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Mémoire de Master

Thème :

GDOF comparisons with other AHRS Orientation Filters

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Ce modeste travail est dédié à:

Mes chers parents

Mes chers frères et mes deux chères sœurs

La mémoire de mon grand-père Yahia

Mon grand-père Mohamed

Toute ma grande famille

Tous ceux qui me sont chers...

B. Smail

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ملخص:

هذا التقرير يعرض معلومات عن فلتر التوجيه بحساب ميل النزول (GDOF) المستعمل في النظم المرجعية لزوايا الموقف والتوجيه (AHRS) و يلخص نتائج منشورات قامت بمقارنة هذا الفلتر بفلترات أخرى معروفة .

كلمات مفتاحية : مصفوفة الدوران ، تمثيل Euler , AHRS, GDOF, Quaternions, EKF, CF

Abstract:

This report shows information about the Gradient Descent Orientation Filter (GDOF) used in Attitude and Heading Reference Systems (AHRS) and resumes results of papers that carried out comparisons of this filter with other well-known ones.

Keywords : AHRS, GDOF, Quaternions, Euler representation, Rotation matix, EKF, CF.

Résumé :

Ce travail présente des informations sur le Filtