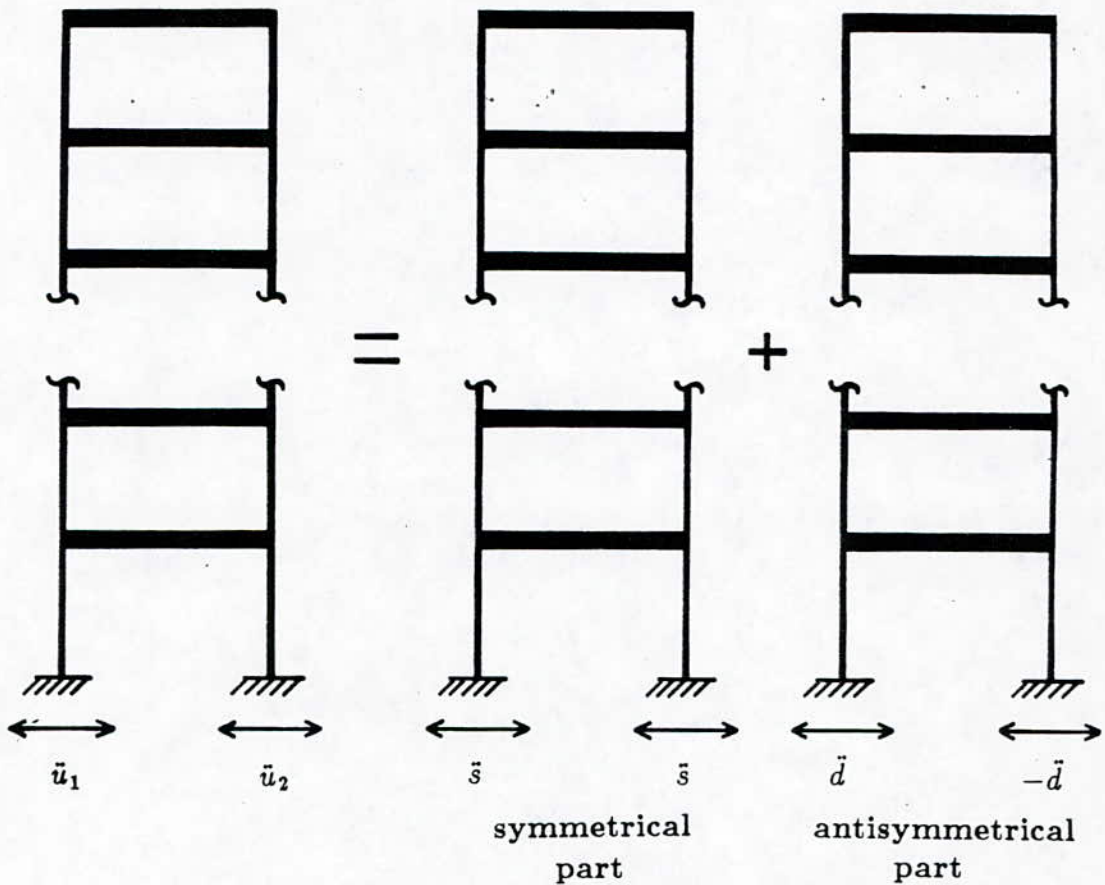
Figure 3.2

Let

$$\begin{cases} \ddot{s} = \dfrac{1}{2}(\ddot{u}_1 + \ddot{u}_2) \\ \ddot{d} = \dfrac{1}{2}(\ddot{u}_1 - \ddot{u}_2) \end{cases} \implies \begin{cases} \ddot{u}_1 = \ddot{s} + \ddot{d} \\ \ddot{u}_2 = \ddot{s} - \ddot{d} \end{cases}$$

Based on this argument, the equations of motion (3.2.3.1.1) could be derived without resorting to the general formulation. However, the above considerations are valid for the present case only, and are not applicable to any general situation.

3. $\ddot{u}_1(t)$ and $\ddot{u}_2(t)$ are fully uncorrelated (i.e. total loss of coherence), $\rho_{12} = 0$, $\rho_{12k} = 0$, and the relation between the response spectra reads as follows:

$$R^p(\omega, \beta) = R(\omega, \beta)\frac{\sqrt{2}}{2}$$

Finally, having examined the extreme situations, one can state that, for the case at hand, the structural response is overpredicted if the assumption of fully correlated excitations is made, while it is underpredicted under the assumption of fully negatively correlated excitations.

3.2.3.2 Multibay Shear Building

Consider the case of a N-story multibay frame structure behaving as a shear beam, subjected to a single horizontal component of partially correlated ground motions (fig. 3.3). For the sake of making the illustration simple and useful, column stiffnesses will be assumed equal.
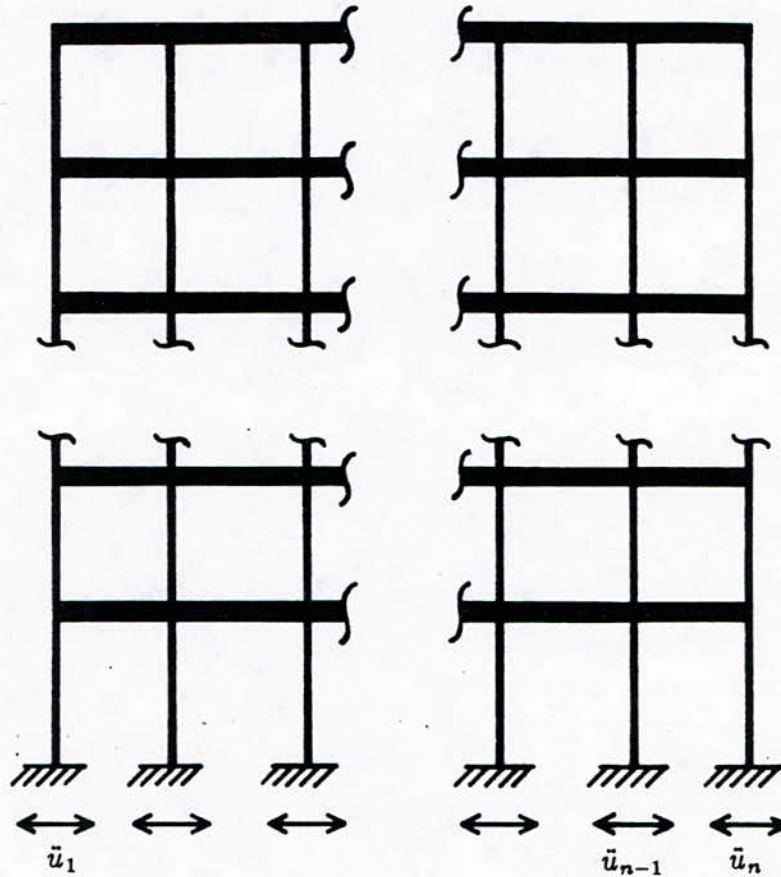
Figure 3.3

The equations of motion for the N degree-of-freedom structural part can be written based on (3.2.1.16):

$$M_s\ddot{V}_s + C_s\dot{V}_s + K_sV_s = M_sK_s^{-1}K_{sb}\ddot{U}_b \qquad (3.2.1.16)$$

Because of the stiffness properties of the structure, (3.2.1.16) reduces for the present case to the following:

$$M_s\ddot{V}_s + C_s\dot{V}_s + K_sV_s = -M_sE\ddot{s} \qquad (3.2.3.2.1)$$

where

$E^T = [1\ 1\ \cdots\ 1]$ is an influence vector.

$\ddot{s}(t) = \dfrac{1}{n}\sum\limits_{i=1}^{n}\ddot{u}_i(t)$ is the static response component.

i indicates the support point.

n is the total number of supports.

In the context of the general formulation, note that:

$$U_s^s = Es$$

which is the pseudo-static (also referred to as static) component of the structural displacement.

Assuming classical damping, and using modal coordinates, one can write the kth modal equation as:

$$\ddot{y}_k + 2\beta_k\omega_k\dot{y}_k + \omega_k^2 y_k = -\gamma_k\ddot{s}(t) \qquad (3.2.3.2.2)$$

where γ_k is the participation factor for mode k.

$$\gamma_k = \frac{\phi_k^T M_s E}{\phi_k^T M_s \phi_k}$$

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(3.2.3.2.2) could be rewritten as:

$$\ddot{y}_k + 2\beta_k \omega_k \dot{y}_k + \omega_k^2 y_k = \sum_{i=1}^{n} \left(\frac{-\gamma_k}{n} \right) \ddot{u}_i \qquad (3.2.3.2.3)$$

Hence, in the context of the general formulation:

$$A_{ki} = -\frac{1}{n}\gamma_k \qquad (3.2.3.2.4)$$

And

$$\gamma_k = -\sum_{i=1}^{n} A_{ki}$$

By applying (3.2.1.61) to this case, the modified response spectrum model can be expressed as follows:

$$R^p(\omega_k, \beta_k) = R(\omega_k, \beta_k) \left\{ \frac{1}{n}[1 + \frac{2}{n}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} \rho_{ijk}] \right\}^{1/2} \qquad (3.2.3.2.5)$$

$$\rho_{ijk} = \frac{\int_0^\infty \rho_{ij} \mid H_k(\omega) \mid^2 G_{\ddot{u}_0}(\omega)d\omega}{\int_0^\infty \mid H_k(\omega) \mid^2 G_{\ddot{u}_0}(\omega)d\omega}$$

ρ_{ij} is the frequency-dependent spatial correlation between stations i and j. Hence, the maximum modal response for equation (3.2.3.2.3) is given by:

$$\overset{p}{Mean}(\max(y_k)) = \mid \gamma_k \mid R^p(\omega_k, \beta_k) \qquad (3.2.3.2.6)$$

which is a similar equation to the case of fully correlated support motions, except for the fact that $R(\omega_k, \beta_k)$ replaces $R^p(\omega_k, \beta_k)$.

Finally, let's examine insightful extreme situations for (3.2.3.2.5):

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1. Support motions are fully correlated: All ρ_{ij}'s $= 1$, the response spectra relation reads as follows:

$$
\begin{aligned}
R^p(\omega, \beta) &= R(\omega, \beta) \left\{ \frac{1}{n} \left[1 + \frac{2}{n} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} 1 \right] \right\}^{1/2} \\
&= R(\omega, \beta) \left\{ \frac{1}{n} \left[1 + \frac{2}{n} C_n^2 \right] \right\}^{1/2} \\
&= R(\omega, \beta) \left[\frac{1}{n} + \frac{2}{n^2} \frac{n!}{(n-2)!2!} \right]^{1/2} \\
&= R(\omega, \beta) \left[\frac{1}{n} + \frac{n-1}{n} \right]^{1/2} = R(\omega, \beta)
\end{aligned}
$$

Hence, $R^p(\omega, \beta) = R(\omega, \beta)$, which is an expected result.

2. Support motions are uncorrelated: All ρ_{ij}'s $= 0$

$$
R^p(\omega, \beta) = R(\omega, \beta) \left\{ \frac{1}{n} [1 + \frac{2}{n} \cdot 0] \right\}^{1/2} = R(\omega, \beta) \sqrt{\frac{1}{n}}
$$

Hence,

$$
R^p(\omega, \beta) = R(\omega, \beta) \sqrt{\frac{1}{n}}
$$

3.3 Continuous Systems: Case of Bridges Subjected to a Single Ground Motion Component

The present section aims at extending the procedure developed for discrete systems to a particular case of elongated continuous systems, namely, that of bridges subjected to a single component of partially correlated seismic excitations. At first, the general formulation for a multispan bridge will be exposed. Then, an application of this formulation to the case of a single span bridge will be presented. Throughout this section, previously derived results will be used as such, without unnecessary redevelopments.

3.3.1 Multispan Bridges

Consider the case of a multispan bridge, modelled as a continous beam, subjected to horizontal transverse partially correlated seismic excitations (fig. 3.4). Note that for the purpose of the present formulation, the bridge could have been subjected to partially correlated vertical support motions as well.
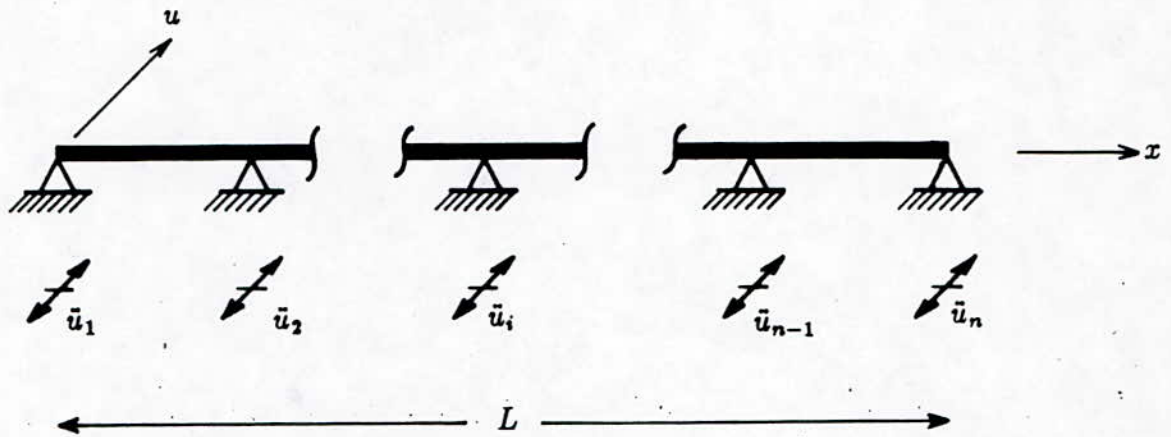
Figure 3.4

There are several ways of solving this problem. The modal superposition method combined with the response spectrum method is commonly used if the structure at hand is subjected to uniform translational excitations. However, if the seismic excitations are imperfectly correlated, then the use of the aforecited approach is invalidated. The aim of the present work, therefore, is to develop a modified response spectrum model, which essentially is an adjustment of the widely used response spectrum to the situation of spatially varying ground motions.

The equation of motion for a beam flexure situation with viscous damping is given by:

$$m\frac{\partial^2 u}{\partial t^2} + c\frac{\partial u}{\partial t} + EI\frac{\partial^4 u}{\partial x^4} = 0 \qquad (3.3.1.1)$$

where:

$u(x,t)$ is the total horizontal transverse displacement of the beam.

m is the mass per unit length assumed to be constant.

c is the viscous damping per unit length assumed to be constant. (It is therefore, implicitely assumed proportional)

EI is the flexural rigidity of the beam assumed to be constant.

Let's first examine the case of uniform support motions:

$$\ddot{u}_i(t) = \ddot{u}_0(t) \qquad i = 1,\cdots,n$$

The displacement of the beam can be decomposed as follows:

$$u(x,t) = u_0(t) + v(x,t) \qquad (3.3.1.2)$$

where:

$u_0(t)$ is a rigid body displacement of the beam, induced by identical support motions applied statically.

$v(x,t)$ is the relative component of the beam displacement.

Substituting (3.3.1.2) into (3.3.1.1), one gets:

$$m\frac{\partial^2 v}{\partial t^2} + c\frac{\partial v}{\partial t} + EI\frac{\partial^4 v}{\partial x^4} = m\frac{\partial^2 u_0}{\partial t^2} + c\frac{\partial u_0}{\partial t} - EI\frac{\partial^4 u_0}{\partial x^4} \qquad (3.3.1.3)$$

$-m\dfrac{\partial^2 u_0}{\partial t^2}$ is the essential part of the effective loading. Indeed, the damping term $c\dfrac{\partial u_0}{\partial t}$ is neglected for practical purposes, and the term $EI\dfrac{\partial^4 u_0}{\partial x^4}$ vanishes from considerations relative to the static case.

Hence, (3.3.1.3) reduces to:

$$m\frac{\partial^2 v}{\partial t^2} + c\frac{\partial v}{\partial t} + EI\frac{\partial^4 v}{\partial x^4} = -m\frac{\partial^2 u_0}{\partial t^2} \tag{3.3.1.4}$$

Let us expand the dynamic part of the displacement $v(x,t)$ in terms of modal coordinates:

$$v(x,t) = \sum_{j=1}^{\infty} \phi_j(x) y_j(t) \tag{3.3.1.5}$$

where:

$\phi_j(x)$ is the jth mode shpae of the structure.

$y_j(t)$ is the jth modal coordinate.

Substituting (3.3.1.5) inot (3.3.1.4), one gets:

$$m\sum_{j=1}^{\infty}\phi_j(x)\frac{d^2 y_j}{dt^2} + c\sum_{j=1}^{\infty}\phi_j(x)\frac{dy_j}{dt} + EI\sum_{j=1}^{\infty}\frac{d^4\phi_j(x)}{dx^4}y_j = -m\frac{\partial^2 u_0}{\partial t^2} \tag{3.3.1.6}$$

Multiplying through by $\phi_k(x)$, integrating over the length of the beam, and using orthogonality of mode shapes, leads to:

$$\left[m\int_0^L \phi_k^2 dx\right]\frac{d^2 y_k}{dt^2} + \left[c\int_0^L \phi_k^2 dx\right]\frac{dy_k}{dt} + \left[EI\int_0^L \frac{d^4\phi_k}{dx^4}\phi_k dx\right]y_k = -m\int_0^L \phi_k \ddot{u}_0 dx \tag{3.3.1.7}$$

(3.3.1.7) is in fact the kth modal equation, which can be rewritten as:

$$\ddot{y}_k + 2\beta_k\omega_k\dot{y}_k + \omega_k^2 y_k = -\gamma_k\ddot{u}_0 \tag{3.3.1.8}$$

where:

β_k is the viscous damping ratio for mode k, such that:

$$2\beta_k\omega_k = \frac{c}{m}$$

ω_k is the natural frequency for mode k, such that:

$$\omega_k^2 = \frac{EI}{m}\frac{d^4\phi_k/dx^4}{\phi_k}$$

γ_k is the participation factor for mode k, such that:

$$\gamma_k = \frac{\int_0^L \phi_k(x)dx}{\int_0^L \phi_k^2(x)dx}$$

The modal equation (3.3.1.8) could be solved by means of the response spectrum method. Indeed, if $R(\omega,\beta)$ is the design response spectrum given at the site of interest, then the mean value of the maximum modal response can be expressed by:

$$Mean\ (max(y_k)) = \mid \gamma_k \mid R(\omega_k, \beta_k) \qquad (3.3.1.9)$$

Alternatively, y_k could be characterized by its spectral density function given by:

$$S_{y_k}(\omega) = \gamma_k^2 \mid H_k(\omega) \mid^2 S_{\ddot{u}_0}(\omega) \qquad (3.3.1.10)$$

where all parameters have already been defined.

Having presented the case of uniform seismic motions, let's examine the more general situation of non uniform seismic motions. The equation of motion is also given

by (3.3.1.1). Similarly to the formulation for discrete systems, the displacement of the beam is decomposed by means of the following:

$$u(x,t) = u_s(x,t) + v(x,t) \qquad (3.3.1.11)$$

where:

$u_s(x,t)$ is the pseudo static displacement induced by the support motions applied statically.

$v(x,t)$ is the dynamic component of the displacement.

$u_s(x,t)$ can be expressed in terms of the support motions as follows:

$$u_s(x,t) = \sum_{i=1}^{n} h_i(x) u_i(t) \qquad (3.3.1.12)$$

where:

i indicates the support point.

n is the total number of supports.

$h_i(x)$ for $i = 1, \cdots, n$ are shape functions obtained form static beam deflection.

By virtue of previous arguments, the equation of motion (3.3.1.1) can be written as:

$$m\frac{\partial^2 v}{\partial t^2} + c\frac{\partial v}{\partial t} + EI\frac{\partial^4 v}{\partial x^4} = -m\frac{\partial^2 u_s}{\partial t^2} \qquad (3.3.1.13)$$

Similarly to the case of uniform seismic motion, modal decompostion of the dynamic component of the displacement can be made, and the kth modal equation reads as follows:

$$\ddot{y}_k + 2\beta_k\omega_k\dot{y}_k + \omega_k^2 y_k = -\frac{\int_0^L \phi_k(x)\ddot{u}_s dx}{\int_0^L \phi_k^2(x)dx} \tag{3.3.1.14}$$

where all parameters have already been defined. Let's substitute (3.3.1.12) into the right hand side of the above equation:

$$-\frac{\int_0^L \phi_k(x)\ddot{u}_s dx}{\int_0^L \phi_k^2(x)dx} = -\frac{\sum_{i=1}^n \int_0^L \phi_k(x)h_i(x)\ddot{u}_i(t)dx}{\int_0^L \phi_k^2(x)dx}$$

Let

$$A_{ki} = -\frac{\int_0^L \phi_k(x)h_i(x)dx}{\int_0^L \phi_k^2(x)dx} \tag{3.3.1.15}$$

Hence, by using (3.3.1.15), the kth modal equation can be rewritten as:

$$\ddot{y}_k + 2\beta_k\omega_k\dot{y}_k + \omega_k^2 y_k = \sum_{i=1}^n A_{ki}\ddot{u}_i \tag{3.3.1.16}$$

which has the same form as the equation arrived at for the case of discrete systems, (3.2.1.19). This equation, as mentioned in earlier sections, cannot be solved by means of the design response spectrum, which was used to solve the equation for the case of uniform seismic loading (3.3.1.8). It is important to note that, since the modal equation for uniform and non uniform seismic inputs for the present case have exactly the same form as those for the case of discrete systems, the modified response spectrum model for the former case will be a replica of that of the latter case, with the use of the relevant parameters of course. In other words, from (3.3.1.8) and (3.3.1.16), it can be shown that:

$$S^p_{y_k}(\omega) = \frac{1}{\gamma_k^2} \left[\sum_{i=1}^{n} A_{ki}^2 + 2\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} A_{ki} A_{kj} \rho_{ij} \right] S_{y_k}(\omega) \qquad (3.3.1.17)$$

which expresses the spectral density function of y_k for the case of partially correlated support motions in terms of its homologous for the case of fully correlated support motions. Notice that (3.3.1.17) is identical to (3.2.1.28), which expresses the above relation for the case of discrete systems. Therefore, by virtue of (3.2.1.61), the modified response spectrum model to be used for the response analysis of the multispan bridge subjected to multiple support excitations is given by:

$$R^p(\omega_k, \beta_k) = R(\omega_k, \beta_k) \left[\sum_{i=1}^{n} A_{ki}^2 + 2\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} A_{ki} A_{kj} \rho_{ijk} \right]^{1/2} \frac{1}{|\gamma_k|} \qquad (3.3.1.18)$$

Recall that:

$$\rho_{ijk} = \frac{\int_0^\infty \rho_{ij} \mid H_k(\omega) \mid^2 G_{\ddot{u}_0}(\omega) d\omega}{\int_0^\infty \mid H_k(\omega) \mid^2 G_{\ddot{u}_0}(\omega) d\omega}$$

A_{ki}'s and γ_k are parameters relative to the present case.

Likewise, based on (3.2.1.62), the standard deviation to be accounted for, in the modified response spectrum model is given by:

$$\sigma(R^p(\omega_k, \beta_k)) = \sigma(R(\omega_k, \beta_k)) \left[\sum_{i=1}^{n} A_{ki}^2 + 2\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} A_{ki} A_{kj} \rho_{ijk} \right]^{1/2} \frac{1}{|\gamma_k|} \qquad (3.3.1.19)$$

Note that (3.2.1.68), (3.2.1.69), ..., (3.2.1.73) are used to handle the situation where a mode under consideration does not contribute to the structural response due to a

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uniform seismic input.

In conclusion, a general modified response spectrum model has been arrived at. It can handle both, discrete systems and the case of continous system presented herein, for the seismic analysis under spatially varying ground motions.

3.3.2 Single Span Bridges

The present section aims at illustrating the use of the modified response spectrum model, through a relatively simple, but very insightful case study. Consider a single span extended bridge, modeled as a simple beam, and subjected to spatially varying horizontal transverse support motions (fig. 3.5).
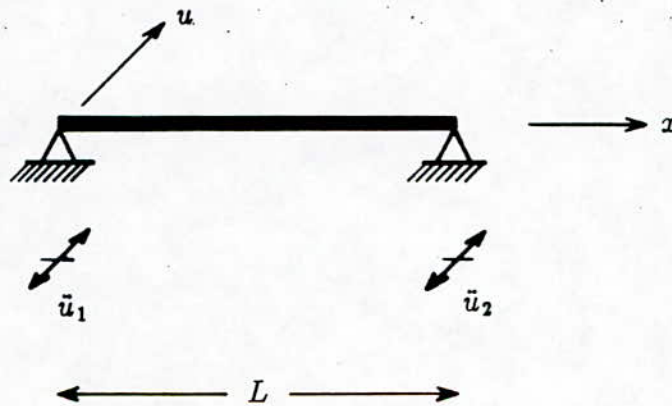
Figure 3.5

Since the structure at hand is not subjected to uniform translational excitations, the use of the modal superpostion method combined with the response spectrum method is invalidated. Let's, then, use the modified response spectrum model, which accounts for the spatial character of the seismic input, for the computation of modal responses.

The modes and frequencies of a simple beam in flexure are given by:

mode shapes:

$$\phi_k(x) = \sqrt{\frac{2}{L}} \sin \frac{k\pi x}{L} \qquad (3.3.2.1)$$

so that

$$\int_0^L \phi_k(x)\phi_l(x)dx = \delta_{kl} \qquad (3.3.2.2)$$

where δ_{kl} is Kroenecker symbol given by:

$$\delta_{kl} = \begin{cases} 1 & \text{if} k = 1 \\ \\ 0 & \text{if} k \neq l \end{cases}$$

frequencies:

$$\omega_k = k^2\pi^2\sqrt{\frac{EI}{mL^4}} \qquad (3.3.2.3)$$

where EI and m have already been defined.

The participation factor of a mode is given by:

$$\gamma_k = \frac{\int_0^L \phi_k(x)dx}{\int_0^L \phi_k^2(x)dx} = \int_0^L \phi_k(x)dx$$

using (3.3.2.1), one gets:

$$\gamma_k = \frac{\sqrt{2L}}{k\pi}[1 - (-1)^k] = \begin{cases} 0 & \text{if } k \text{ is even} \\ \\ 2\frac{\sqrt{2L}}{k\pi} & \text{if } k \text{ is odd} \end{cases} \qquad (3.3.2.4)$$

In other words, if k is even, which means that the mode under consideration is anti-symmetrical with respect to midspan, then this very mode will not contribute to the

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structural response under uniform support excitations. However, if k is odd, which means that the mode in question is symmetrical with respect to midspan, then there will be a contribution from this mode to the structural response under uniform support excitations.

The equation of motion is given by (3.3.1.13):

$$m\frac{\partial^2 v}{\partial t^2} + c\frac{\partial v}{\partial t} + EI\frac{\partial^4 v}{\partial x^4} = -m\frac{\partial^2 u_s}{\partial t^2} \qquad (3.3.1.13)$$

where:

$$u_s(x,t) = \sum_{i=1}^{2} h_i(x)u_i(t) \qquad (3.3.2.5)$$

$$h_i(x) = 1 - \frac{x}{L} \qquad (3.3.2.6)$$

$$h_2(x) = \frac{x}{L} \qquad (3.3.2.7)$$

Using modal coordinates, the kth modal equation can be written as:

$$\ddot{y}_k + 2\beta_k\omega_k\dot{y}_k + \omega_k^2 y_k = \sum_{i=1}^{2} A_{ki}\ddot{u}_i \qquad (3.3.2.8)$$

where, by virtue of (3.3.1.15), A_{k1} and A_{k2} are given by:

$$A_{k1} = -\frac{\sqrt{2L}}{k\pi} \qquad \forall k \qquad (3.3.2.9)$$

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$$A_{k2} = \begin{cases} -\dfrac{\sqrt{2L}}{k\pi} & \text{if } k \text{ is odd} \\ \dfrac{\sqrt{2L}}{k\pi} & \text{if } k \text{ is even} \end{cases} \qquad (3.3.2.10)$$

The last two equations confirm (3.3.2.4) since,

$$\gamma_k = -\sum_{i=1}^{2} A_{ki}$$

Hence, the kth modal equation can be rewritten as:

$$\ddot{y}_k + 2\beta_k\omega_k\dot{y}_k + \omega_k^2 y_k = -\frac{\sqrt{2L}}{k\pi} \begin{cases} \ddot{u}_1 + \ddot{u}_2 & \text{if } k \text{ is odd} \\ \ddot{u}_1 - \ddot{u}_2 & \text{if } k \text{ is even} \end{cases} \qquad (3.3.2.11)$$

And by virtue of earlier development, the modified response spectrum model will be given by:

$$R^p(\omega_k, \beta_k) = R(\omega_k, \beta_k) \begin{cases} \sqrt{\dfrac{1 + \rho_{12k}}{2}} & \text{if } k \text{ is odd} \\ \dfrac{2\sqrt{2L}}{k\pi}\sqrt{\dfrac{1 - \rho_{12k}}{2}} & \text{if } k \text{ is even} \end{cases} \qquad (3.3.2.12)$$

Hence,

$$\overset{p}{Mean}(\max(y_k)) = R^p(\omega_k, \beta_k) \begin{cases} \gamma_k & \text{if } k \text{ is odd} \\ 1 & \text{if } k \text{ is even} \end{cases} \qquad (3.3.2.13)$$

which can be rewritten as:

$$\overset{p}{Mean}(\max(y_k)) = R(\omega_k, \beta_k) \begin{cases} \dfrac{2\sqrt{2L}}{k\pi}\sqrt{\dfrac{1 + \rho_{12k}}{2}} & \text{if } k \text{ is odd} \\ \dfrac{2\sqrt{2L}}{k\pi}\sqrt{\dfrac{1 - \rho_{12k}}{2}} & \text{if } k \text{ is even} \end{cases} \qquad (3.3.2.14)$$

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Analogously,

$$\sigma^p(\max(y_k)) = \sigma(R(\omega_k, \beta_k)) \begin{cases} \dfrac{2\sqrt{2L}}{k\pi}\sqrt{\dfrac{1+\rho_{12k}}{2}} & \text{if } k \text{ is odd} \\[3mm] \dfrac{2\sqrt{2L}}{k\pi}\sqrt{\dfrac{1-\rho_{12k}}{2}} & \text{if } k \text{ is even} \end{cases} \qquad (3.3.2.15)$$

Therefore, the use of the modified response spectrum model for the computation of modal responses under non uniform seismic input has been illustrated.

Let's now examine this problem from another viewpoint so as to get more physical insight. Namely, let us decompose the seismic loading into a symmetrical part (rigid body component), and an antisymmetrical part (non rigid body component) (fig. 3.6).

symmetrical
part

antisymmetrical
part
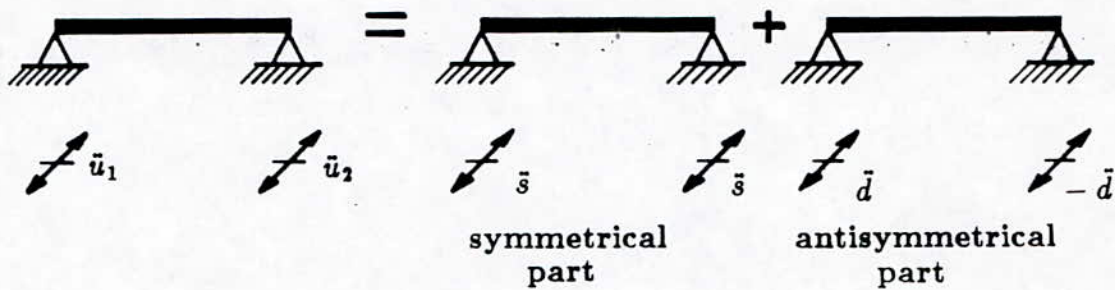
Figure 3.6

where:

$$\bar{s}(t) = \frac{1}{2}(\ddot{u}_1(t) + \ddot{u}_2(t)) \qquad (3.3.2.16)$$

$$\bar{d}(t) = \frac{1}{2}(\ddot{u}_1(t) - \ddot{u}_2(t)) \qquad (3.3.2.17)$$

In the present case, both parts of the loading will cause a structural response. In fact, in the context of a mode superposition analysis, the response at any arbitrary point receives contributions from both symmetrical and antisymmetrical modes (with respect

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to midspan). Symmetrical modes with respect to midspan will be exclusively excited by the symmetrical part of the loading, while antisymmetrical modes will be exclusively excited by the antisymmetrical part. Hence, in the case of partially correlated excitations, there will be a modal response at any given mode. It is worth mentioning at this point that such a fact would not be captured by the "classical" response spectrum method since it is based on the assumption that the seismic excitation is a uniform translation (full correlation), which inherently ignores the contribution of antisymmetrical modes to the total response.

Following the decomposition of the seismic loading, let's decompose any modal response as:

$$y_k = y_{k1} + y_{k2} \tag{3.3.2.18}$$

where:

y_k is the kth modal response.

y_{k1} is the contribution to the kth modal response from the symmetrical part of the loading.

y_{k2} is the contribution to the kth modal response from the antisymmetrical part of the loading.

Using (3.3.1.14) y_{k1} and y_{k2} will be such that:

$$\ddot{y}_{k1} + 2\beta_k\omega_k\dot{y}_{k1} + \omega_k^2 y_{k1} = -\left[\int_0^L \phi_k(x)dx\right]\ddot{s}(t) \tag{3.3.2.19}$$

$$\ddot{y}_{k2} + 2\beta_k\omega_k\dot{y}_{k2} + \omega_k^2 y_{k2} = -\left[\int_0^L \phi_k(x)\left(1 - \frac{2x}{L}\right)\right]\bar{d}(t) \qquad (3.3.2.20)$$

$$\int_0^L \phi_k(x)dx = \frac{\sqrt{2L}}{k\pi}[1 - (-1)^n] = \begin{cases} \dfrac{2\sqrt{2L}}{k\pi} & \text{if } k \text{ is odd} \\[2ex] 0 & \text{if } k \text{ is even} \end{cases} \qquad (3.3.2.4)$$

$$\int_0^L \phi_k(x)\left(1 - \frac{2x}{L}\right)dx = \frac{\sqrt{2L}}{k\pi}[1 + (-1)^k] = \begin{cases} 0 & \text{if } k \text{ is odd} \\[2ex] \dfrac{2\sqrt{2L}}{k\pi} & \text{if } k \text{ is even} \end{cases}$$

Therefore, y_{k1} and y_{k2} are mutually exclusive. Alternatively, if k is odd, the mode is symmetrical with respect to midspan, then:

$$y_k = y_{k1} \quad \text{and} \quad y_{k2} = 0$$

which means no contribution from the antisymmetrical part of the loading, which is expected. On the other hand, if k is even, the mode is antisymmetrical, then:

$$y_k = y_{k2} \quad \text{and} \quad y_{k1} = 0$$

which means no contribution from the symmetrical part of the loading, which is also expected. Therefore, the kth modal equation can be written as:

$$\ddot{y}_k + 2\beta_k\omega_k\dot{y}_k + \omega_k^2 y_k = -\frac{2\sqrt{2L}}{k\pi}\begin{cases} \bar{s}(t) & \text{if } k \text{ is odd} \\[2ex] \bar{d}(t) & \text{if } k \text{ is even} \end{cases} \qquad (3.3.2.21)$$

which is exactly identical to the equation (3.3.2.11), derived based on the general formulation.

Before closing this section, let's examine extreme situation for (3.3.2.14):

$$Mean^p(\max(y_k)) = 2\frac{\sqrt{2L}}{k\pi}R(\omega_k, \beta_k) \begin{cases} \sqrt{\dfrac{1+\rho_{12k}}{2}} & \text{if } k \text{ is odd} \\[2ex] \sqrt{\dfrac{1-\rho_{12k}}{2}} & \text{if } k \text{ is even} \end{cases} \tag{3.3.2.14}$$

1. $\tilde{u}_1(t)$ and $\tilde{u}_2(t)$ are identical (fully correlated), then $\rho_{12} = 1$ and $\rho_{12k} = 1$, and (3.3.2.14) reads as follows:

$$Mean^p(\max(y_k)) = 2\frac{\sqrt{2L}}{k\pi}R(\omega_k, \beta_k) \begin{cases} 1 & \text{if } k \text{ is odd} \\[2ex] 0 & \text{if } k \text{ is even} \end{cases}$$

which is an expected result.

2. $\tilde{u}_1(t)$ and $\tilde{u}_2(t)$ are fully negatively correlated, $\rho_{12} = -1$, $\rho_{12k} = -1$, and (3.3.2.14) becomes:

$$Mean^p(\max(y_k)) = 2\frac{\sqrt{2L}}{k\pi}R(\omega_k, \beta_k) \begin{cases} 0 & \text{if } k \text{ is odd} \\[2ex] 1 & \text{if } k \text{ is even} \end{cases}$$

which also is expected.

3. $\tilde{u}_1(t)$ and $\tilde{u}_2(t)$ are fully uncorrelated, $\rho_{12} = 0$, $\rho_{12k} = 0$, and (3.3.2.14) takes the form of:

$$Mean^p(\max(y_k)) = 2\frac{\sqrt{2L}}{k\pi}R(\omega_k, \beta_k) \begin{cases} \dfrac{\sqrt{2}}{2} & \text{if } k \text{ is odd} \\[2ex] \dfrac{\sqrt{2}}{2} & \text{if } k \text{ is even} \end{cases}$$

Chapter 4

Effect of Spatial Variation of Seismic Excitations
on Modal Cross-Correlation

4.1 Introduction

The modified response spectrum model developed in chapter 3 is a means for the computation of modal responses for the case of partially correlated seismic excitations. This model takes the form of an adjustment of the classical response spectrum, to the situation of non uniform seismic input. The computation of a physical (as opposed to modal) response of interest, should it be for the case of uniform or non uniform seismic excitations, is based on the combination of the classical or modified response spectrum model, with the modal superposition method. In practice, this computation procedure translates into the use of combination rules for maximum modal responses. For the case of uniform support excitations, a number of combination rules, with different levels of sophistication, has been proposed. The simplest rule is the so-called SAV, which stands for sum of absolute values, and which generally is overconservative. A combination rule, which is widely used in practice, is the SRSS rule. It stands for square root of sum of squares, and it is more suitable than the previous one. However, it may be on the unconservative side for closely spaced modes situations, such as seismic analyses of nuclear power plants. To overcome this shortcoming of the SRSS rule, more elaborated rules, which account for modal cross-correlations, exist. Different expressions for the

cross-correlations coefficients, based on different considerations, have been suggested by Wilson et al. [52], Rosenblueth et al. [38], and Der Kiureghian [14], to cite a few only. Among the aforementioned formulations, the one proposed by the latter author, based on a random vibration analysis, may be considered to be the most rigorous.

The present chapter aims at proposing expressions for the modal cross-correlation coefficients for the case of multiple support excitations, thereby providing a combination rule for the maximum modal responses given by the modified response spectrum model. It is important to mention that the work spirit of this chapter is similar to that of the previous one. Namely that, the expressions for the modal cross-correlation coefficients sought after, will take the form of an adjustment, or an extension, of the homologous coefficients for the case of uniform support excitations. The formulation will, at first, account for a single ground motion component, and will, afterwards, be extended to multicomponent ground motion.

4.2 Case of a Single Ground Motion Component

Let's first go over the derivation of the existing expressions of modal cross-correlation coefficients for the case of uniform support excitations. The following formulation is due to Der Kiureghian [14]. The dynamic response of a linear, multidegree-of-freedom system, discrete or continuous, classically damped, subjected to a single component of uniform translational ground motion, can be determined by solving the modal equation of motion (3.2.1.7 or 3.3.1.8):

$$\ddot{y}_k + 2\beta_k\omega_k\dot{y}_k + \omega_k^2 y_k = -\gamma_k\ddot{u}_0(t) \tag{4.2.1}$$

where:

k is the mode number.

γ_k is the participation factor for mode k.

Any response quantity $z(t)$ linearly related to the modal coordinates can be found using the relation:

$$z(t) = \sum_k L_k y_k(t) \tag{4.2.2}$$

where L_k's are constant, and depend on the response of interest. From [9] and [14], the one-sided spectral density function of the response of interest, $G_z(\omega)$ is given by:

$$G_z(\omega) = \sum_k \sum_l L_k L_l H_k^*(\omega) H_l(\omega) \gamma_k \gamma_l G_{\ddot{u}_0}(\omega) \tag{4.2.3}$$

where:

k and l indicate modes.

* indicates the complex conjugate.

$H_k(\omega)$ is the transfer function (for displacement response) of mode k given by:

$$H_k(\omega) = \frac{1}{\omega_k^2 - \omega^2 + 2i\beta_k\omega_k\omega}$$

$G_{\ddot{u}_0}(\omega)$ is the one-sided spectral density function of the input excitation.

Define E_k and E_l such that:

$$E_k = L_k \gamma_k \qquad (4.2.4)$$

$$E_l = L_l \gamma_l \qquad (4.2.5)$$

where E_k and E_l may be considered effective participation factors for modes k and l respectively. Hence, (4.2.3) can be rewritten as;

$$G_z(\omega) = \sum_k \sum_l E_k E_l H_k^*(\omega) H_l(\omega) G_{\ddot{u}_0}(\omega) \qquad (4.2.6)$$

Incidentally, by switching k and l in the double summation, the real parts of $H_k^*(\omega) H_l(\omega)$ will remain, while the imaginary parts will cancel out.

In order to evaluate the mean value of the peak response, it is necessary to compute the spectral moments of the response. The mth spectral moment of $z(t)$ is given by:

$$\lambda_m = \int_0^\infty \omega^m G_z(\omega) d\omega \qquad (4.2.7)$$

Substituting (4.2.6) into (4.2.7), one gets:

$$\lambda_m = \sum_k \sum_l E_k E_l \text{ Real} \int_0^\infty \omega^m H_k^*(\omega) H_l(\omega) G_{\ddot{u}_0}(\omega) d\omega \qquad (4.2.8)$$

Let

$$\lambda_{m,kl} = \text{ Real} \int_0^\infty \omega^m H_k^*(\omega) H_l(\omega) G_{\ddot{u}_0}(\omega) d\omega \qquad (4.2.9)$$

where $\lambda_{m,kl}$ is the mth cross-spectral moment associated with modes k and l. Hence, (4.2.8) can be rewritten as:

$$\lambda_m = \sum_k \sum_l E_k E_l \lambda_{m,kl} \qquad (4.2.10)$$

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Let

$$\epsilon_{m,kl} = \frac{\lambda_{m,kl}}{\sqrt{\lambda_{m,kk}\lambda_{m,ll}}} \qquad (4.2.11)$$

where $\epsilon_{m,kl}$ is the mth cross-correlation coefficient between modes k and l. Hence, (4.2.8) can take the following form:

$$\lambda_m = \sum_k \sum_l E_k E_l \epsilon_{m,kl} \sqrt{\lambda_{m,kk}\lambda_{m,ll}} \qquad (4.2.12)$$

where:

$$\lambda_{m,kk} = \int_0^\infty \omega^m \mid H_k(\omega) \mid^2 G_{\ddot{u}_0}(\omega) d\omega \qquad (4.2.13)$$

Note that $\epsilon_{0,kl}$ is the cross-correlation coefficient between the kth and the lth modal responses. And that $\epsilon_{2,kl}$ is the cross-correlation coefficient between the kth and the lth modal "velocities" (time derivatives of the response of interest). $\epsilon_{1,kl}$ has no obvious physical interpretation. However, it behaves similarly to cross-correlation coefficients [14]. It is also shown in [14] that:

$$\text{If} \quad \frac{\omega_k}{\omega_l} \longrightarrow 1, \text{then} \quad \epsilon_{m,kl} \longrightarrow 1, \quad m = 0,1,2$$

$$\text{If} \quad \frac{\omega_k}{\omega_l} \longrightarrow 0, \text{then} \quad \epsilon_{m,kl} \longrightarrow 0, \quad m = 0,1,2$$

The mean value of the maximum response and the zeroth spectral moment of the response are related as follows:

$$\lambda_0 = \left[\frac{Mean\ (max(z))}{p}\right]^2 \qquad (4.2.14)$$

where p is a peak factor. Hence,

$$\lambda_{0,kk} = \left[\frac{R(\omega_k, \beta_k)}{p_k} \right]^2 \qquad (4.2.15)$$

where:

$R(\omega, \beta)$ is the mean response spectrum given at the site of interest.

p_k is a peak factor.

Therefore, by writing (4.2.12) for $m = 0$,

$$\lambda_0 = \sum_k \sum_l E_k E_l \epsilon_{0,kl} \sqrt{\lambda_{0,kk} \lambda_{0,ll}} \qquad (4.2.16)$$

and substituting (4.2.14) and (4.2.15) into (4.2.16), and by simplifying peak factors

for practical purposes [14] and [41], one gets:

$$[Mean(max(z))]^2 = \sum_k \sum_l \epsilon_{0,kl} E_k E_l R(\omega_k, \beta_k) R(\omega_l, \beta_l) \qquad (4.2.17)$$

which can be rewritten as:

$$Mean(max(z)) = [\sum_k \sum_l \epsilon_{0,kl} \, Mean(max(z_k)) Mean(max(z_l))]^{1/2} \qquad (4.2.18)$$

where:

$$Mean(max(z_k)) = E_k R(\omega_k, \beta_k)$$

$$Mean(max(z_l)) = E_l R(\omega_l, \beta_l)$$

which are the mean values of the maximum k^{th} and l^{th} modal responses, respectively

(they may be negative).

For structures with well-separated frequencies, the coefficients $\epsilon_{0,kl}$ vanish for $k \neq l$, and (4.2.18) becomes:

$$Mean(max(z)) = [\sum_k (Mean(max(z_k)))^2]^{1/2} \qquad (4.2.19)$$

which is the SRSS rule for modal combination.

At present, let us extend this existing formulation to the case of multiple support excitations. The goal which is aimed at, is to get an analogous equation to (4.2.18) that would read as follows:

$$\overset{p}{Mean}(\max(z)) = \left[\sum_k \sum_l \epsilon^p_{0,kl} \overset{p}{Mean}(\max(z_k)) \overset{p}{Mean}(\max(z_l)) \right]^{1/2}.$$

where:

The superscript p stands for partially correlated seismic excitations.

$$\overset{p}{Mean}(\max(z_k)) = E^p_k R^p(\omega_k, \beta_k)$$

$$\overset{p}{Mean}(\max(z_l)) = E^p_l R^p(\omega_l, \beta_l)$$

$R^p(\omega, \beta)$ is the modified response spectrum model.

$\epsilon^p_{0,kl}$ is the modified cross-correlation coefficient between modes k and l for the case of non uniform support motions. Ideally, it should be expressed as the product of $\epsilon_{0,kl}$ and a correction factor, that accounts for the spatial character of the seismic input.

Let's start the formulation by writing the kth modal equation for a linear, mutlidegree-of-freedom, multiply-supported system, discrete or continous, classically damped, subjected to a single component of non uniform translational ground motion (3.2.1.19 or 3.3.1.16):

$$\ddot{y}_k + 2\beta_k\omega_k\dot{y}_k + \omega_k^2 y_k = \sum_{i=1}^{n} A_{ki}\ddot{u}_i \qquad (4.2.20)$$

where all parameters have already been defined.

Any response quantity $z(t)$ linearly related to the modal coordinates can be expressed as

$$z(t) = \sum_k L_k y_k(t) \qquad (4.2.21)$$

where L_k's are constant, depending on the response of interest. Let

$$\ddot{s}_k(t) = \sum_{i=1}^{n} A_{ki}\ddot{u}_i(t) \qquad (4.2.22)$$

The spectral density function of the response, $S_z^p(\omega)$ is given by:

$$S_z^p(\omega) = \sum_k \sum_l L_k L_l H_k^*(\omega) H_l(\omega) S_{\ddot{s}_k \ddot{s}_l}(\omega) \qquad (4.2.23)$$

where the superscript p stands for partially correlated seismic excitations. By switching k and l in the double summation, the real parts of $H_k^*(\omega)H_l(\omega)S_{\ddot{s}_k\ddot{s}_l}(\omega)$ will remain, while the imaginary parts will cancel out. In order to evaluate spectral moments of the response, one needs to express the one-sided spectral density function of the response, $G_z^p(\omega)$. $G_z^p(\omega)$ is such that:

$$\int_0^\infty G_z^p(\omega)d\omega = \int_{-\infty}^\infty S_z^p(\omega)d\omega \qquad (4.2.24)$$

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Hence, let's express the right hand side of (4.2.24):

$$\int_{-\infty}^{\infty} S_z^p(\omega) d\omega = \int_{-\infty}^{\infty} \sum_k \sum_l L_k L_l H_k^*(\omega) H_l(\omega) S_{\ddot{s}_k \ddot{s}_l}(\omega) d\omega$$

$$= \int_0^{\infty} \sum_k \sum_l L_k L_l 2 \ Real \ [H_k^*(\omega) H_l(\omega) S_{\ddot{s}_k \ddot{s}_l}(\omega)] d\omega$$

Therefore,

$$G_z^p(\omega) = 2 \sum_k \sum_l L_k L_l Real \ [H_k^*(\omega) H_l(\omega) S_{\ddot{s}_k \ddot{s}_l}(\omega)] \tag{4.2.25}$$

Let's now express spectral moments:

$$\lambda_m^p = \int_0^{\infty} \omega^m G_z^p(\omega) d\omega \tag{4.2.26}$$

Substituting (4.2.25) into (4.2.26), one gets:

$$\lambda_m^p = \sum_k \sum_l L_k L_l 2 \ Real \int_0^{\infty} \omega^m H_k^*(\omega) H_l(\omega) S_{\ddot{s}_k \ddot{s}_l}(\omega) d\omega \tag{4.2.27}$$

Analogously to the case of uniform excitations, let:

$$\lambda_{m,kl}^p = 2 \ Real \int_0^{\infty} \omega^m H_k^*(\omega) H_l(\omega) S_{\ddot{s}_k \ddot{s}_l}(\omega) d\omega \tag{4.2.28}$$

and hence,

$$\lambda_m^p = \sum_k \sum_l L_k L_l \lambda_{m,kl}^p \tag{4.2.29}$$

where $\lambda_{m,kl}^p$ is the mth cross-spectral moment associated with modes k and l for the case of non uniform seismic input. Let's also define the mth cross-correlation coefficient between modes k and l for the case of non uniform seismic input:

$$\epsilon_{m,kl}^p = \frac{\lambda_{m,kl}^p}{\sqrt{\lambda_{m,kk}^p \lambda_{m,ll}^p}} \tag{4.2.30}$$

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Therefore, (4.2.29) could be rewritten as:

$$\lambda_m^P = \sum_k \sum_l L_k L_l \epsilon_{m,kl}^P \sqrt{\lambda_{m,kk}^P \lambda_{m,ll}^P} \qquad (4.2.31)$$

(4.2.31) could rewritten for $m = 0$ (zeroth spectral moment) as:

$$\lambda_0^P = \sum_k \sum_l L_k L_l \epsilon_{0,kl}^P \sqrt{\lambda_{0,kk}^P \lambda_{0,ll}^P} \qquad (4.2.32)$$

where:

$$\lambda_{0,kk}^P = 2 \int_0^\infty \mid H_k(\omega) \mid^2 S_{\ddot{s}_k}(\omega) d\omega$$

which is equivalent to

$$\lambda_{0,kk}^P = \int_0^\infty \mid H_k(\omega) \mid^2 G_{\ddot{s}_k}(\omega) d\omega \qquad (4.2.33)$$

From previous arguments:

$$\lambda_0^P = \left[\frac{M\overset{P}{e}an(\max(z))}{p} \right]^2 \qquad (4.2.34)$$

$$\lambda_{0,kk}^P = \left[\frac{M\overset{P}{e}an(\max(y_k))}{p_k} \right]^2 \qquad (4.2.35)$$

where p and p_k are peak factors. By virtue of [14] and [42], (4.2.32) leads to:

$$M\overset{P}{e}an(\max(z)) = [\sum_k \sum_l \epsilon_{0,kl}^P L_k L_l M\overset{P}{e}an(\max(y_k)) M\overset{P}{e}an(\max(y_l))]^{1/2} \qquad (4.2.36)$$

From (3.2.1.56),

$$M\overset{P}{e}an(\max(y_k)) = \mid \gamma_k \mid R^P(\omega_k, \beta_k) \qquad (4.2.37)$$

$$M\overset{P}{e}an(\max(y_l)) = \mid \gamma_l \mid R^P(\omega_l, \beta_l) \qquad (4.2.38)$$

where $R^P(\omega, \beta)$ is the modified response spectrum model.

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Define:

$$E_k^p = |\gamma_k| \, L_k = \begin{cases} E_k & \text{if } \gamma_k > 0 \\ \\ -E_k & \text{if } \gamma_k < 0 \end{cases} \tag{4.2.39}$$

$$E_l^p = |\gamma_l| \, L_l = \begin{cases} E_l & \text{if } \gamma_l > 0 \\ \\ -E_l & \text{if } \gamma_l < 0 \end{cases} \tag{4.2.40}$$

By using (4.2.39), (4.2.40), (4.2.37), and (4.2.38) into (4.2.36), one gets:

$$M\overset{p}{e}an(\max(z)) = \left[\sum_k \sum_l \epsilon_{0,kl}^p E_k^p E_L^p R^p(\omega_k, \beta_k) R^p(\omega_l, \beta_l) \right]^{1/2} \tag{4.2.41}$$

which can be rewritten as:

$$M\overset{p}{e}an(\max(z)) = \left[\sum_k \sum_l \epsilon_{0,kl}^p M\overset{p}{e}an(\max(z_k)) M\overset{p}{e}an(\max(z_l)) \right]^{1/2} \tag{4.2.42}$$

where

$$M\overset{p}{e}an(\max(z_k)) = E_k^p R^p(\omega_k, \beta_k) \tag{4.2.43}$$

$$M\overset{p}{e}an(\max(z_l)) = E_l^p R^p(\omega_l, \beta_l) \tag{4.2.44}$$

Notice that if $\gamma_k = 0$ (or $\gamma_l = 0$), then (4.2.41) still holds true, with:

$$M\overset{p}{e}an(\max(z_k)) = L_k R^p(\omega_k, \beta_k) \tag{4.2.45}$$

where $R^p(\omega_k, \beta_k)$ is given by (3.2.1.72), which addresses the case of no modal participation under uniform seismic input.

Hence, (4.2.42) represents the equation that was aimed at. At present, let's express $\epsilon_{0,kl}^p$ in terms of $\epsilon_{0,kl}$. Let's first express $\lambda_{0,kl}^p$ in terms of $\lambda_{0,kl}$. From (4.2.28):

$$\lambda_{0,kl}^p = 2 \, Real \int_0^\infty H_k^*(\omega) H_l(\omega) S_{\ddot{s}_k \ddot{s}_l}(\omega) d\omega \tag{4.2.46}$$

The first step is to express $S_{\ddot{s}_k \ddot{s}_l}(\omega)$ in terms of auto-spectra and cross-spectra of the ground acceleration at the different supports. The cross correlation $R_{\ddot{s}_k \ddot{s}_l}(\tau)$ can be written as:

$$
\begin{aligned}
R_{\ddot{s}_k \ddot{s}_l}(\tau) &= E\left[\ddot{s}_k(t)\ddot{s}_l(t+\tau)\right] \\
&= E\left[\left(\sum_{i=1}^{n} A_{ki}\ddot{u}_i(t)\right)\left(\sum_{i=1}^{n} A_{li}\ddot{u}_i(t+\tau)\right)\right] \\
&= E\left[\sum_{i=1}^{n}\sum_{j=1}^{n} A_{ki} A_{lj}\ddot{u}_i(t)\ddot{u}_j(t+\tau)\right]
\end{aligned}
$$

Exchanging the order of summation and expectation, $R_{\ddot{s}_k \ddot{s}_l}(\tau)$ reads as:

$$
R_{\ddot{s}_k \ddot{s}_l}(\tau) = \sum_{i=1}^{n}\sum_{j=1}^{n} A_{ki} A_{lj} R_{\ddot{u}_i \ddot{u}_j}(\tau) \tag{4.2.47}
$$

where:

    i and j denote support points.

    k and l denote mode numbers.

By taking Fourier transforms in (4.2.47), the cross-spectral density function, $S_{\ddot{s}_k \ddot{s}_l}(\omega)$ is given by:

$$
S_{\ddot{s}_k \ddot{s}_l}(\omega) = \sum_{i=1}^{n}\sum_{j=1}^{n} A_{ki} A_{lj} S_{\ddot{u}_i \ddot{u}_j}(\omega) \tag{4.2.48}
$$

Hence, $\lambda_{0,kl}^p$ can be written as:

$$
\begin{aligned}
\lambda_{0,kl}^p &= 2\ Real\ \int_0^{\infty} H_k^*(\omega) H_l(\omega) \sum_{i=1}^{n}\sum_{j=1}^{n} A_{ki} A_{lj} S_{\ddot{u}_i \ddot{u}_j}(\omega)\, d\omega \\
&= \sum_{i=1}^{n}\sum_{j=1}^{n} A_{ki} A_{lj} 2\ Real\ \int_0^{\infty} H_k^*(\omega) H_l(\omega) S_{\ddot{u}_i \ddot{u}_j}(\omega)\, d\omega \\
&= \sum_{i=1}^{n}\sum_{j=1}^{n} A_{ki} A_{lj} \int_{-\infty}^{\infty} H_k^*(\omega) H_l(\omega) S_{\ddot{u}_i \ddot{u}_j}(\omega)\, d\omega
\end{aligned}
$$

Hence,

$$
\lambda_{0,kl}^p = \sum_{i=1}^{n}\sum_{j=1}^{n} A_{ki} A_{lj} \int_{-\infty}^{\infty} H_k^*(\omega) H_l(\omega) S_{\ddot{u}_i \ddot{u}_j}(\omega)\, d\omega \tag{4.2.49}
$$

Recall that:

$$S_{\ddot{u}_i \ddot{u}_j}(\omega) = r_{ij}(\omega, d_{ij}) S_{\ddot{u}_0}(\omega) \qquad (3.2.1.25)$$

where r_{ij} is the coherency function between stations i and j.

Define:

$$r_{ijkl} = \frac{\int_{-\infty}^{\infty} H_k^*(\omega) H_l(\omega) S_{\ddot{u}_i \ddot{u}_j}(\omega) d\omega}{\int_{-\infty}^{\infty} H_k^*(\omega) H_l(\omega) S_{\ddot{u}_0}(\omega) d\omega} \qquad (4.2.50)$$

where:

$$\int_{-\infty}^{\infty} H_k^*(\omega) H_l(\omega) S_{\ddot{u}_0}(\omega) = Real \int_0^{\infty} H_k^*(\omega) H_l(\omega) G_{\ddot{u}_0}(\omega) d\omega = \lambda_{0,kl}$$

Hence,

$$r_{ijkl} = \frac{\int_{-\infty}^{\infty} H_k^*(\omega) H_l(\omega) S_{\ddot{u}_i \ddot{u}_j}(\omega) d\omega}{\lambda_{0,kl}} \qquad (4.2.51)$$

r_{ijkl} can be thought of as the ratio of the covariance of the relative responses of 2 oscillators (ω_k, β_k) and (ω_l, β_l), respectively subjected to the partially correlated excitations $\ddot{u}_i(t)$ and $\ddot{u}_j(t)$, to the homologous covariance for the case of fully correlated excitations.

Hence, $\lambda_{0,kl}^p$ can be written as:

$$\lambda_{0,kl}^p = \sum_{i=1}^{n} \sum_{j=1}^{n} A_{ki} A_{lj} r_{ijkl} \lambda_{0,kl} \qquad (4.2.52)$$

or as:

$$\lambda_{0,kl}^p = \lambda_{0,kl} \sum_{i=1}^{n} \sum_{j=1}^{n} A_{ki} A_{lj} r_{ijkl} \qquad (4.2.53)$$

Therefore, $\epsilon_{0,kl}^p$ can be expressed as:

$$\epsilon_{0,kl}^p = \frac{\lambda_{0,kl}^p}{\sqrt{\lambda_{0,kk}^p \lambda_{0,ll}^p}} \quad \text{from (4.2.30)} \qquad (4.2.54)$$

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$$= \frac{\lambda_{0,kl}}{\sqrt{\lambda_{0,kk}\lambda_{0,ll}}} \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} A_{ki}A_{lj}r_{ijkl}}{\left\{ (\sum_{i=1}^{n}\sum_{j=1}^{n} A_{ki}A_{kj}r_{ijkk})(\sum_{i=1}^{n}\sum_{j=1}^{n} A_{li}A_{lj}r_{ijll}) \right\}^{1/2}}$$

Using $\epsilon_{0,kl} = \dfrac{\lambda_{0,kl}}{\sqrt{\lambda_{0,kl}\lambda_{0,ll}}}$ in the above equation, one gets

$$\epsilon_{0,kl}^{p} = \epsilon_{0,kl} \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} A_{ki}A_{lj}r_{ijkl}}{\left\{ (\sum_{i=1}^{n}\sum_{j=1}^{n} A_{ki}A_{kj}r_{ijkk})(\sum_{i=1}^{n}\sum_{j=1}^{n} A_{li}A_{lj}r_{ijll}) \right\}^{1/2}} \tag{4.2.55}$$

Notice that r_{ijkk} can be rewritten as:

$$r_{ijkk} = \frac{\int_{-\infty}^{\infty} |H_k(\omega)|^2 \, S_{\ddot{u}_i\ddot{u}_j}(\omega)d\omega}{\int_{-\infty}^{\infty} |H_k(\omega)|^2 \, S_{\ddot{u}_0}(\omega)d\omega}$$

$\Longrightarrow$

$$r_{ijkk} = \frac{\int_{0}^{\infty} \rho_{ij} |H_k(\omega)|^2 \, G_{\ddot{u}_0}(\omega)d\omega}{\int_{0}^{\infty} |H_k(\omega)|^2 \, G_{\ddot{u}_0}(\omega)d\omega} \tag{4.2.56}$$

where ρ_{ij} is the frequency dependent spatial correlation coefficient of the seismic motion between stations i and j. It is the real part of the coherency function r_{ij}. From (4.2.56), one remarks that r_{ijkk} is nothing but ρ_{ijk}.

Hence,

$$r_{ijkk} = \rho_{ijk} \tag{4.2.57}$$

Similarly,

$$r_{ijll} = \rho_{ijl} \tag{4.2.58}$$

Therefore, (4.2.55) could be rewritten as:

$$\epsilon^p_{0,kl} = \epsilon_{0,kl} \frac{\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n} A_{ki}A_{lj}r_{ijkl}}{\left\{ \left(\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n} A_{ki}A_{kj}\rho_{ijk}\right)\left(\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n} A_{li}A_{lj}\rho_{ijl}\right)\right\}^{1/2}} \tag{4.2.59}$$

Finally, the combination rule has been extended to the case of partially correlated excitations. And the modal cross-correlation coefficient for such a case has been expressed in terms of the homologous coefficient for the case of fully correlated (identical) excitations.

As a closing part of this section, let's examine extreme cases for (4.2.55) or (4.2.59).

1. Support motions are fully correlated:

All $r_{ijkl} = 1$, (4.2.55) reads as:

$$\epsilon^p_{0,kl} = \epsilon_{0,kl} \frac{\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n} A_{ki}A_{lj}}{\left\{ \left(\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n} A_{ki}A_{kj}\right)\left(\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n} A_{li}A_{lj}\right)\right\}^{1/2}}$$

$$= \epsilon^p_{0,kl} \frac{\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n} A_{ki}A_{lj}}{\left\{ \left(\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n} A_{ki}A_{lj}\right)^2\right\}^{1/2}}$$

$$= \epsilon_{0,kl}$$

Hence,

$$\epsilon^p_{0,kl} = \epsilon_{0,kl}$$

which is an expected result.

2. Support motions are mutually uncorrelated: All $r_{ij} = 0$

$$\lambda_{0,kl}^p = \sum_{i=1}^n A_{ki} A_{li} \lambda_{0,kl} \qquad \text{from} \qquad (4.2.49)$$

Hence,

$$\epsilon_{0,kl}^p = \epsilon_{0,kl} \frac{\displaystyle\sum_{i=1}^n A_{ki} A_{li}}{\left\{ (\displaystyle\sum_{i=1}^n A_{ki}^2)(\displaystyle\sum_{i=1}^n A_{li}^2) \right\}^{1/2}}$$

Since

$$\left(\sum_{i=1}^n A_{ki} A_{li} \right)^2 \le \left(\sum_{i=1}^n A_{ki}^2 \right) \left(\sum_{i=1}^n A_{li}^2 \right)$$

$\epsilon_{0,kl}^p \le \epsilon_{0,kl}$ in this case.

4.3 Case of Multicomponent Ground Motion

Having presented a modal combination rule, and expressed a modal cross-correlation coefficient for the case of a single component of non uniform translational ground motion, let's extend this formulation to the situation of non uniform uncorrelated multicomponent ground motion. Prior to tackling the problem of non uniform multicomponent ground motion, let's extend the existing formulation for a single uniform ground motion component [14] to the case of uniform uncorrelated multicomponent ground motion.

A typical modal equation for a linear, three-dimensional, lumped mass, multidegree of freedom, multiply-supported system, subjected to uniform, three-dimensional translational seismic excitations reads as follows: (3.2.2.3)

$$\ddot{y}_k + 2\beta_k \omega_k \dot{y}_k + \omega_k^2 y_k = -\sum_{q=1}^{3} \gamma_{kq} \ddot{u}_{0q} \qquad (4.3.1)$$

where:

q indicates the ground motion component

k indicates the mode number

All parameters have already been defined.

Any response quantity $z(t)$ linearly related to the modal coordinates can be expressed as:

$$z(t) = \sum_k L_k y_k(t) \qquad (4.3.2)$$

where L_k's are constant, and depend on the response of interest.

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Let

$$\ddot{s}_k(t) = -\sum_{q=1}^{3} \gamma_{kq} \ddot{u}_{0q}(t) \tag{4.3.3}$$

The spectral density function of the response, $S_z(\omega)$ is given by:

$$S_z(\omega) = \sum_k \sum_l L_k L_l H_k^*(\omega) H_l(\omega) S_{\ddot{s}_k \ddot{s}_l}(\omega) \tag{4.3.4}$$

Let's express $S_{\ddot{s}_k \ddot{s}_l}(\omega)$.

The cross-correlation function $R_{\ddot{s}_k \ddot{s}_l}(\tau)$ is given by:

$$
\begin{aligned}
R_{\ddot{s}_k \ddot{s}_l}(\tau) &= E\left[\ddot{s}_k(t)\ddot{s}_l(t+\tau)\right] \\
&= E\left[\left(\sum_{q=1}^{3}\gamma_{kq}\ddot{u}_{0q}(t)\right)\left(\sum_{q=1}^{3}\gamma_{lq}\ddot{u}_{0q}(t+\tau)\right)\right] \\
&= E\left[\sum_{q=1}^{3}\sum_{r=1}^{3}\gamma_{kq}\gamma_{lr}\ddot{u}_{0q}(t)\ddot{u}_{0r}(t+\tau)\right]
\end{aligned}
$$

Exchanging the order of summation and expectation, $R_{\ddot{s}_k \ddot{s}_l}(\tau)$ can be written as:

$$R_{\ddot{s}_k \ddot{s}_l}(\tau) = \sum_{q=1}^{3}\sum_{r=1}^{3}\gamma_{kq}\gamma_{lr}R_{\ddot{u}_{0q}\ddot{u}_{0r}}(\tau) \tag{4.3.5}$$

Since the ground motion components are assumed uncorrelated, (4.3.5) reduces to:

$$R_{\ddot{s}_k \ddot{s}_l}(\tau) = \sum_{q=1}^{3}\gamma_{kq}\gamma_{lq}R_{\ddot{u}_{0q}}(\tau) \tag{4.3.6}$$

And by taking Fourier transform, one gets:

$$S_{\ddot{s}_k \ddot{s}_l}(\omega) = \sum_{q=1}^{3}\gamma_{kq}\gamma_{lq}S_{\ddot{u}_{0q}}(\omega) \tag{4.3.7}$$

Therefore, (4.3.4) may be rewritten as:

$$S_z(\omega) = \sum_k \sum_l L_k L_l H_k^*(\omega) H_l(\omega) \sum_{q=1}^{3}\gamma_{kq}\gamma_{lq}S_{\ddot{u}_{0q}}(\omega)$$

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$$S_z(\omega) = \sum_{q=1}^{3}\sum_{k}\sum_{l} L_k L_l H_k^*(\omega) H_l(\omega) \gamma_{kq}\gamma_{lq} S_{\ddot{u}_{0_q}}(\omega) \qquad (4.3.8)$$

From (4.3.8), the one-sided spectral density function of the response can be expressed

as:

$$G_z(\omega) = \sum_{q=1}^{3}\sum_{k}\sum_{l} L_k L_l H_k^*(\omega) H_l(\omega) \gamma_{kq}\gamma_{lq} G_{\ddot{u}_{0_q}}(\omega) \qquad (4.3.9)$$

Let:

$$E_{kq} = L_k \gamma_{kq} \qquad (4.3.10)$$

$$E_{lq} = L_l \gamma_{lq} \qquad (4.3.11)$$

where E_{kq} and E_{lq} can be viewed as effective participation factors.

Hence, (4.3.9) becomes:

$$G_z(\omega) = \sum_{q=1}^{3}\sum_{k}\sum_{l} E_{kq} E_{lq} H_k^*(\omega) H_l(\omega) G_{\ddot{u}_{0_q}}(\omega) \qquad (4.3.12)$$

The spectral moments of the response are then given by:

$$\lambda_m = \int_0^\infty \omega^m G_z(\omega)\,d\omega \qquad (4.3.13)$$

$$\lambda_m = \sum_{q=1}^{3}\sum_{k}\sum_{l} E_{kq} E_{lq}\ Real \int_0^\infty \omega^m H_k^*(\omega) H_l(\omega) G_{\ddot{u}_{0_q}}(\omega)\,d\omega \qquad (4.3.14)$$

Similarly to the case of a single ground motion component, define $\lambda_{mq,kl}$ by:

$$\lambda_{mq,kl} = Real \int_0^\infty \omega^m H_k^*(\omega) H_l(\omega) G_{\ddot{u}_{0_q}}(\omega)\,d\omega \qquad (4.3.15)$$

where $\lambda_{mq,kl}$ is the mth cross-spectral moment associated with modes k and l for the

q^{th} ground motion component. Hence,

$$\lambda_m = \sum_{q=1}^{3}\sum_{k}\sum_{l} E_{kq} E_{lq} \lambda_{mq,kl} \qquad (4.3.16)$$

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The zeroth spectral moments are of interest. For $m = 0$, (4.3.16) reads as:

$$\lambda_0 = \sum_{q=1}^{3} \sum_k \sum_l E_{kq} E_{lq} \lambda_{0q,kl} \tag{4.3.17}$$

Define $\epsilon_{0q,kl}$ by:

$$\epsilon_{0q,kl} = \frac{\lambda_{oq,kl}}{\sqrt{\lambda_{oq,kk} \lambda_{0q,ll}}} \tag{4.3.18}$$

$\epsilon_{0q,kl}$ is the modal response cross-correlation coefficient between modes k and l for the qth ground motion component. Therefore,

$$\lambda_0 = \sum_{q=1}^{3} \sum_k \sum_l E_{kq} E_{lq} \epsilon_{0q,kl} \sqrt{\lambda_{0q,kk} \lambda_{0q,ll}} \tag{4.3.19}$$

By virtue of arguments made in the previous-section, (4.3.19) leads to:

$$Mean(max(z)) = [\sum_{q=1}^{3} \sum_k \sum_l E_{kq} E_{lq} \epsilon_{0q,kl} R_q(\omega_k, \beta_k) R_q(\omega_l, \beta_l)]^{1/2} \tag{4.3.20}$$

where $R_q(\omega, \beta)$ is the mean response spectrum of the qth ground motion component given at the site of interest.

Define:

$$Mean(max(z_{kq})) = E_{kq} R_q(\omega_k, \beta_k) \tag{4.3.21}$$

$$Mean(max(z_{lq})) = E_{lq} R_q(\omega_l, \beta_l) \tag{4.3.22}$$

(4.3.21) and (4.3.22) give the mean values of the maximum response due to modes k and l respectively, associated with the qth ground motion component.

Finally, (4.3.20) may be rewritten as:

$$Mean(max(z)) = [\sum_{q=1}^{3} \sum_k \sum_l \epsilon_{0q,kl} Mean(max(z_{kq})) Mean(max(z_{lq}))]^{1/2} \tag{4.3.23}$$

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(4.3.20) and (4.3.23) provide a modal combination rule for the case of uniform uncorrelated multicomponent translational ground motion.

At present, let's extend this formulation to the case of non uniform uncorrelated multicomponent translational ground motion. Similarly to the previous section, the goal is to arrive at equations analogous to (4.3.20) and (4.3.23).

A typical modal equation for a linear, three-dimensional lumped mass, multi-degree of freedom, multiply-supported system subjected to spatially varying three-dimensional seismic excitations, reads as follows: (3.2.2.19)

$$\ddot{y}_k + 2\beta_k\omega_k\dot{y}_k + \omega_k^2 y_k = \sum_{q=1}^{3}\sum_{i=1}^{n} A_{kqi}\ddot{u}_{qi} \tag{4.3.24}$$

where:

q indicates the ground motion component.

i indicates the support number

k indicates the mode number.

Recall from (3.2.2.20) that:

$$\gamma_{kq} = -\sum_{i=1}^{n} A_{kqi} \tag{4.3.25}$$

Any response quantity $z(t)$ linearly related to the modal coordinates can be found using:

$$z(t) = \sum_{k} L_k y_k(t) \tag{4.3.26}$$

Let $\ddot{s}_k(t)$ and $\ddot{s}_{kq}(t)$ be given by:

$$\ddot{s}_k(t) = \sum_{q=1}^{3}\sum_{i=1}^{n} A_{kqi}\ddot{u}_{qi}(t) \tag{4.3.27}$$

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$$\ddot{s}_{kq}(t) = \sum_{i=1}^{n} A_{kqi} \ddot{u}_{qi}(t) \qquad (4.3.28)$$

or equivalently,

$$\ddot{s}_k(t) = \sum_{q=1}^{3} \ddot{s}_{kq}(t) \qquad (4.3.29)$$

The spectral density function of the response $S_z^p(\omega)$, is given by:

$$S_z^p(\omega) = \sum_k \sum_l L_k L_l H_k^*(\omega) H_l(\omega) S_{\ddot{s}_k \ddot{s}_l}(\omega) \qquad (4.3.30)$$

where p stands for the case of partially correlated seismic excitations.

Let's express $S_{\ddot{s}_k \ddot{s}_l}(\omega)$. The cross-correlation function $R_{\ddot{s}_k \ddot{s}_l}(\tau)$ is given by:

$$
\begin{aligned}
R_{\ddot{s}_k \ddot{s}_l}(\tau) &= E\left[\ddot{s}_k(t)\ddot{s}_l(t+\tau)\right] \\
&= E\left[\left(\sum_{q=1}^{3}\ddot{s}_{kq}(t)\right)\left(\sum_{q=1}^{3}\ddot{s}_{lq}(t+\tau)\right)\right] \\
&= E\left[\sum_{q=1}^{3}\sum_{r=1}^{3}\ddot{s}_{kq}(t)\ddot{s}_{lr}(t+\tau)\right]
\end{aligned}
$$

switching orders between summation and expectation, $R_{\ddot{s}_k \ddot{s}_l}(\tau)$ may be written as:

$$R_{\ddot{s}_k \ddot{s}_l}(\tau) = \sum_{q=1}^{3}\sum_{r=1}^{3} E[\ddot{s}_{kq}(t)\ddot{s}_{lr}(t+\tau)] \qquad (4.3.31)$$

Assuming uncorrelated ground motion components, (4.3.31) becomes:

$$R_{\ddot{s}_k \ddot{s}_l}(\tau) = \sum_{q=1}^{3} R_{\ddot{s}_{kq}\ddot{s}_{lq}}(\tau) \qquad (4.3.32)$$

Taking Fourier transforms, $S_{\ddot{s}_k \ddot{s}_l}(\omega)$ can be written as:

$$S_{\ddot{s}_k \ddot{s}_l}(\omega) = \sum_{q=1}^{3} S_{\ddot{s}_{kq}\ddot{s}_{lq}}(\omega) \qquad (4.3.33)$$

Substituting (4.3.33) into (4.3.30), $S_z^p(\omega)$ can be expressed as:

$$S_z^p(\omega) = \sum_k \sum_l L_k L_l H_k^*(\omega) H_l(\omega) \sum_{q=1}^{3} S_{\ddot{s}_{kq}\ddot{s}_{lq}}(\omega)$$

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$$S_z^p(\omega) = \sum_{q=1}^{3}\sum_k \sum_l L_k L_l H_k^*(\omega) H_l(\omega) S_{\ddot{s}_{k_q} \ddot{s}_{l_q}}(\omega) \tag{4.3.34}$$

And the one-sided spectral density function, $G_z^p(\omega)$, will be given by:

$$G_z^p(\omega) = \sum_{q=1}^{3}\sum_k \sum_l L_k L_l 2\ Real\ [H_k^*(\omega) H_l(\omega) S_{\ddot{s}_{k_q} \ddot{s}_{l_q}}(\omega)] \tag{4.3.35}$$

From (4.3.37), the spectral moments will read as:

$$\lambda_m^p = \int_0^\infty \omega^m G_z^p(\omega) d\omega \tag{4.3.36}$$

or

$$\lambda_m^p = \sum_{q=1}^{3}\sum_k \sum_l L_k L_l 2\ Real \int_0^\infty \omega^m H_k^*(\omega) H_l(\omega) S_{\ddot{s}_{k_q} \ddot{s}_{l_q}}(\omega) d\omega \tag{4.3.37}$$

similarly to the uniform seismic input case, define:

$$\lambda_{mq,kl}^p = 2\ Real \int_0^\infty \omega^m H_k^*(\omega) H_l(\omega) S_{\ddot{s}_{k_q} \ddot{s}_{l_q}}(\omega) d\omega \tag{4.3.38}$$

where $\lambda_{mq,kl}^p$ is the mth cross-spectral moment associated with modes k and l for the spatially varying q^{th} ground motion component.

(4.3.37) can be rewritten as:

$$\lambda_m^p = \sum_{q=1}^{3}\sum_k \sum_l L_k L_l \lambda_{mq,kl}^p \tag{4.3.39}$$

which for zeroth moments can be written as:

$$\lambda_0^p = \sum_{q=1}^{3}\sum_k \sum_l L_k L_l \lambda_{0q,kl}^p \tag{4.3.40}$$

Define:

$$\epsilon_{0q,kl}^p = \frac{\lambda_{oq,kl}^p}{\sqrt{\lambda_{oq,kk}^p \lambda_{0q,ll}^p}} \tag{4.3.41}$$

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which is the modal response cross-correlation coefficient between modes k and l relative to the spatially varying q^{th} ground motion component.

Substituting (4.3.41) into (4.3.40),

$$\lambda_0^p = \sum_{q=1}^{3}\sum_{k}\sum_{l}\epsilon_{0q,kl}^p L_k L_l \sqrt{\lambda_{0q,kk}^p \lambda_{0q,ll}^p} \qquad (4.3.42)$$

Let's now make use of the following:

$$\lambda_0^p = \left[\frac{\overset{p}{Mean}(\max(z))}{p}\right]^2 \qquad (4.3.43)$$

$$\lambda_{0q,kk}^p = \left[\frac{\gamma_{kq}R_q^p(\omega_k,\beta_k)}{p_k}\right]^2 \qquad (4.3.44)$$

$$\lambda_{0q,ll}^p = \left[\frac{\gamma_{lq}R_l^p(\omega_l,\beta_l)}{p_l}\right]^2 \qquad (4.3.45)$$

where:

p, p_k, and p_l are peak factors

$R_q^p(\omega,\beta)$ is the modified response spectrum model for component q. (3.2.2.37)

(4.3.42) can be rewritten as:

$$\overset{p}{Mean}(\max(z)) = \left[\sum_{q=1}^{3}\sum_{k}\sum_{l}\epsilon_{0q,kl}^p L_k L_l \mid \gamma_{kq} \mid \gamma_{lq} \mid R_q^p(\omega_k,\beta_k)R_q^p(\omega_l,\beta_l)\right]^{1/2} \qquad (4.3.46)$$

Define:

$$E_{kq}^p = \mid \gamma_{kq} \mid L_k = \begin{cases} E_{kq} & \text{if } \gamma_{kq} > 0 \\ \\ -E_{kq} & \text{if } \gamma_{kq} < 0 \end{cases} \qquad (4.3.47)$$

$$E_{lq}^p = \mid \gamma_{lq} \mid L_l = \begin{cases} E_{lq} & \text{if } \gamma_{lq} > 0 \\ \\ -E_{lq} & \text{if } \gamma_{lq} < 0 \end{cases} \qquad (4.3.48)$$

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Using (4.3.47) and (4.3.48), (4.3.46) may be rewritten as:

$$\overset{p}{Mean}(\max(z)) = \left[\sum_{q=1}^{3} \sum_{k} \sum_{l} \epsilon^p_{0q,kl} E^p_{kq} R^p_q(\omega_k, \beta_k) E^p_{lq} R^p_q(\omega_l, \beta_l) \right]^{1/2} \tag{4.3.49}$$

Define:

$$\overset{p}{Mean}(\max(z_{kq})) = E^p_{kq} R^p_q(\omega_k, \beta_k) \tag{4.3.50}$$

$$\overset{p}{Mean}(\max(z_{lq})) = E^p_{lq} R^p_q(\omega_l, \beta_l) \tag{4.3.51}$$

(4.3.50) and (4.3.51) give the mean value of the maximum response of interest due to modes k and l respectively, associated with ground motion component q. (4.3.49) can be rewritten as:

$$\overset{p}{Mean}(\max(z)) = \left[\sum_{q=1}^{3} \sum_{k} \sum_{l} \epsilon^p_{0q,kl} \overset{p}{Mean}(\max(z_{kq})) \overset{p}{Mean}(\max(z_{lq})) \right]^{1/2} \tag{4.3.52}$$

Notice that (4.3.49) and (4.3.52) are analogous to (4.3.20) and (4.3.23), which was the goal aimed at.

Prior to expressing $\epsilon^p_{0q,kl}$ in terms of $\epsilon_{0q,kl}$, which is the last task in this section, let's mention that (4.3.52) can handle the case of no participation of a given mode under a given component of uniform excitations.

Say

$$\gamma_{kq} = 0,$$

then,

$$\overset{p}{Mean}(\max(z_{kq})) = L_k R^p_q(\omega_k, \beta_k) \tag{4.3.53}$$

where $R^p_q(\omega_k, \beta_k)$ is given by (3.2.2.47).

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Let's now express $\epsilon^p_{0q,kl}$ in terms of $\epsilon_{0q,kl}$. At first, let's express $\lambda^p_{0q,kl}$ in terms of $\lambda_{0q,kl}$.

$$\lambda^p_{0q,kl} = 2 \; Real \int_0^\infty H_k^*(\omega) H_l(\omega) S_{\ddot{s}_{kq}\ddot{s}_{lq}}(\omega) \tag{4.3.54}$$

Let's write $S_{\ddot{s}_{kq}\ddot{s}_{lq}}(\omega)$ out:

The cross-correlation function $R_{\ddot{s}_{kq}\ddot{s}_{lq}}(\tau)$ is given by:

$$
\begin{aligned}
R_{\ddot{s}_{kq}\ddot{s}_{lq}}(\tau) &= E\left[\ddot{s}_{kq}(t)\ddot{s}_{lq}(t+\tau)\right] \\
&= E\left[(\sum_{i=1}^n A_{kqi}\ddot{u}_{qi}(t))(\sum_{i=1}^n A_{lqi}\ddot{u}_{qi}(t+\tau))\right] \\
&= E\left[\sum_{i=1}^n \sum_{j=1}^n A_{kqi} A_{lqj} \ddot{u}_{qi}(t)\ddot{u}_{qj}(t+\tau)\right]
\end{aligned}
$$

which leads to:

$$R_{\ddot{s}_{kq}\ddot{s}_{lq}}(\tau) = \sum_{i=1}^n \sum_{j=1}^n A_{kqi} A_{lqj} R_{\ddot{u}_{qi}\ddot{u}_{qj}}(\tau) \tag{4.3.55}$$

Taking Fourier transforms in (4.3.55),

$$S_{\ddot{s}_{kq}\ddot{s}_{lq}}(\omega) = \sum_{i=1}^n \sum_{j=1}^n A_{kqi} A_{lqj} S_{\ddot{u}_{qi}\ddot{u}_{qj}}(\omega) \tag{4.3.56}$$

Hence,

$$
\begin{aligned}
\lambda^p_{0q,kl} &= 2 \; Real \int_0^\infty H_k^*(\omega) H_l(\omega) \sum_{i=1}^n \sum_{j=1}^n A_{kqi} A_{lqj} S_{\ddot{u}_{qi}\ddot{u}_{qj}}(\omega)\,d\omega \\
&= \sum_{i=1}^n \sum_{j=1}^n A_{kqi} A_{lqj} 2 \; Real \int_0^\infty H_k^*(\omega) H_l(\omega) S_{\ddot{u}_{qi}\ddot{u}_{qj}}(\omega)\,d\omega \\
\lambda^p_{0q,kl} &= \sum_{i=1}^n \sum_{j=1}^n A_{kqi} A_{lqj} \int_0^\infty H_k^*(\omega) H_l(\omega) S_{\ddot{u}_{qi}\ddot{u}_{qj}}(\omega)\,d\omega
\end{aligned}
\tag{4.3.57}
$$

Analogously to the case of a single ground motion component, define:

$$r_{qijkl} = \frac{\int_{-\infty}^\infty H_k^*(\omega) H_l(\omega) S_{\ddot{u}_{qi}\ddot{u}_{qj}}(\omega)\,d\omega}{\int_{-\infty}^\infty H_k^*(\omega) H_l(\omega) S_{\ddot{u}_{0q}}(\omega)\,d\omega} \tag{4.3.58}$$

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$$r_{qijkl} = \frac{\int_{-\infty}^{\infty} H_k^*(\omega) H_l(\omega) S_{\ddot{u}_{qi}\ddot{u}_{qj}}(\omega) d\omega}{\lambda_{0q,kl}} \qquad (4.3.59)$$

where r_{qij} is such that:

$$S_{\ddot{u}_{qi}\ddot{u}_{qj}}(\omega) = r_{qij} S_{\ddot{u}_{0q}}(\omega) \qquad (4.3.60)$$

It is the coherency function between stations i and j along the qth component of ground motion.

Hence,

$$\lambda_{0q,kl}^p = \lambda_{0q,kl} \sum_{i=1}^{n} \sum_{j=1}^{n} A_{kqi} A_{lqj} r_{qijkl} \qquad (4.3.61)$$

Consequently,

$$\lambda_{0q,kk}^p = \lambda_{0q,kk} \sum_{i=1}^{n} \sum_{j=1}^{n} A_{kqi} A_{kqj} r_{qijkk} \qquad (4.3.62)$$

$$\lambda_{0q,ll}^p = \lambda_{0q,ll} \sum_{i=1}^{n} \sum_{j=1}^{n} A_{lqi} A_{lqj} r_{qijll} \qquad (4.4.63)$$

Note that:

$$r_{qijkk} = \frac{\int_0^{\infty} \rho_{qij} \mid H_k(\omega) \mid^2 G_{\ddot{u}_{0q}}(\omega) d\omega}{\int_0^{\infty} \mid H_k(\omega) \mid^2 G_{\ddot{u}_{0q}}(\omega) d\omega} \qquad (4.3.64)$$

$$r_{qijll} = \frac{\int_0^{\infty} \rho_{qij} \mid H_l(\omega) \mid^2 G_{\ddot{u}_{0q}}(\omega) d\omega}{\int_0^{\infty} \mid H_l(\omega) \mid^2 G_{\ddot{u}_{0q}}(\omega) d\omega} \qquad (4.4.65)$$

Comparing (4.3.62) and (4.3.63) with (3.2.2.35), one remarks that:

$$r_{qijkk} = \rho_{qijk} \qquad (4.3.66)$$

$$r_{qijll} = \rho_{qijl} \qquad (4.3.67)$$

where ρ_{qij} is the real part of r_{qij}.

Therefore, $\epsilon_{0q,kl}^{p}$ can be written as:

$$\epsilon_{0q,kl}^{p} = \frac{\lambda_{oq,kl}}{\sqrt{\lambda_{oq,kk}\lambda_{0q,ll}}} \frac{\sum_{i=1}^{n}\sum_{j=1}^{n}A_{kqi}A_{lqj}r_{qijkl}}{\left\{(\sum_{i=1}^{n}\sum_{j=1}^{n}A_{kqi}A_{kqj}\rho_{qijk})(\sum_{i=1}^{n}\sum_{j=1}^{n}A_{lqi}A_{lqj}\rho_{qijl})\right\}^{1/2}} \qquad (4.3.68)$$

since

$$\epsilon_{0q,kl} = \frac{\lambda_{oq,kl}}{\sqrt{\lambda_{oq,kk}\lambda_{0q,ll}}} \qquad (4.3.18)$$

$\epsilon_{0q,kl}^{p}$ can be expressed as:

$$\epsilon_{0q,kl}^{p} = \epsilon_{0q,kl} \frac{\sum_{i=1}^{n}\sum_{j=1}^{n}A_{kqi}A_{lqj}r_{qijkl}}{\left\{(\sum_{i=1}^{n}\sum_{j=1}^{n}A_{kqi}A_{kqj}\rho_{qijk})(\sum_{i=1}^{n}\sum_{j=1}^{n}A_{lqi}A_{lqj}\rho_{qijl})\right\}^{1/2}} \qquad (4.3.69)$$

Finally, a modal combination rule has been proposed for the case of uncorrelated non uniform multicomponent translational seismic excitations (4.3.49) and (4.3.52). In this combination rule, the modal cross-correlation coefficients have been expressed in terms of the homologous coefficients for the case of uniform seismic input (4.3.69).

Chapter 5

Validation of the Theoretical Response Spectrum Model
Through Digital Simulation of Seismic Ground Motion

5.1 Introduction

The modified response spectrum model developed in Chapter 3 is a means for the computation of modal responses for the case of structures subjected to imperfectly correlated seismic excitations. This model is the result of a theoretical development based upon the probabilistic modeling of earthquake ground motions by means of a homogeneous Gaussian space-time random field. The purpose of the present chapter is to validate the model through digital simulation of seismic ground motion. In other words, a sample of a random field will be simulated, and the response spectrum, corresponding to a given effective seismic loading expressed as a combination of support motions (temporal processes), will be computed and compared with the prediction given by the modified response spectrum model. In the context of the present thesis, a modified response spectrum model has been derived to handle general situations. However, for the purposes of this chapter, two cases will be examined; namely those of structures having two and three support points, and subjected to a single ground motion component.

5.2 Simulation of Unidimensional Homogeneous Gaussian Space-Time Random Fields Using Spectral Representation

The Spectral Representation Method proposed by Shinozuka et al. [41] is used for the simulation of an unidimensional homogeneous Gaussian space-time random field which is viewed as a collection of temporal random processes at each of the spatial locations. This section aims at presenting, very concisely, the essence of the aforementioned method.

As its name indicates, the first step in this approach is the specification of a target cross-spectral density matrix:

$$[S(\omega)] = \begin{bmatrix} S_{11}(\omega) & S_{12}(\omega) & \cdots & S_{1n}(\omega) \\ S_{21}(\omega) & S_{22}(\omega) & \cdots & S_{2n}(\omega) \\ & & & \\ S_{n1}(\omega) & S_{n2}(\omega) & & S_{nn}(\omega) \end{bmatrix} \tag{5.2.1}$$

where:

n is the number of spatial locations (support points).

$S_{ij}(\omega)$ is the cross-spectral density function between locations i and j.

Note that due to the assumption of homogeneity, the target cross-spectral density matrix $[S(\omega)]$ is Hermitian.

The following step in the method is to find a matrix $[H(\omega)]$ such that:

$$[S(\omega)] = [H(\omega)][H^*(\omega)]^T \tag{5.2.2}$$

where:

* indicates complex conjugate

T indicates transpose

To find $[H(\omega)]$ in an efficient way, we assume it to be a lower triangular matrix. In other words, a Choleski decomposition of the matrix $[S(\omega)]$ is sought after.

It turns out that the elements of the matrix $[H(\omega)]$ are given by:

$$H_{kk}(\omega) = \left[\frac{D_k(\omega)}{D_{k-1}(\omega)}\right]^{1/2} \tag{5.2.3}$$

where:

$k = 1, 2, ..., n$

$D_k(\omega)$ is the kth principal minor of $[S(\omega)]$ with $D_0(\omega)$ being defined as unity.

and

$$H_{jk}(\omega) = H_{kk}(\omega)\frac{[S(\omega)]\begin{pmatrix} 1, 2, ..., k-1, j \\ \\ 1, 2, ..., k-1, k \end{pmatrix}}{D_k(\omega)} \tag{5.2.4}$$

where:

$k = 1, 2, ..., n$

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$$j = k+1, ..., n$$

$$[S(\omega)] \begin{pmatrix} 1, 2, ..., k-1, j \\ \\ 1, 2, ..., k-1, k \end{pmatrix}$$

is the determinant of a submatrix of $[S(\omega)]$ obtained by deleting all elements except the $(1, 2, ..., k-1, j)$ th rows and $(1, 2, ..., k-1, k)$ th columns.

It is worth noting that the above decomposition is valid only when the matrix $[S(\omega)]$ is Hermitian and positive definite. Since the cross-spectral density matrix $[S(\omega)]$ is known to be only non-negative definite, special consideration is needed in those cases where $[S(\omega)]$ has a zero principal minor [41].

Once the matrix $[H(\omega)]$ is determined, the last step in the method is to simulate the field $u_j(t)$ $(j = 1, 2, ..., n)$ by the following series:

$$u_j(t) = 2 \sum_{m=1}^{j} \sum_{l=1}^{N} \mid H_{jm}(\omega_l) \mid \sqrt{\Delta\omega} \cos(\omega_l t + \theta_{jm}(\omega_l) + \phi_{ml}) \qquad (5.2.5)$$

where:

$\Delta\omega$ is the frequency increment.

$\omega_l = l\Delta\omega$

N is such that $N\Delta\omega$ is an upper cut-off frequency.

$\theta_{jm}(\omega_l)$ is a phase spectrum given by:

$$\theta_{jm}(\omega_l) = \tan^{-1} \left[\frac{Im H_{jm}(\omega_l)}{Re H_{jm}(\omega_l)} \right] \qquad (5.2.6)$$

ϕ_{ml} are independent random phase angles uniformly distributed over the range

$(0, 2\pi)$.

It is worth mentioning that the digital generation of sample functions (5.2.5) can be efficiently performed by means of the FFT technique.

Finally, it is to be noted that the simulated processes are asymptotically Gaussian as N becomes large by virtue of the central limit theorem. Also, it may be shown that in terms of ensemble averages:

$E[u_j(t)] = 0$

$E[u_j(t)u_k(t+\tau)] = R_{jk}(\tau)$ is the Wiener-Khintchine transform of $S_{jk}(\omega)$.

So long as the assumption of independent random phase angles uniformly distributed over the range $(0, 2\pi)$ is valid.

5.3   Case of Two Support Points

The situation that is of interest in the present section is that of a N-story single bay frame structure behaving as a shear beam, subjected to a single horizontal component of partially correlated ground motions. For the sake of simplicity of illustration, column stiffnesses will be assumed equal (fig. 3.1). As was arrived at in section 3.2.3.1, the modified response spectrum model is expressed as follows:

$$R^p(\omega_k, \beta_k) = R(\omega_k, \beta_k) \left[\frac{1 + \rho_{12k}}{2}\right]^{1/2} \tag{3.2.3.1.7}$$

where:

$R(\omega, \beta)$ is the response spectrum at the site of interest.

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$$\rho_{12k} = \frac{\int_0^\infty \rho_{12} \mid H_k(\omega) \mid^2 G_{\ddot{u}_0}(\omega) d\omega}{\int_0^\infty \mid H_k(\omega) \mid^2 G_{\ddot{u}_0}(\omega) d\omega}$$

ρ_{12} is the frequency-dependent spatial correlation coefficient between stations 1 and 2.

Incidentally, it is worth mentioning that (3.2.3.1.7) expresses the modified response spectrum model for the case of symmetrical modes with respect to midspan in a simply supported bridge (3.3.2.12).

It was also shown that the effective seismic loading, so far as the floor responses are concerned, is $\ddot{s}(t) = \frac{1}{2}(\ddot{u}_1(t) + \ddot{u}_2(t))$. Therefore, the aim herein is to examine how well the modified response spectrum model given by (3.2.3.1.7) predicts the response spectrum corresponding to $\ddot{s}(t)$. To do so, $\ddot{u}_1(t)$ and $\ddot{u}_2(t)$ will be simulated as part of an unidimensional homogeneous Gaussian random field, using spectral representation.

The target cross-spectral density matrix is given by:

$$[S(\omega)] = \begin{bmatrix} S_{11}(\omega) & S_{12}(\omega) \\ \\ S_{21}(\omega) & S_{22}(\omega) \end{bmatrix} \tag{5.3.1}$$

where the assumption of a common auto spectrum to all spatial locations is maintained in order to render the validation process meaningful. Hence,

$$S_{11}(\omega) = S_{22}(\omega) = S(\omega) \tag{5.3.2}$$

Also,

$$S_{12}(\omega) = S_{21}^*(\omega) = r_{12}(\omega, D_{12})S(\omega) \tag{5.3.3}$$

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where r_{12} is the coherency function.

By virtue of (5.2.3) and (5.2.4), the elements of the matrix $[H(\omega)]$ are given by:

$$H_{11}(\omega) = \sqrt{S(\omega)} \tag{5.3.4}$$

$$H_{21}(\omega) = \sqrt{S(\omega)}r_{21} \tag{5.3.5}$$

$$H_{22}(\omega) = \sqrt{S(\omega)}\sqrt{1-\mid r_{21}\mid^2} \tag{5.3.6}$$

And (5.2.5) shows that $\ddot{u}_1(t)$ and $\ddot{u}_2(t)$ can be simulated by the following series:

$$\ddot{u}_1(t) = 2\sum_{j=1}^{N}\mid H_{11}(\omega_j)\mid\sqrt{\Delta\omega}\cos(\omega_j t + \phi_{1j}) \tag{5.3.7}$$

$$\ddot{u}_2(t) = 2\sum_{j=1}^{N}\mid H_{21}(\omega_j)\mid\sqrt{\Delta\omega}\cos(\omega_j t + \theta_{21}(\omega_j) + \phi_{1j})$$

$$+\mid H_{22}(\omega_j)\mid\sqrt{\Delta\omega}\cos(\omega_j t + \phi_{2j}) \tag{5.3.8}$$

where:

$N\Delta\omega$ is an upper cut-off frequency

$\omega_j = j\Delta\omega$

$$\theta_{21}(\omega) = \tan^{-1}\left[\frac{ImH_{21}(\omega)}{ReH_{21}(\omega)}\right]$$

ϕ_{ij} and ϕ_{2j} are independent random phase angles uniformly distributed between 0 and 2π.

A Kanai-Tajimi type of autospectrum has been chosen:

$$S(\omega) = S_0 \frac{\omega_g^4 + 4\beta_g^2\omega_g^2\omega^2}{(\omega_g^2 - \omega^2)^2 + 4\beta_g^2\omega_g^2\omega^2}$$ (5.3.9)

where: ω_g and β_g are the ground characteristics chosen to be:

$\omega_g = 6\pi\,(rd/s)$

$\beta_g = 0.5$

S_0 is a constant equal to $\frac{1}{3600}$ so that the peak acceleration is about 0.5g.

A model for the coherency function proposed by Loh and Yeh [30] has been adopted:

$$r(\omega, D) = \exp(-\alpha \frac{\omega\,|\,D\,|}{2\pi V})\exp(i\omega\frac{D}{V})$$ (5.3.10)

α is a constant equal to 0.125

D is the distance between the two spatial locations

V is the phase velocity of the dominant wave (or the apparent wave-propagation velocity between the two spatial locations).

The FFT (Fast Fourier Transform) technique is used for the computation of $\ddot{u}_1(t)$ and $\ddot{u}_2(t)$. The Fourier transform at zero frequency is taken to be null so as to yield zero-mean temporal processes. For the cases studied, the number of samplings has been $2^{11} = 2048$, with a time increment of 0.01 sec. These data lead to a time history duration of 20.48 sec. It is to be noted that the dominant frequency chosen in the autospectra, 3hz, is much smaller than the Nyquist frequency, 50 hz in these cases, which rules out aliasing problems. For the sake of making the obtained time histories more realistic, they are windowed, once computed, by means of the following function:

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$$W(t) = \begin{cases} \dfrac{t}{0.1T} & 0 < t < 0.1T \quad \text{(build-up phase)} \\[2mm] 1 & 0.1T < t < 0.9T \quad \text{(strong phase)} \\[2mm] \dfrac{-t}{0.1T} & 0.9T < t < T \quad \text{(die-out phase)} \end{cases}$$

where T is the duration of the time history.

In this section, three case studies are looked at:

1. $V = 500m/s$ and $D = 200m$.

The windowed acceleration time histories at locations 1 and 2 are shown in fig. 5.1 and fig. 5.2. The pseudo-acceleration spectra at locations 1 and 2, the pseudo-velocity spectra at locations 1 and 2, and the relative displacement spectra at locations 1 and 2 are shown in fig. 5.3, fig. 5.4, and fig. 5.5 respectively. It can be observed that spectra at both locations are close, which is in agreement with the assumption of spatially homogeneous spectra made prior to developing the model. Exact spectra for $\ddot{s}(t) = \dfrac{1}{2}(\ddot{u}_1(t) + \ddot{u}_2(t))$ are computed and compared against the model predictions given by (3.2.3.1.7). Satisfactory model predictions for the three spectra are shown in fig. 5.6, fig. 5.7, and fig. 5.8. Incidentally, a damping ratio equal to 1% has been used for all spectra. In fig. 5.9, fig. 5.10, and fig. 5.11, an assessment of the effect of assuming full coherence is aimed at. In other words, the same exact spectra as those in fig. 5.6, fig. 5.7, and fig. 5.8 are compared against model predictions which disregard loss of coherence and account for wave travelling effects exclusively. Alternatively, in fig. 5.6, fig. 5.7, and fig. 5.8, ρ_{12}, the frequency dependent spatial correlation coefficient has

been taken as:

$$\rho_{12}(\omega, D) = \exp(-\alpha \frac{\omega \mid D \mid}{2\pi V}) \cos \omega \frac{D}{V} \qquad (5.3.11)$$

which rightly accounts for both loss of coherence effects (exponentially decaying function), and wave travelling effects. Whereas in fig. 5.9, fig. 5.10, and fig. 5.11, ρ_{12} has been taken as:

$$\rho_{12} = \cos \omega \frac{D}{V} \qquad (5.3.12)$$

which, wrongly but purposely accounts for wave travelling effects only.

Upon observation of these figures, one can see that, for the case at hand, the impact of neglecting loss of coherence effects is practically null, except for those frequencies where $\frac{\omega D}{V}$ is an odd multiple of π, ($\cos \frac{\omega D}{V} = -1$), and where the model results slightly underpredict the exact ones. The justification of this observation is that at those very frequencies, the model is applied to a situation where the components of $\ddot{u}_1(t)$ and $\ddot{u}_2(t)$ are fully coherent but 180 degrees out of phase, and hence, cancellation effects are important. However, it is worth noting that if a stronger decay were considered in the coherency function, the above conclusions could be changed.
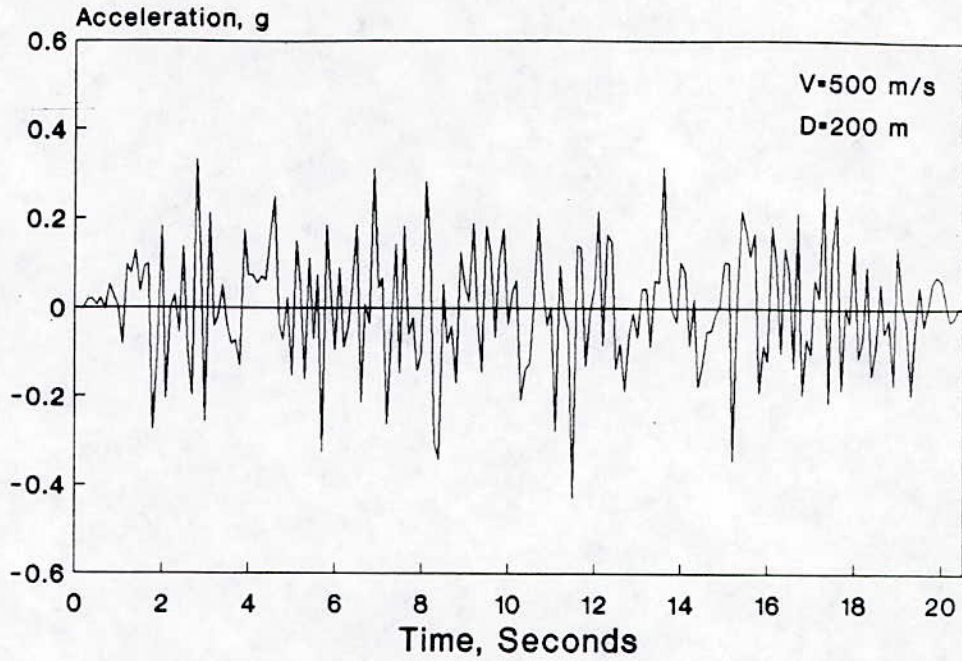
Figure 5.1 Windowed Acceleration Time History at Location 1
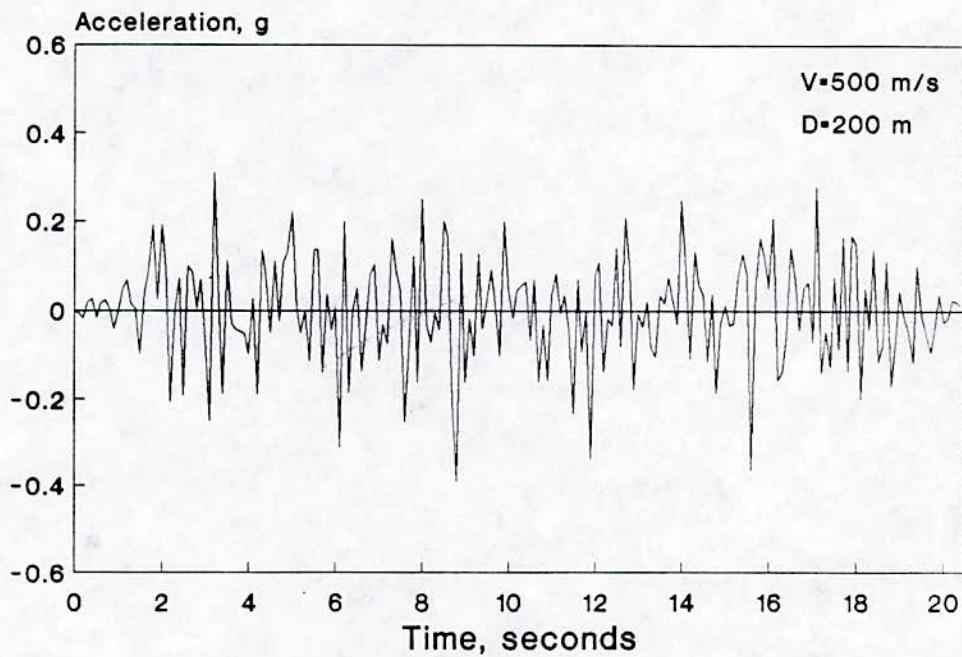
Figure 5.2 Windowed Acceleration Time History at Location 2

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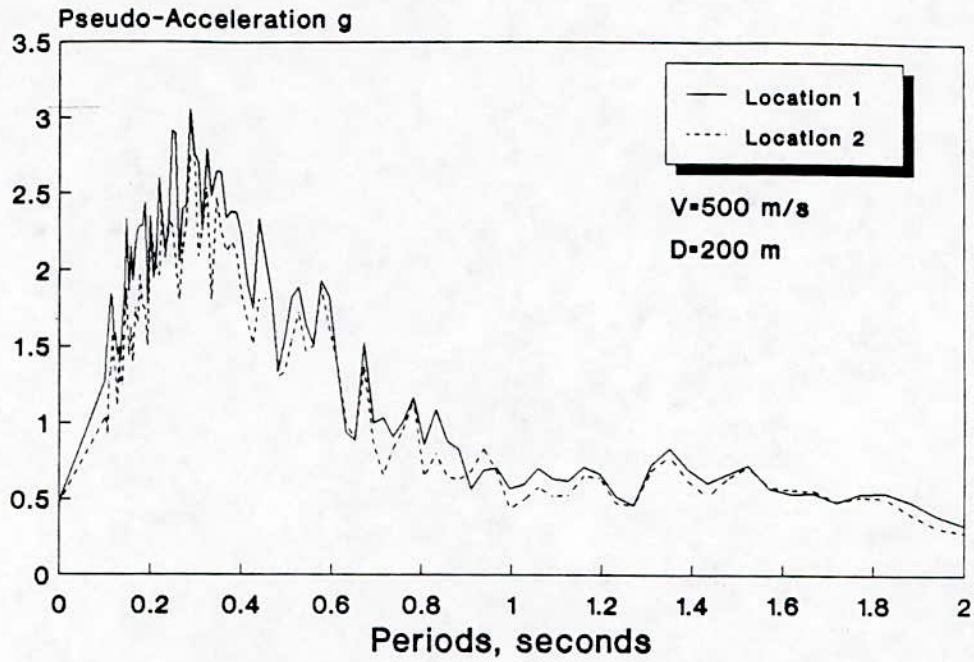
Figure 5.3 Acceleration Spectra at Locations 1 and 2
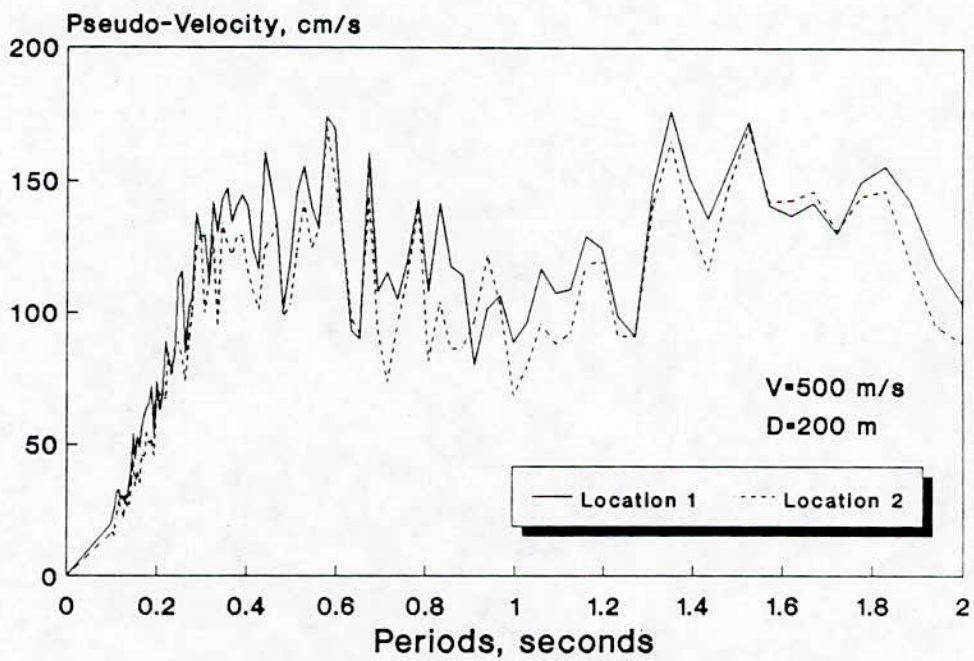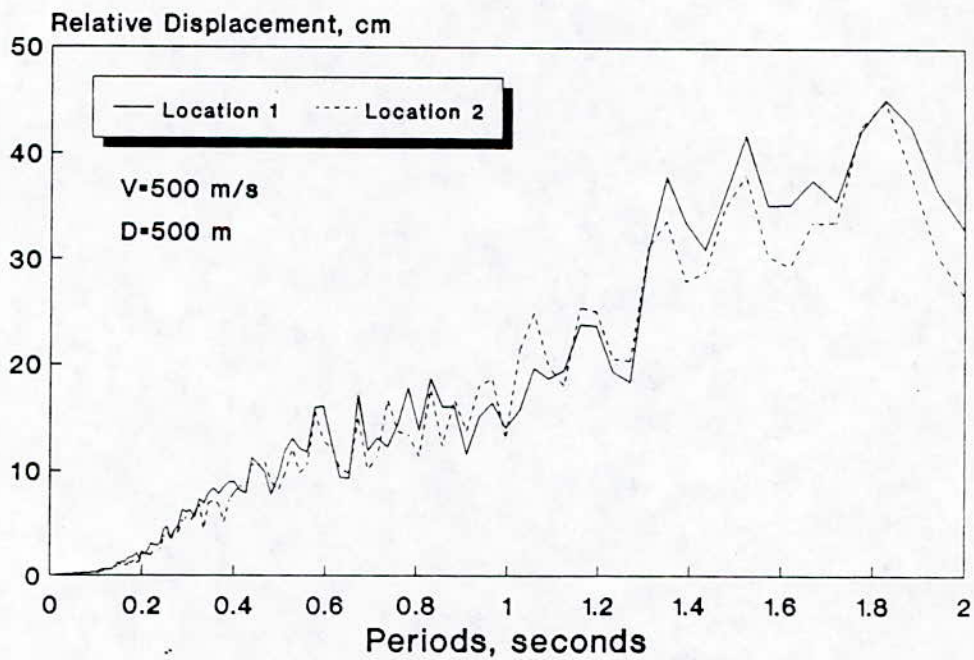

Figure 5.4 Velocity Spectra at Locations 1 and 2

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Relative Displacements, cm

V=500 m/s
D=200 m

Location 1 ----- Location 2

Periods, seconds
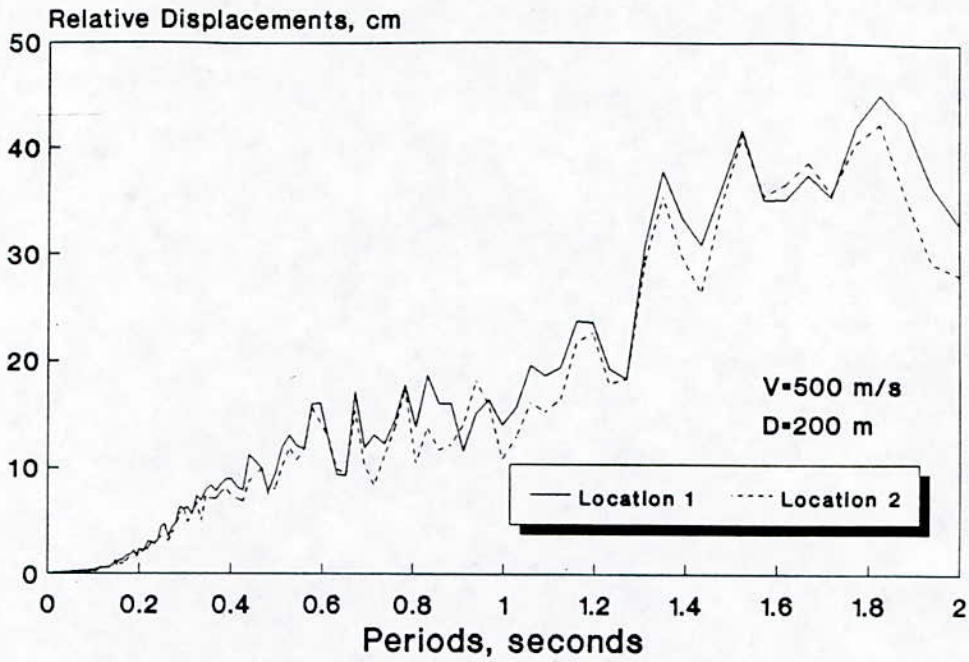
Figure 5.5 Displacement Spectra at Locations 1 and 2

Pseudo-Acceleration, g

Exact Prediction ----- Model Prediction

V=500 m/s
D=200 m

Periods, seconds
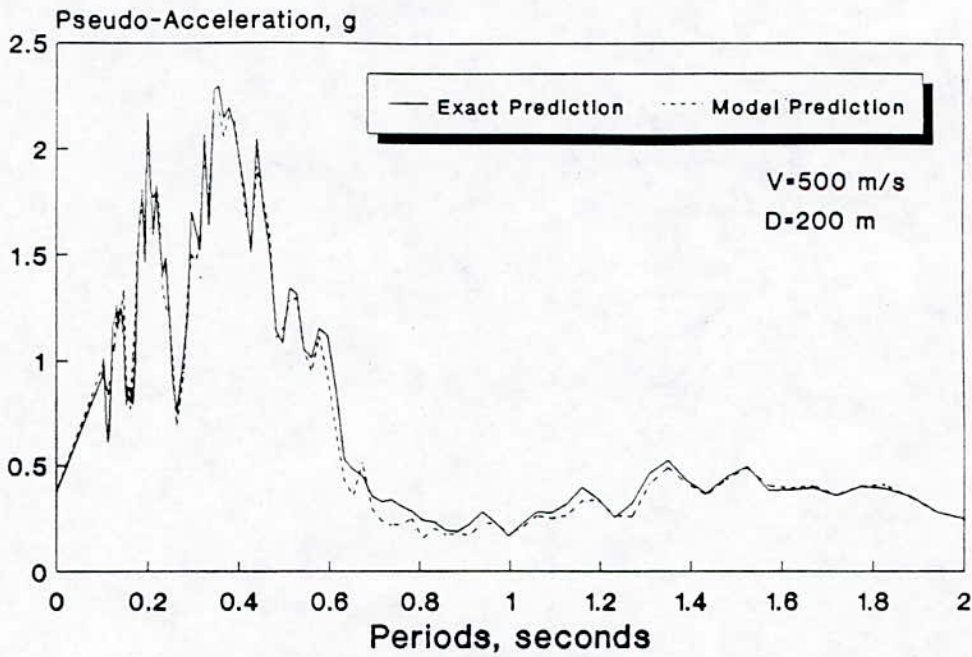
Figure 5.6 Acceleration Spectra: Exact vs Model Prediction

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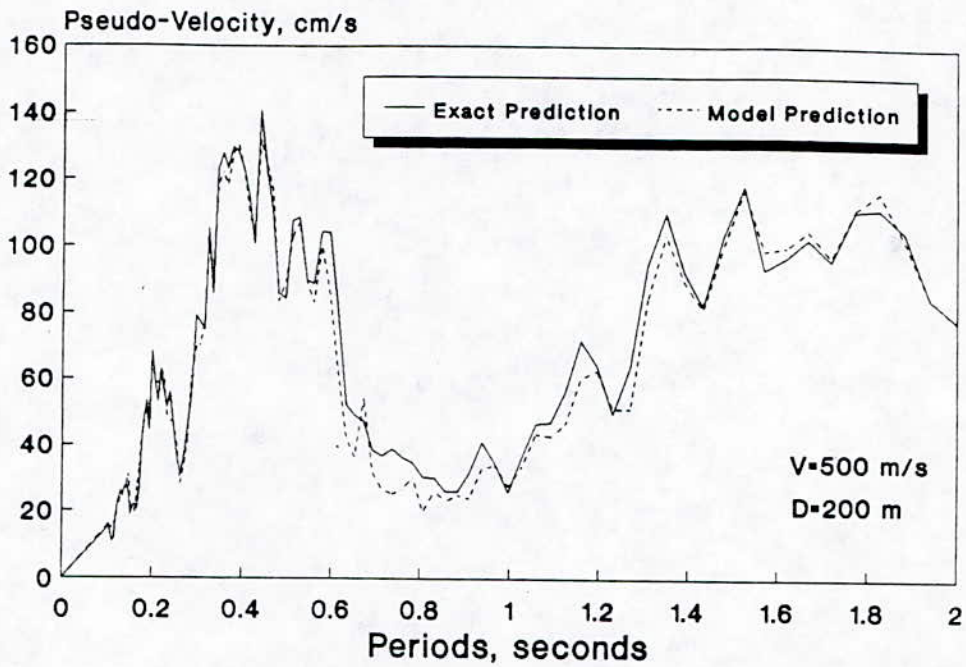
Figure 5.7 Velocity Spectra: Exact vs Model Prediction
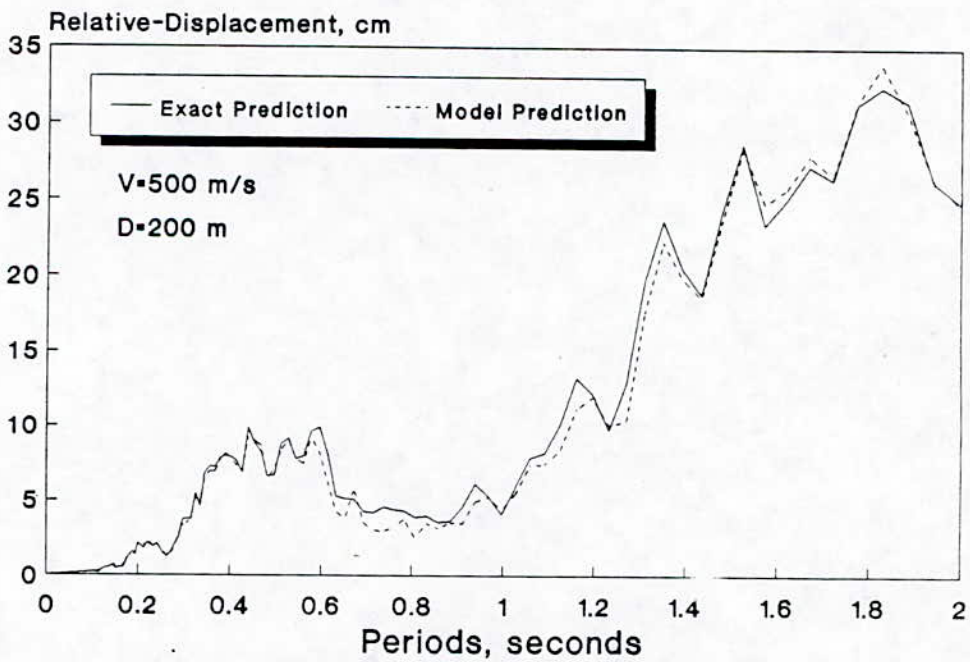
Figure 5.8 Displacement Spectra: Exact vs Model Prediction

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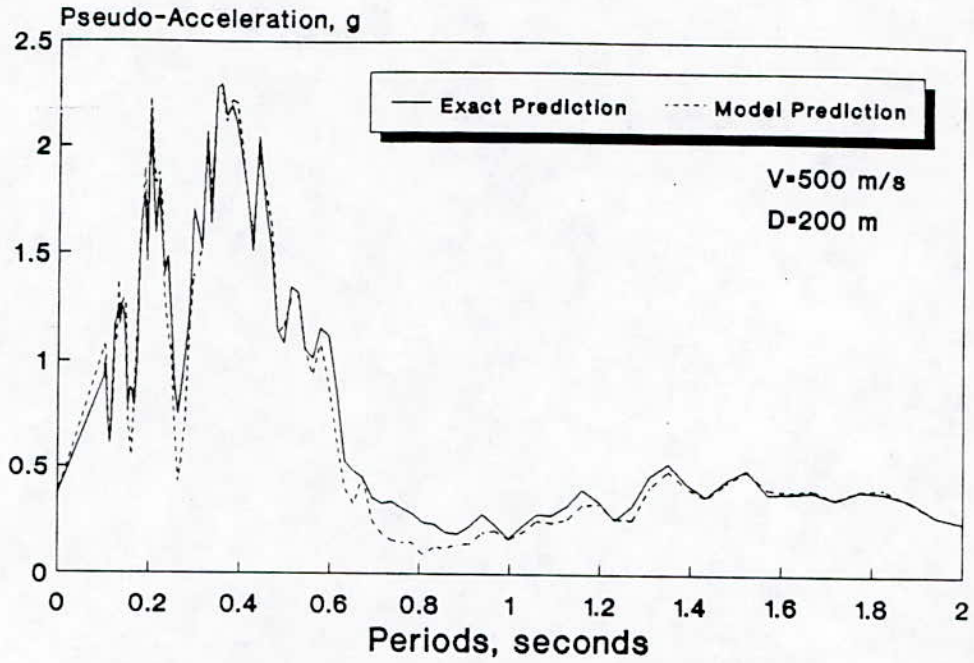
Figure 5.9 Effect of Full Coherence on Acceleration Spectra
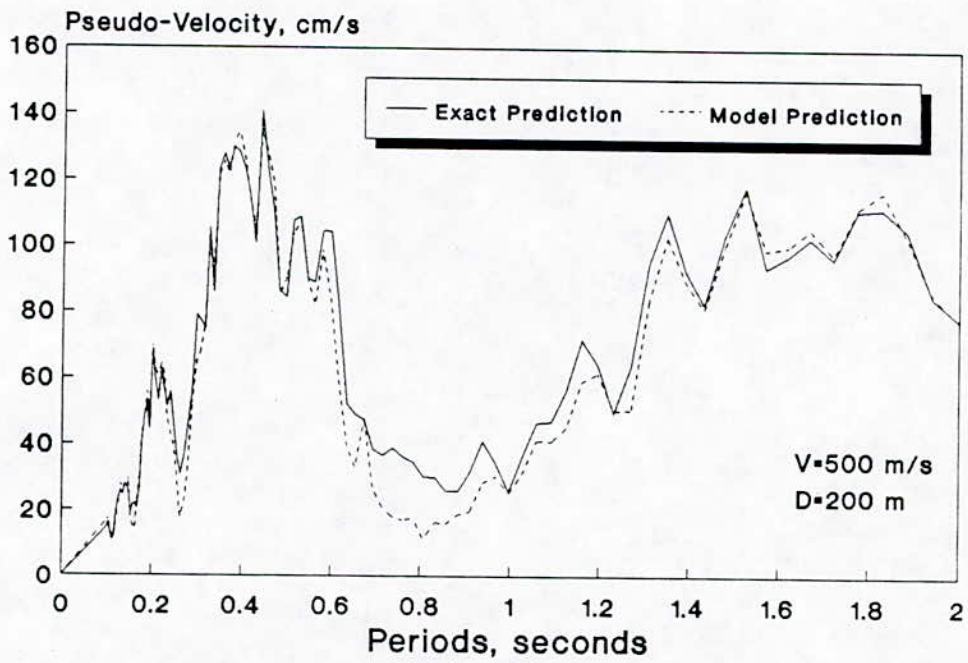
Figure 5.10 Effect of Full Coherence on Velocity Spectra

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Relative Displacements, cm
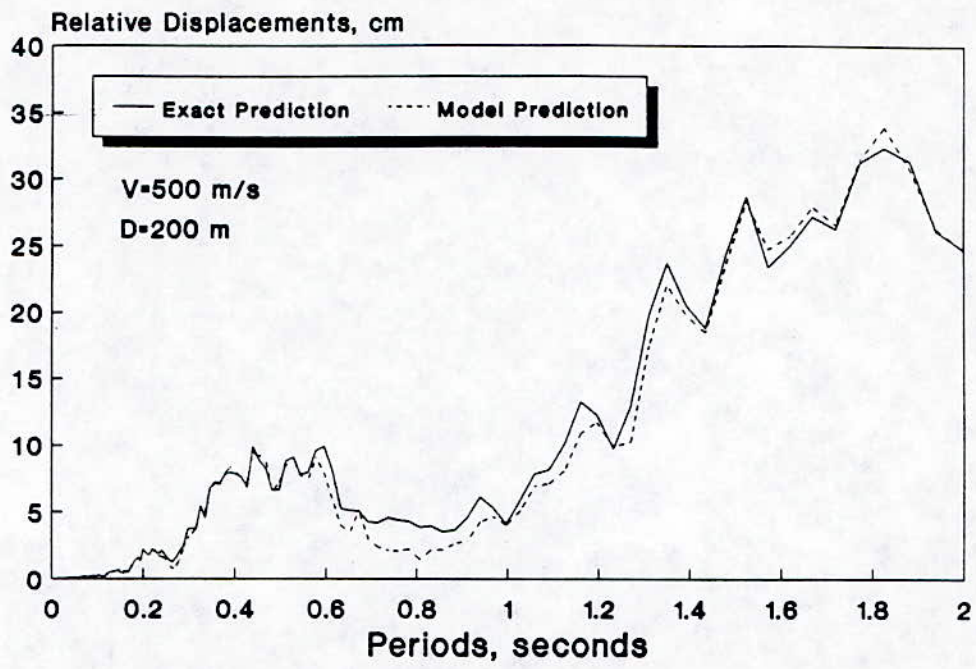
Figure 5.11 Effect of Full Coherence on Displacement Spectra

2. $V = 500m/s$ and $D = 500m$

Apart from the fact that the separating distance has been increased to $D = 500m$ in this case study, the results observed are very similar to those of the previous one. The figures are in a sequence analogous to that of case study 1.
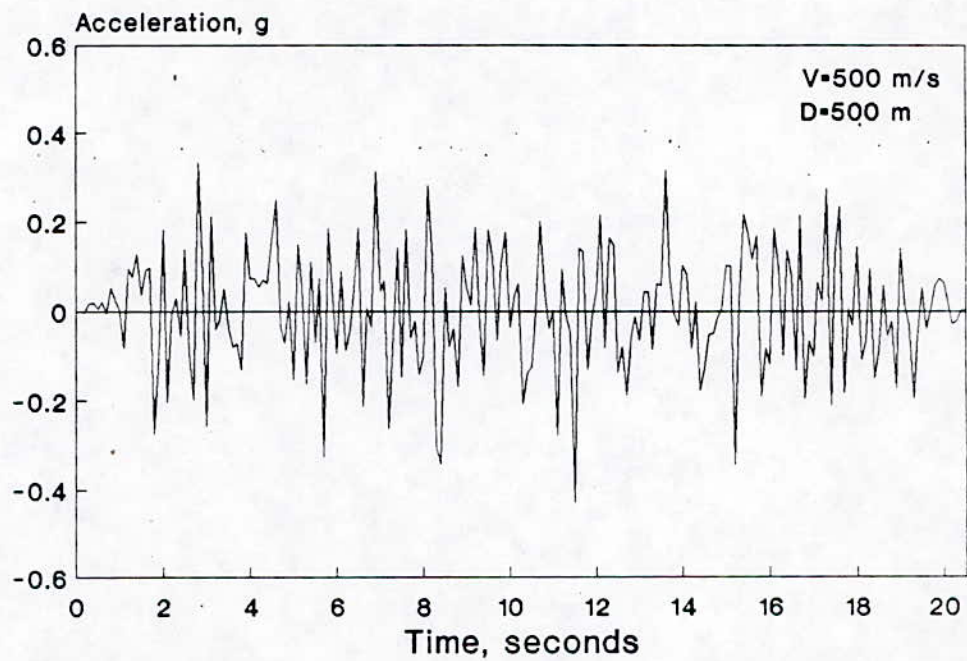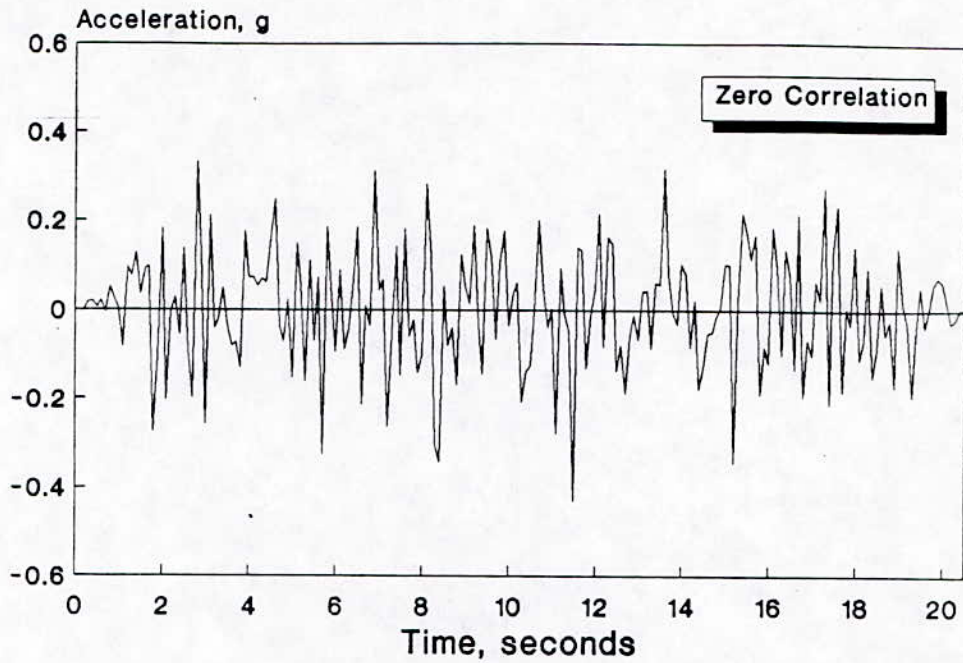

Figure 5.12 Windowed Acceleration Time History at Location 1
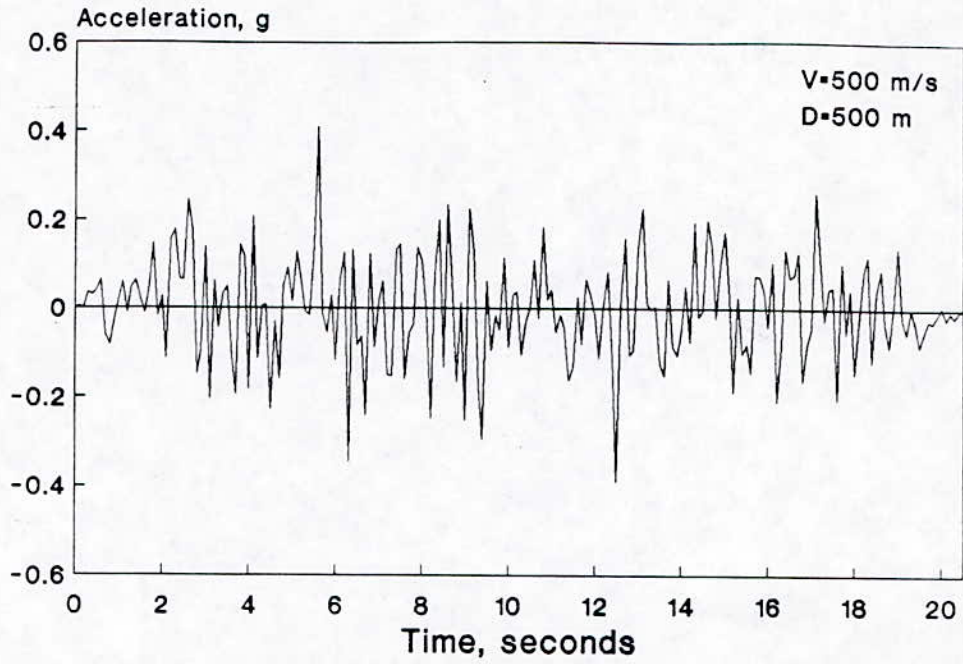
Figure 5.13 Windowed Acceleration Time History at Location 2
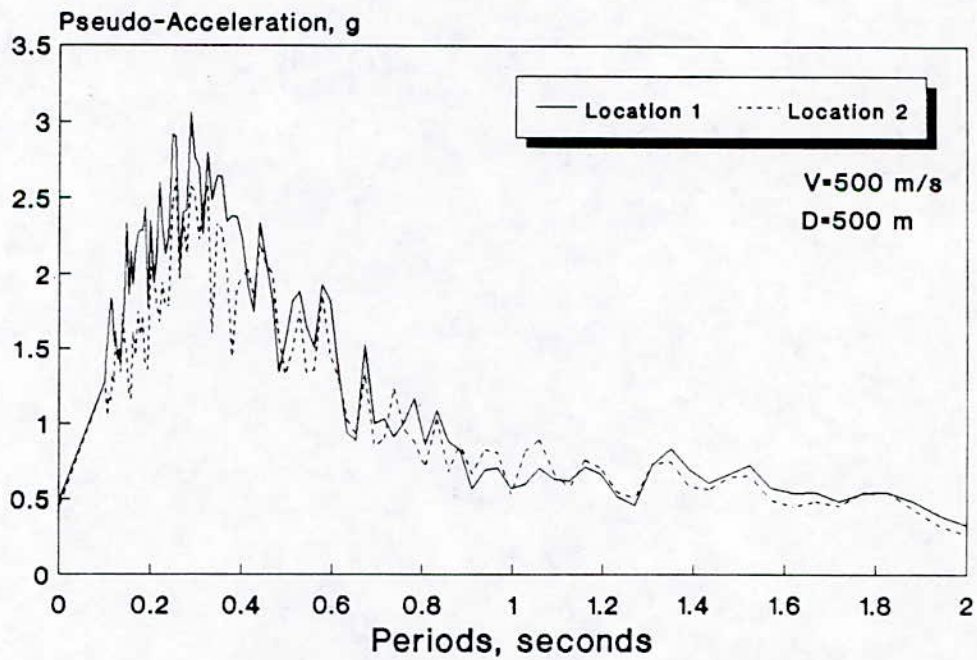
Figure 5.14 Acceleration Spectra at Locations 1 and 2

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Pseudo-Velocity, cm/s
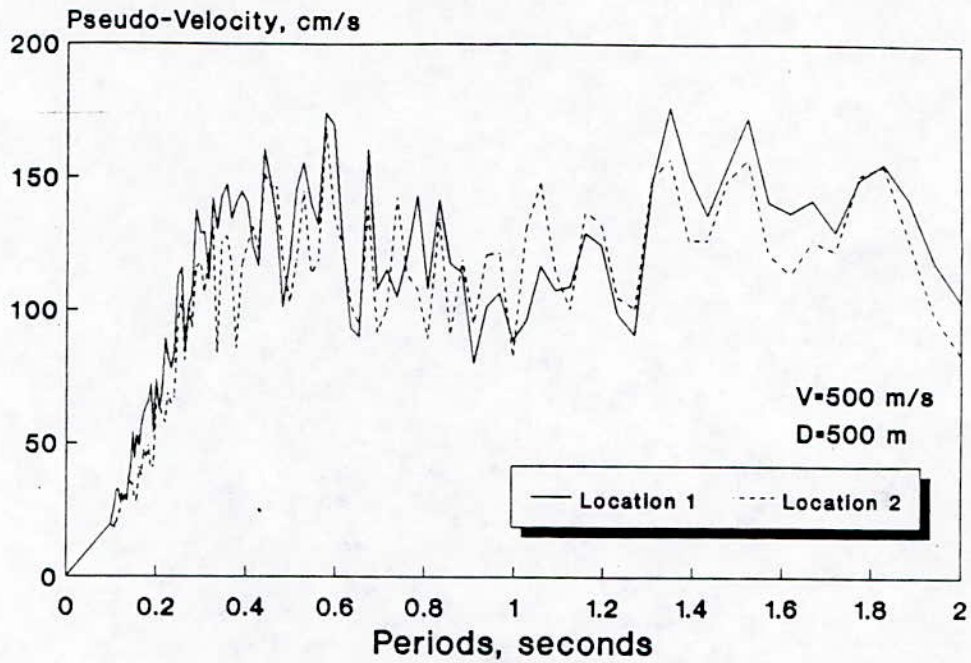
Figure 5.15 Velocity Spectra at Locations 1 and 2

Relative Displacement, cm

Figure 5.16 Displacement Spectra at Locations 1 and 2

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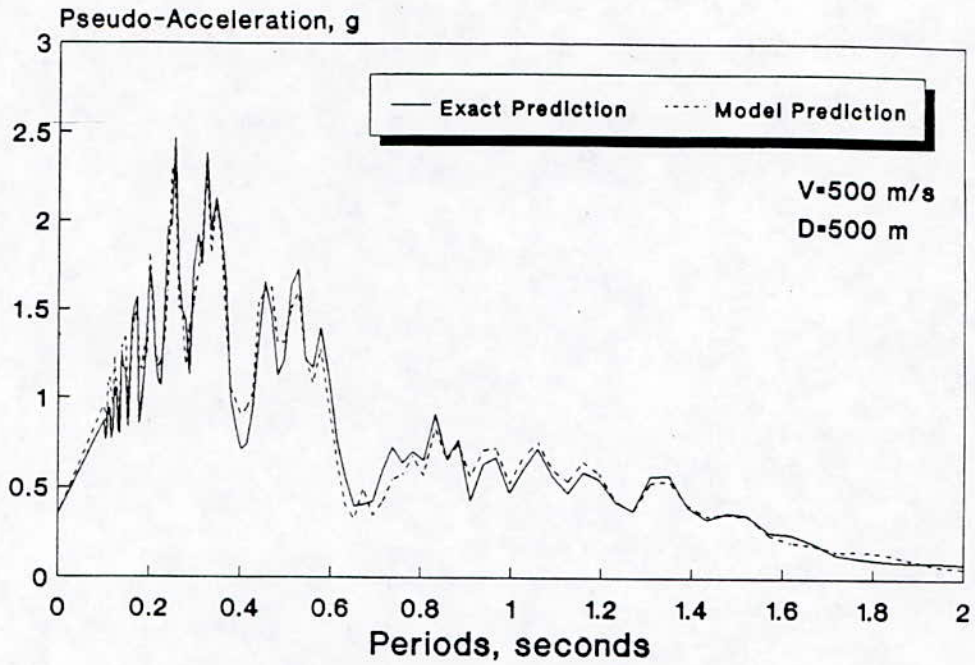
Figure 5.17 Acceleration Spectra: Exact vs Model Prediction
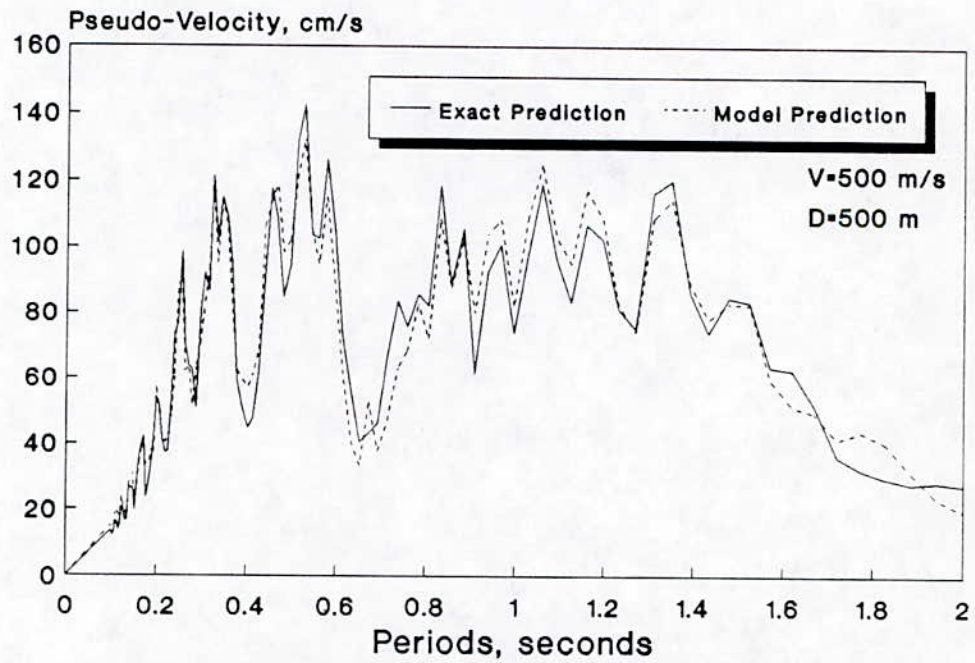
Figure 5.18 Velocity Spectra: Exact vs Model Prediction
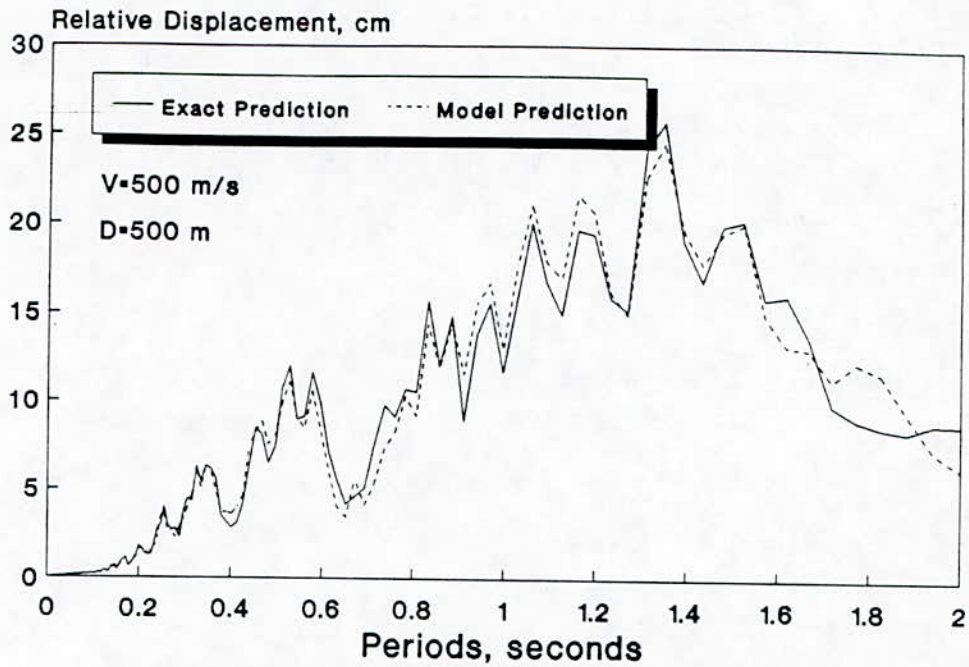
Figure 5.19 Displacement Spectra: Exact vs Model Prediction
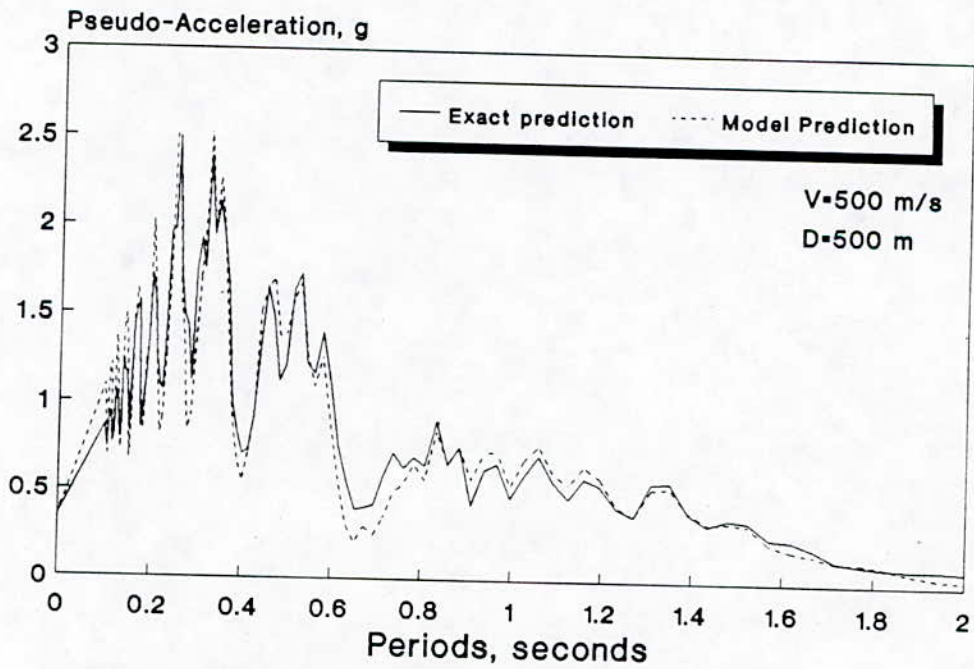
Figure 5.20 Effect of Full Coherence on Acceleration Spectra

Pseudo-Velocity, cm/s
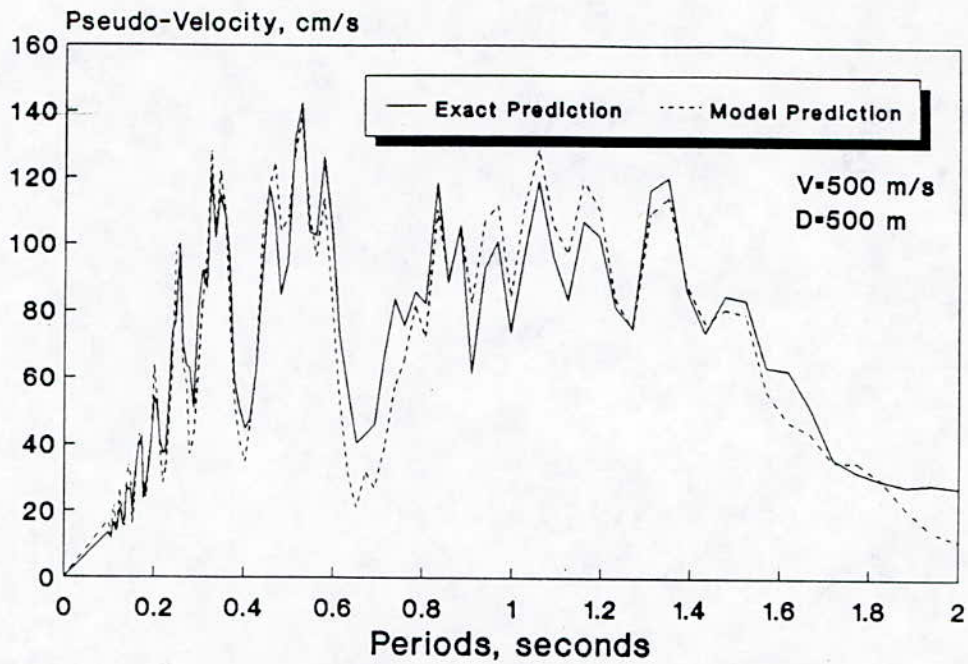
Figure 5.21 Effect of Full Coherence on Velocity Spectra

Relative Displacement, cm
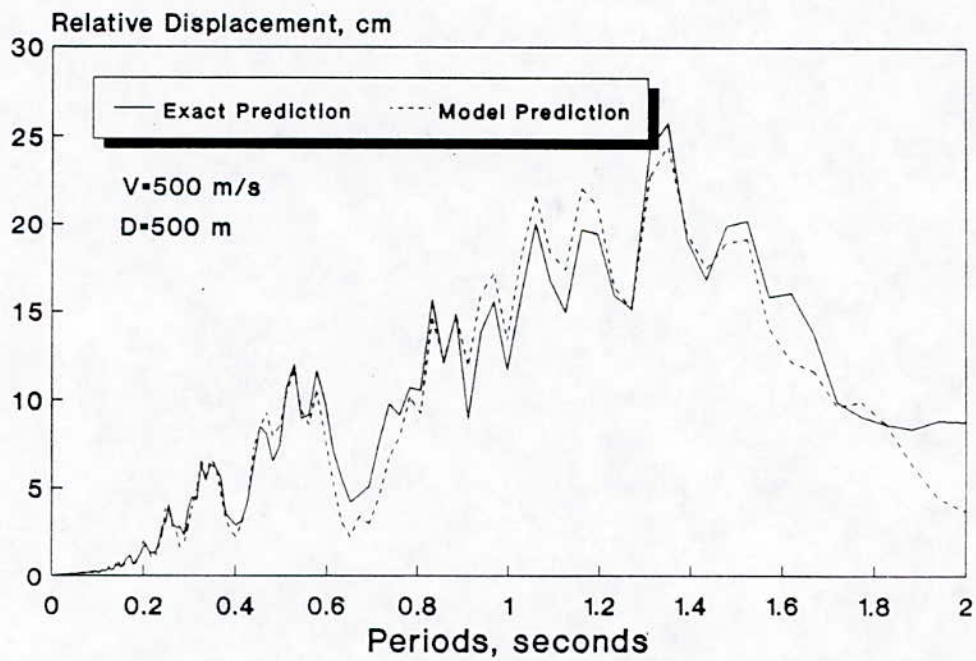
Figure 5.22 Effect of Full Coherence on Displacement Spectra

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3. The third case that has been examined is one where the support motions at both supports are uncorrelated. In other words, the target cross-spectral density matrix has been set to be:

$$[S(\omega)] = \begin{bmatrix} S(\omega) & 0 \\ 0 & S(\omega) \end{bmatrix}$$

(5.3.13)

Hence,

$$H_{11} = \sqrt{S(\omega)}$$

(5.3.14)

$$H_{21} = 0$$

(5.3.15)

$$H_{22} = \sqrt{S(\omega)}$$

(5.3.16)

By virtue of the fact that $\rho_{12} = 0$, (3.2.3.1.7) reads as follows:

$$R^p(\omega, \beta) = R(\omega, \beta)\frac{\sqrt{2}}{2}$$

(5.3.17)

The windowed acceleration time histories at locations 1 and 2 are displayed in fig. 5.23 and fig. 5.24. The pseudo-acceleration, pseudo-velocity, and relative-displacement spectra at locations 1 and 2 are shown in fig. 5.25, fig. 5.26, and fig. 5.27, respectively. And finally, the model predictions are proved to be satisfactory by comparison to the exact spectra, as it is shown in fig. 5.28, fig. 5.29, and fig. 5.30.

Figure 5.23 Windowed Acceleration Time History at Location 1
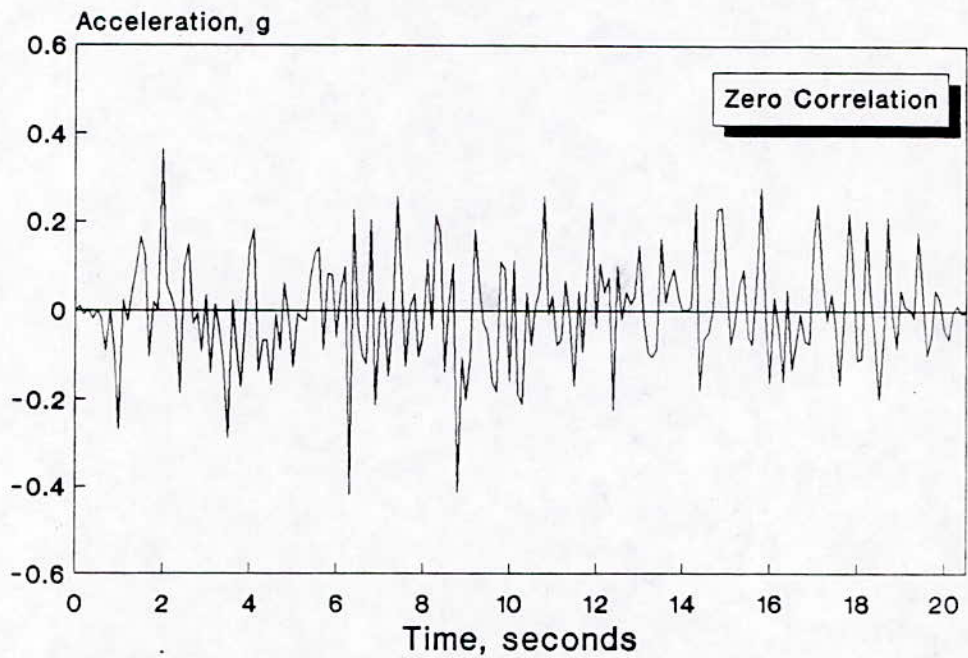
Figure 5.24 Windowed Acceleration Time History at Location 2

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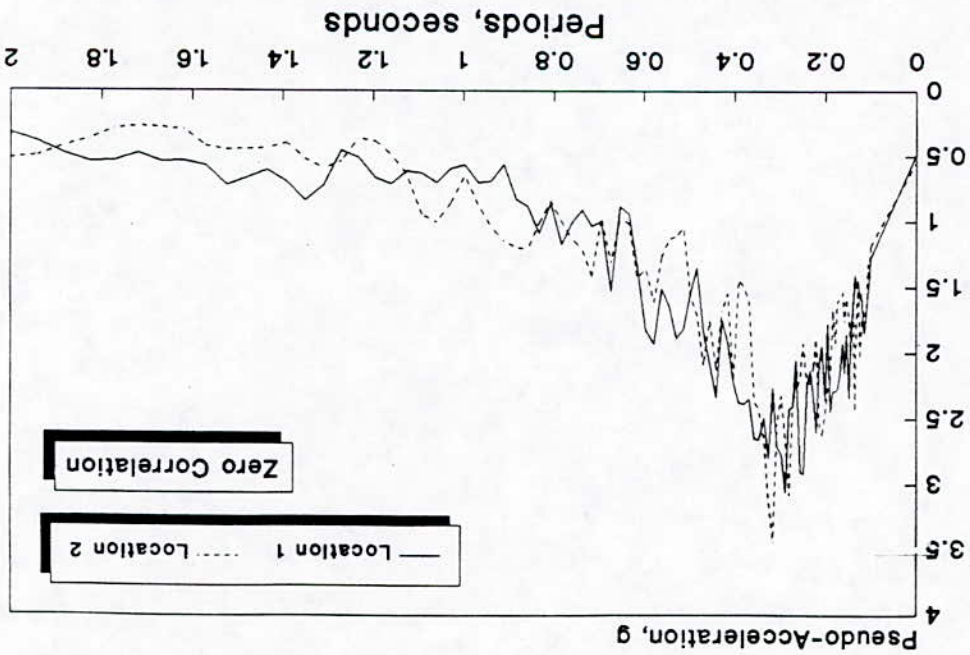**Figure 5.26 Velocity Spectra at Locations 1 and 2**

Periods, seconds

Pseudo-Velocity, cm/s

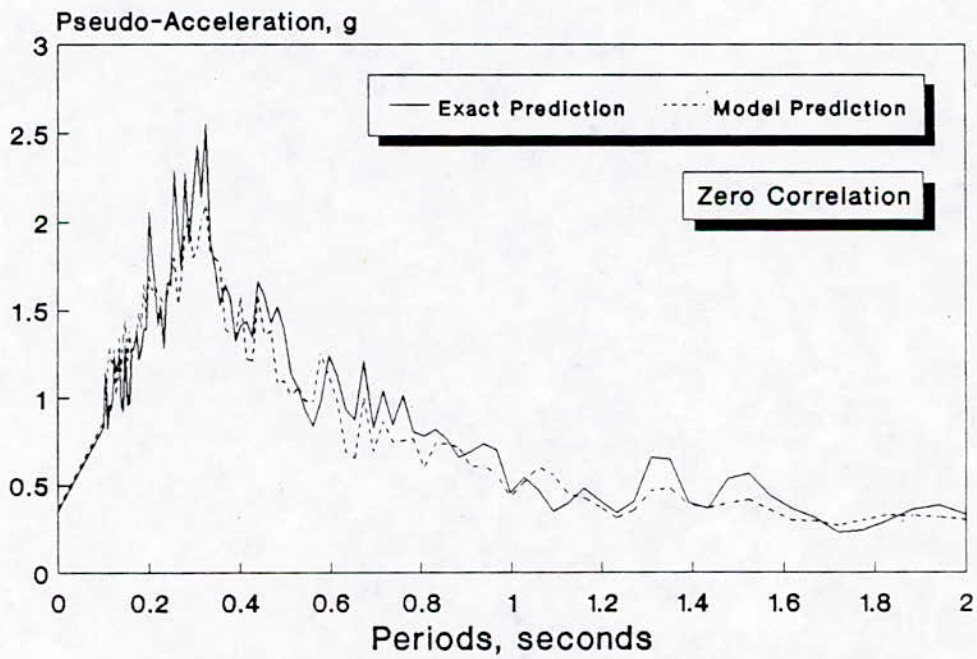Zero Correlation

Location 1 —— Location 2 ……
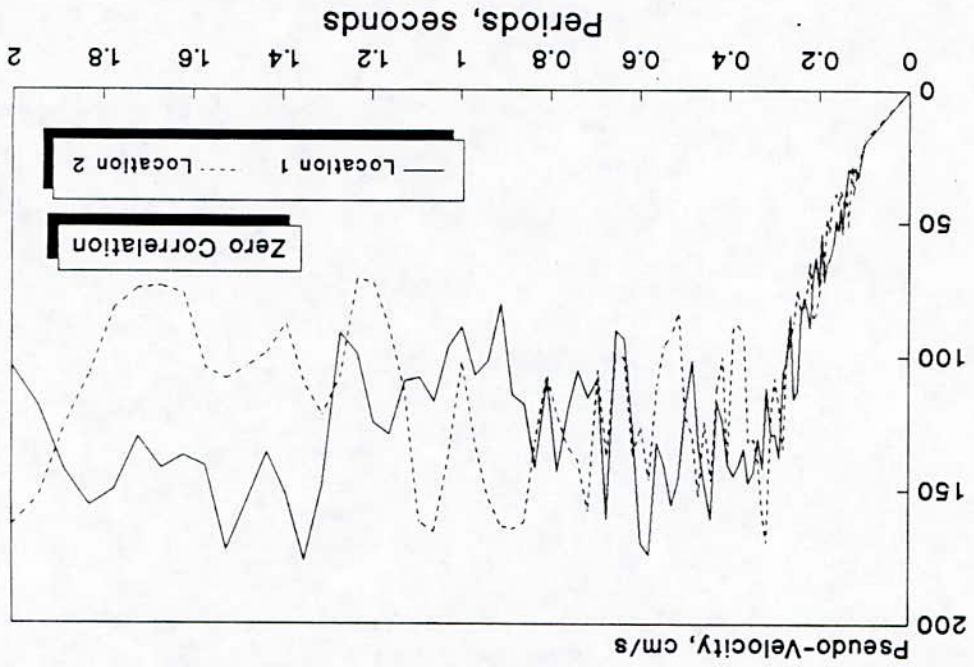
Figure 5.25 Acceleration Spectra at Locations 1 and 2

Periods, seconds

Pseudo-Acceleration, g

Zero Correlation

Location 1 —— Location 2 ……

Relative Displacement, cm
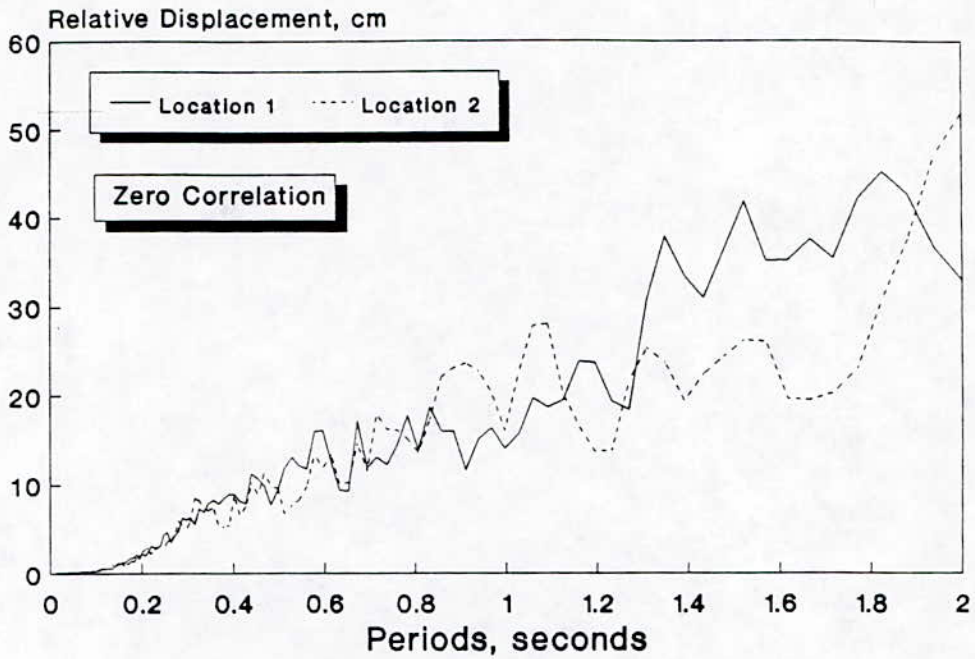
Figure 5.27 Displacement Spectra at Locations 1 and 2

Pseudo-Acceleration, g

Figure 5.28 Acceleration Spectra : Exact vs Model Prediction

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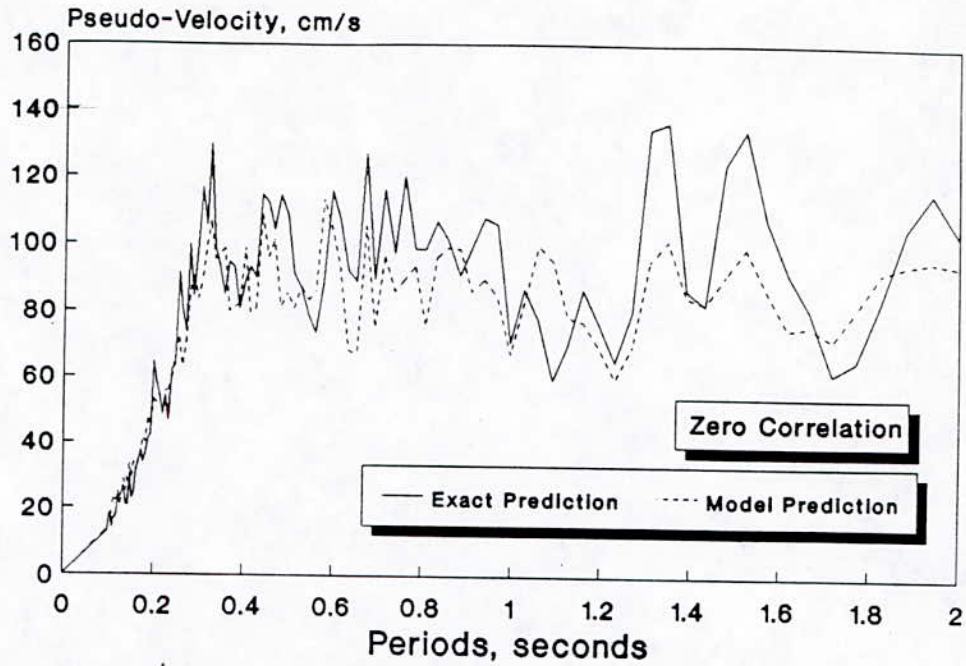
Figure 5.29 Velocity Spectra: Exact vs Model Prediction
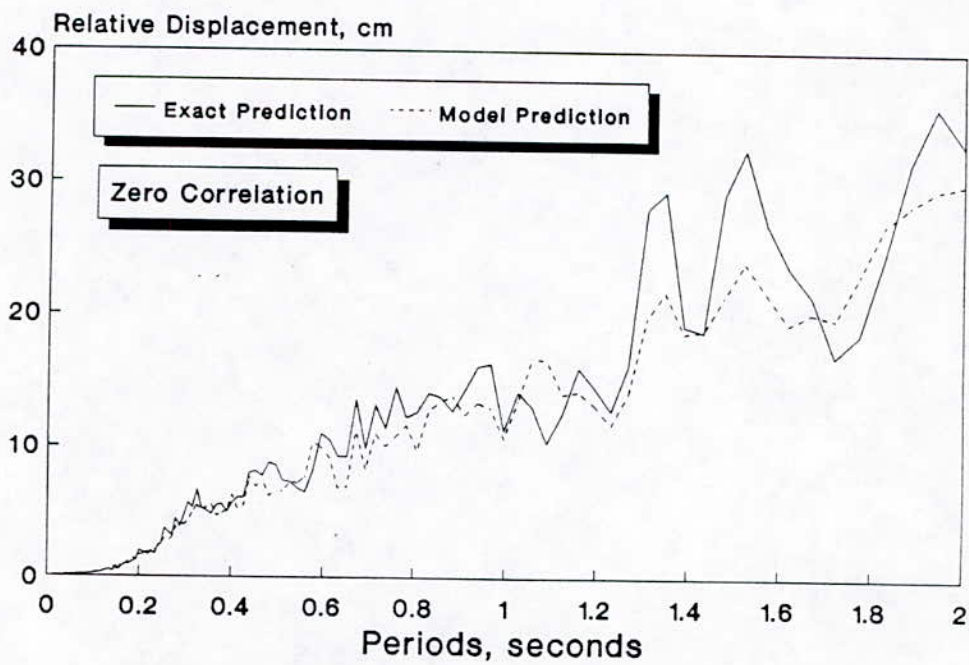
Figure 5.30 Displacement Spectra: Exact vs Model Prediction

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5.4   Case of Three Support Points

The situation under examination in the present section is that of a N-story double bay (three support points) frame structure behaving as a shear beam subjected to a single horizontal component of partially correlated ground motions. Similarly to the previous section, column stiffnesses will be assumed equal for the sake of illustration. As was arrived at in section (3.2.3.2), the modified response spectrum model is expressed as follows:

$$R^p(\omega_k, \beta_k) = R(\omega_k, \beta_k) \left[\frac{1}{n}(1 + \frac{2}{n}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n} \rho_{ijk}) \right]^{1/2} \tag{3.2.3.2.5}$$

which yields, after setting n to 3:

$$R^p(\omega_k, \beta_k) = R(\omega_k, \beta_k) \left[\frac{1}{3} + \frac{2}{9}(\rho_{12k} + \rho_{23k} + \rho_{13k}) \right]^{1/2} \tag{5.4.1}$$

where:

$$\rho_{ijk} = \frac{\int_0^\infty \rho_{ij} \mid H_k(\omega) \mid^2 G_{\ddot{u}_0} d\omega}{\int_0^\infty \mid H_k(\omega) \mid^2 G_{\ddot{u}_0} d\omega}$$

The effective seismic loading, so far as the floor responses are concerned, is $\ddot{s}(t) = \frac{1}{3}(\ddot{u}_1(t) + \ddot{u}_2(t) + \ddot{u}_3(t))$. Therefore, the goal that is aimed at in this section is to examine how well the modified response spectrum model given by (5.4.1) predicts the response spectrum corresponding to $\ddot{s}(t)$.

Similarly to the previous section, $\ddot{u}_1(t)$, $\ddot{u}_2(t)$ and $\ddot{u}_3(t)$ will be generated as part of an unidimensional homogeneous Gaussian random field, using spectral representation.

The target cross-spectral density matrix is given by:

$$[S(\omega)] = \begin{bmatrix} S_{11}(\omega) & S_{12}(\omega) & S_{13}(\omega) \\ S_{21}(\omega) & S_{22}(\omega) & S_{23}(\omega) \\ S_{31}(\omega) & S_{32}(\omega) & S_{33}(\omega) \end{bmatrix} \tag{5.4.2}$$

where:

$$S_{11}(\omega) = S_{22}(\omega) = S_{33}(\omega) = S(\omega) \tag{5.4.3}$$

$$S_{ij}(\omega) = S_{ji}^*(\omega) = r_{ij}(\omega, D_{ij})S(\omega) \tag{5.4.4}$$

By virtue of (5.2.3) and (5.2.4), the elements of the matrix $[H(\omega)]$ are given by:

$$H_{11}(\omega) = \sqrt{S(\omega)} \tag{5.4.5}$$

$$H_{21}(\omega) = \sqrt{S(\omega)}r_{21} \tag{5.4.6}$$

$$H_{22}(\omega) = \sqrt{S(\omega)}\sqrt{1 - |r_{21}|^2} \tag{5.4.7}$$

$$H_{31}(\omega) = \sqrt{S(\omega)}r_{31} \tag{5.4.8}$$

$$H_{32}(\omega) = \frac{r_{32} - r_{31}r_{12}}{\sqrt{1 - |r_{12}|^2}}\sqrt{S(\omega)} \tag{5.4.9}$$

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$$H_{33}(\omega) = \left[\frac{1- \mid r_{12} \mid^2 - \mid r_{13} \mid^2 - \mid r_{23} \mid^2 + 2 \mid r_{12} \mid \mid r_{23} \mid \mid r_{13} \mid \cos(\theta_{12} + \theta_{23} - \theta_{13})}{1- \mid r_{12} \mid^2} \right]^{1/2} \sqrt{S(\omega)}$$

(5.4.10)

where r_{ij} is the coherency function between stations i and j. And θ_{ij} is the corresponding phase spectrum.

(5.2.5) shows that $\ddot{u}_1(t)$, $\ddot{u}_2(t)$, and $\ddot{u}_3(t)$ can be simulated by the following series:

$$\ddot{u}_1(t) = 2\sum_{j=1}^{N} \mid H_{11}(\omega_j) \mid \sqrt{\Delta\omega} \cos(\omega_j t + \phi_{1j})$$

(5.4.11)

$$\ddot{u}_2(t) = 2\sum_{j=1}^{N} \mid H_{21}(\omega_j) \mid \sqrt{\Delta\omega} \cos(\omega_j t + \theta_{21}(\omega_j) + \phi_{1j})$$

$$+ \mid H_{22}(\omega_j) \mid \sqrt{\Delta\omega} \cos(\omega_j t + \phi_{2j})$$

(5.4.12)

$$\ddot{u}_3(t) = 2\sum_{j=1}^{N} \mid H_{31}(\omega_j) \mid \sqrt{\Delta\omega} \cos(\omega_j t + \theta_{31}(\omega_j) + \phi_{1j})$$

$$+ \mid H_{32}(\omega_j) \mid \sqrt{\Delta\omega} \cos(\omega_j t + \theta_{32}(\omega_j) + \phi_{2j})$$

$$+ \mid H_{33}(\omega_j) \mid \sqrt{\Delta\omega} \cos(\omega_j t + \phi_{3j})$$

(5.4.13)

where N, ω_j and θ_{ij} have already been defined. ϕ_{1j}, ϕ_{2j} and ϕ_{3j} are independent random phase angles uniformly distributed between 0 and 2π.

The same autospectrum and coherency function models have been taken in this section as in the previous one. The FFT technique has been used for the generation of time histories using the same characteristics (sampling rate, etc.) as earlier. And

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the same windowing process has been applied. Response spectra have been computed

accounting for a damping ratio of 1% .

The case studied in this section is as follows:

$V = 500m/s$

$D_{12} = 200m, D_{23} = 300m, D_{13} = 500m.$

Windowed acceleration time histories at locations 1, 2, and 3 are displayed in fig. 5.31,

fig. 5.32, and fig. 5.33. The spatial homogeneity of the spectra is verified in fig. 5.34,

fig. 5.35, and fig. 5.36. Finally, exact spectra for $\bar{s}(t) = \frac{1}{3}(\ddot{u}_1(t) + \ddot{u}_2(t) + \ddot{u}_3(t))$ are

computed and compared against the model predictions given by (5.4.1) which prove

to be satisfactory as can be seen from fig. 5.37, fig. 5.38, and fig. 5.39.
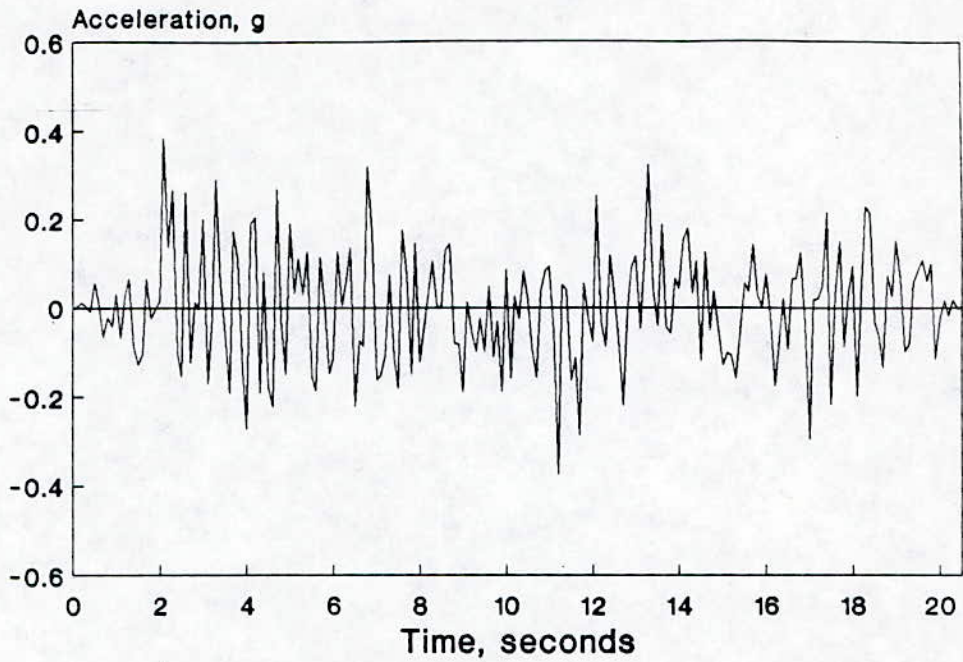
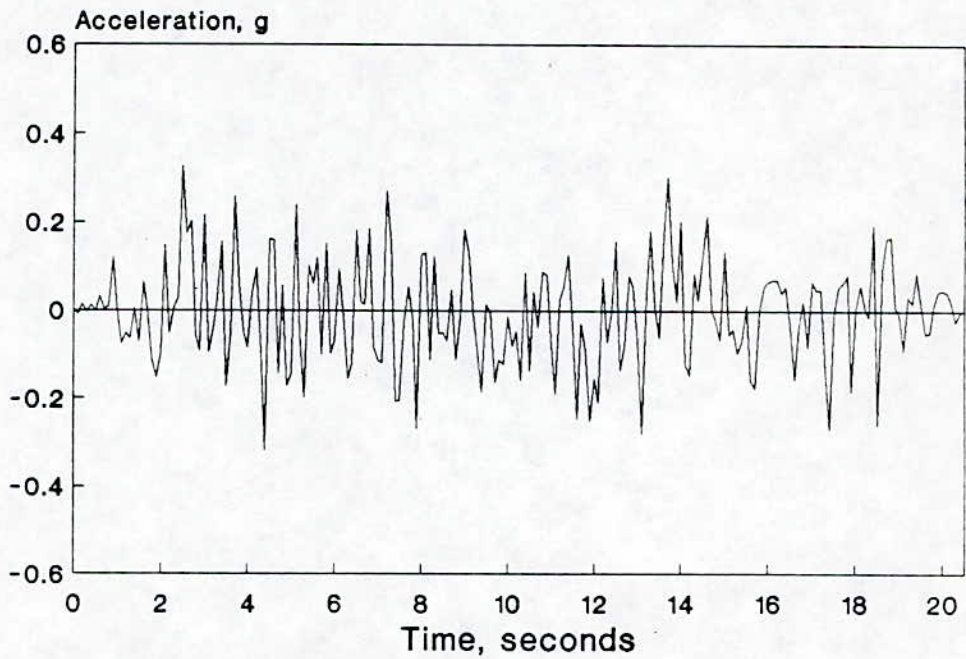Figure 5.31 Windowed Acceleration Time History at Location 1

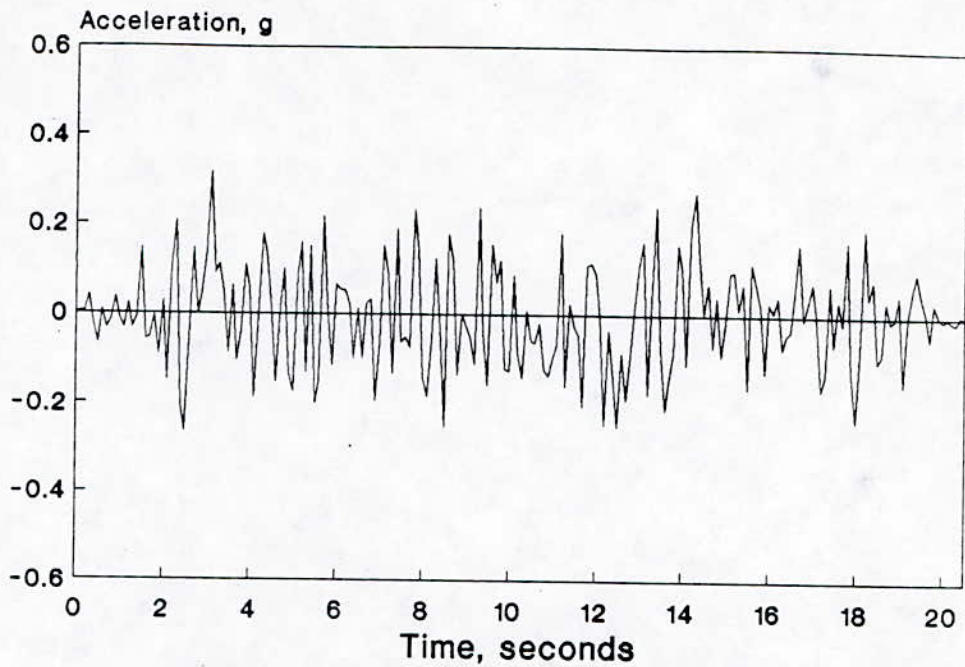Figure 5.32 Windowed Acceleration Time History at Location 2

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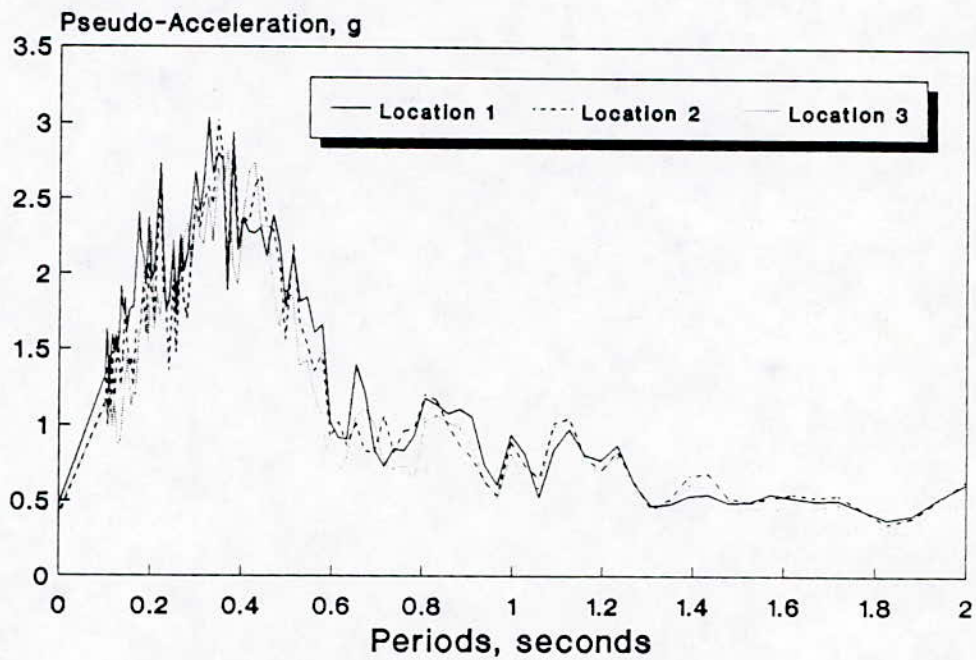Figure 5.33 Windowed Acceleration Time History at Location 3

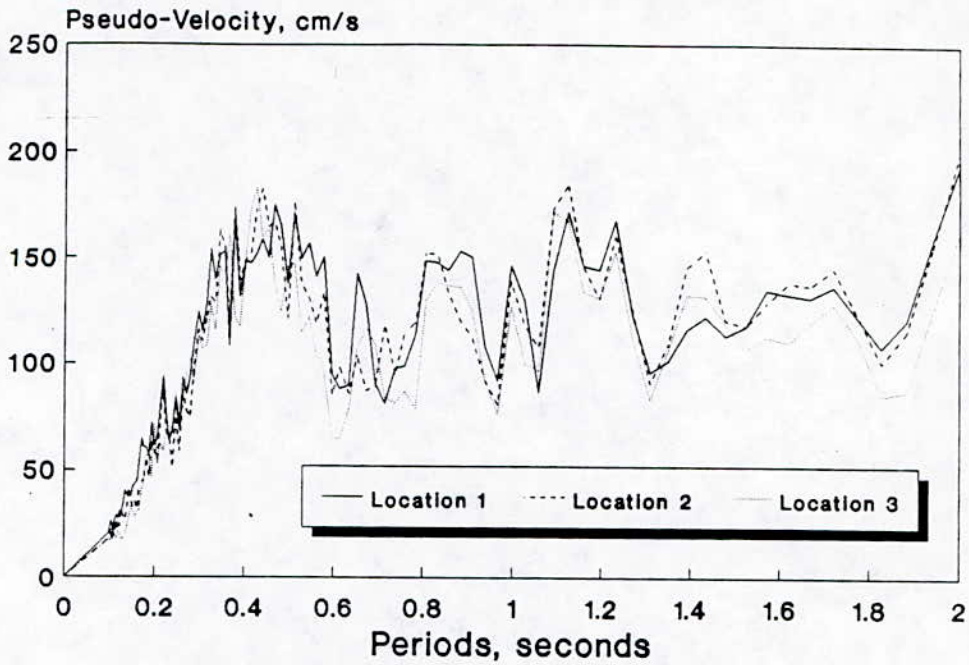Figure 5.34 Acceleration Spectra at Locations 1, 2 and 3

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Pseudo-Velocity, cm/s

Figure 5.35 Velocity Spectra at Locations 1, 2 and 3
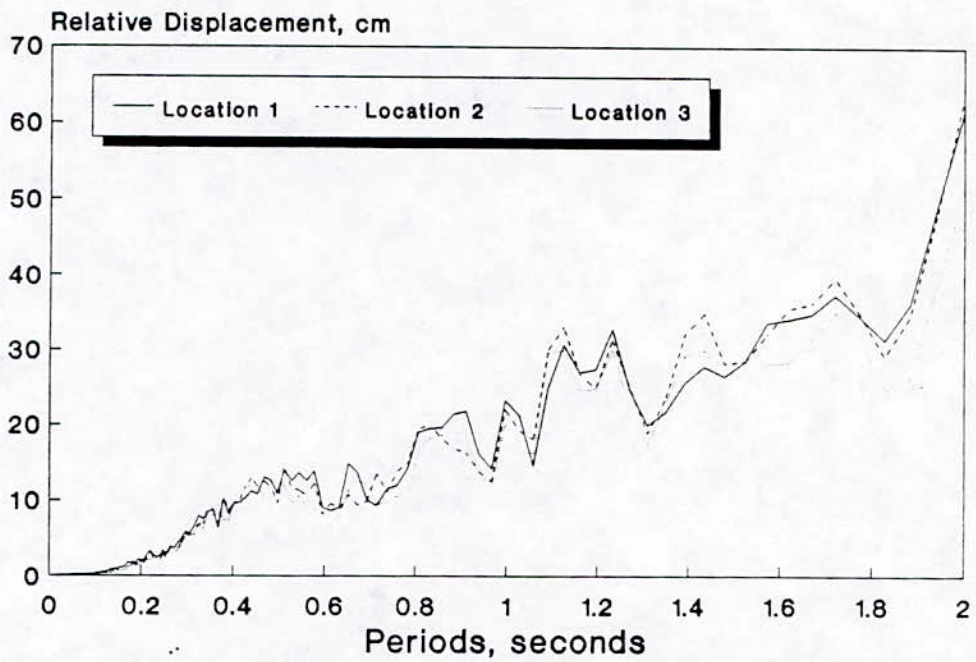
Relative Displacement, cm

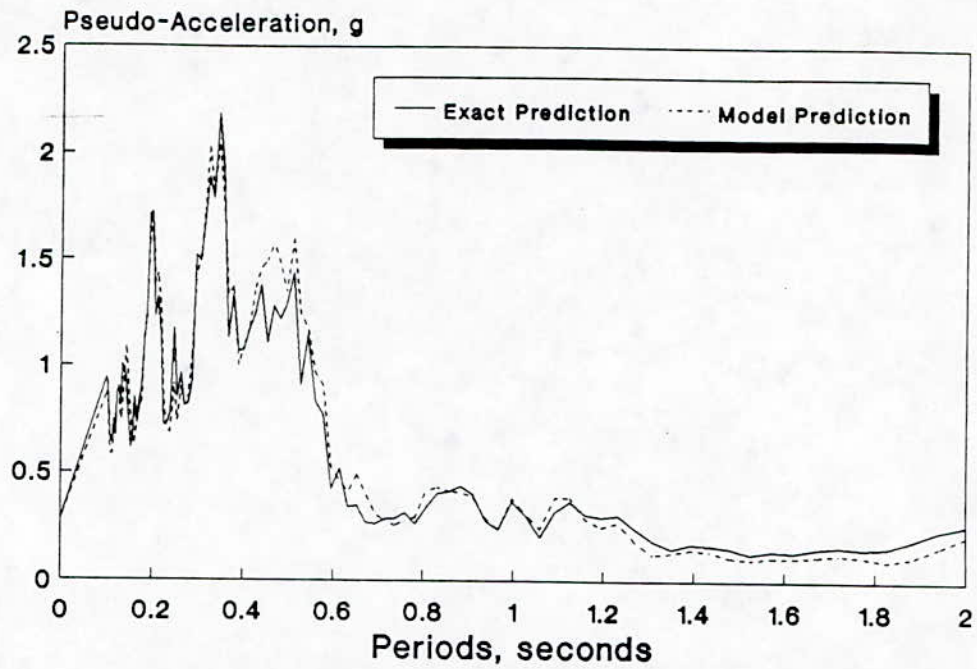Figure 5.36 Displacement Spectra at Locations 1, 2 and 3

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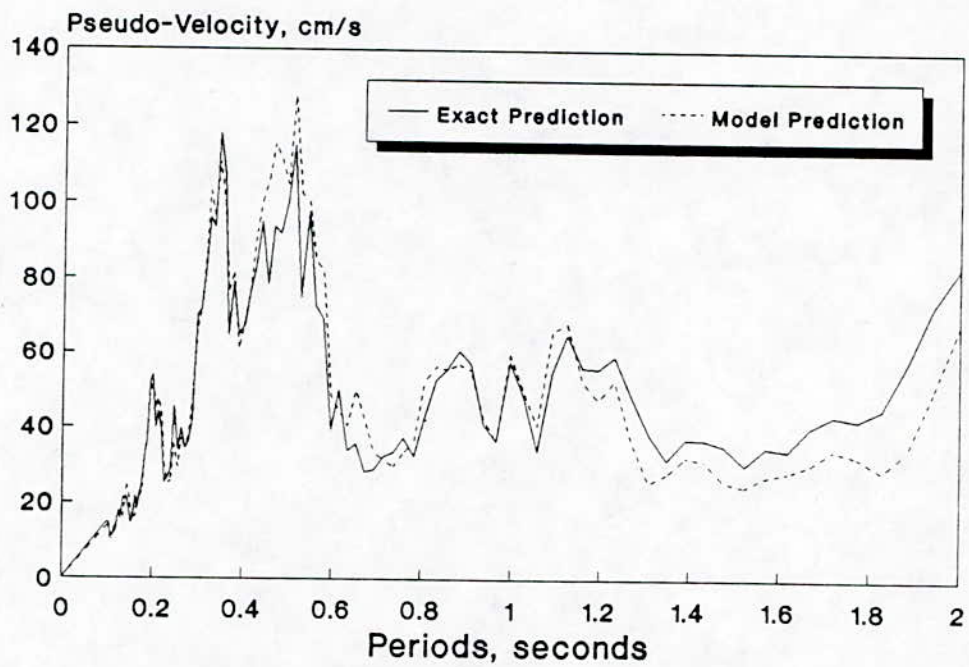Figure 5.37 Acceleration Spectra: Exact vs Model Prediction
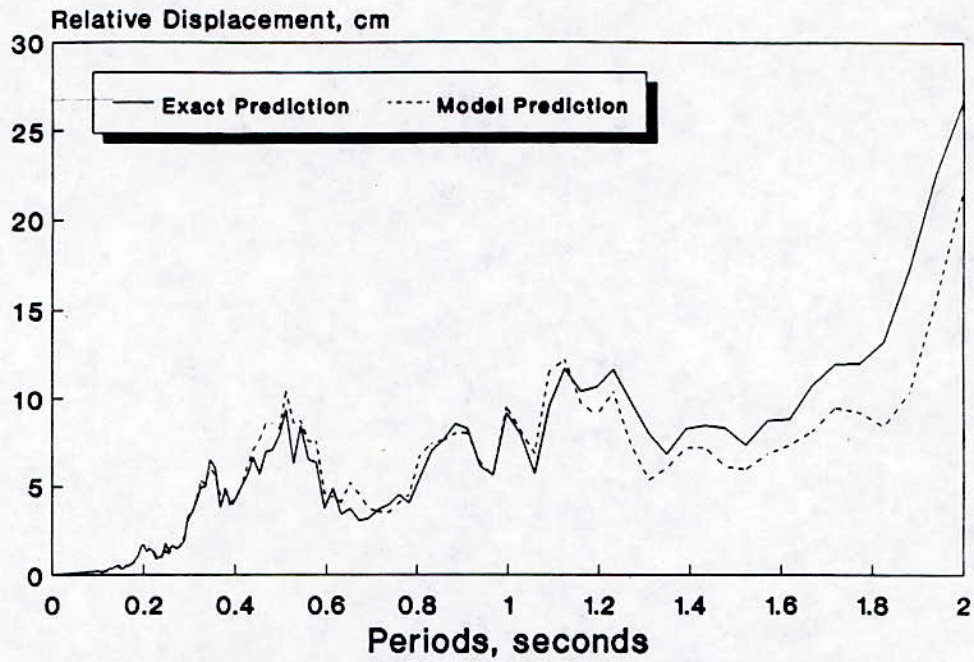

Figure 5.38 Velocity Spectra: Exact vs Model Prediction

Figure 5.39 Displacement Spectra: Exact vs Model Prediction

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Chapter 6

Conclusions and Suggestions

6.1 Conclusions

The spatial variability of seismic ground motion is an important aspect, which should be taken into account at least approximately, for the earthquake resistant design of extended facilities. This situation has motivated the present work, whose main objective is to provide a practical means for the design of elongated structures accounting for partially correlated seismic excitations.

A procedure for the modification of the design response spectrum is proposed in this thesis, which enables addressing the problem of multiply-supported structures subjected to imperfectly correlated seismic excitations. The technique is an extension to the response spectrum method which is a commonly used tool in earthquake resistant design, thereby allowing the inclusion of this potentially important problem into seismic design practice. A modified response spectrum model is developed for the design of extended facilities subjected to single and multicomponent ground motion, and a modal combination rule is proposed for each case. The modification procedure is based on adjusting each spectral value of the given design response spectrum by means of a correction factor which depends on the structural properties and on the characteristics of the wave propagation phenomenon. Finally, the theoretical model is validated through digital simulation of seismic ground motion, whereby model predictions are

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found to be in good agreement with exact results.

6.2 Suggestions for Further Research

A modified response spectrum model for the design of extended structures subjected to spatially varying ground motion has been developed in this thesis. Further research in this area should address the following points.

- Investigation of the potential simplification of the derived model by, numerically or analytically, examining the effect of decomposing the seismic input into a rigid body component (average value of the individual support motions) and a non rigid body component, and neglecting the latter component in the computation of the dynamic part of the structural response.

- Further investigation of the effect of incoherent seismic motions on the response of structures, while the importance of wave travelling effects was recognized about two decades ago.

- Inclusion of soil-structure interaction effects.

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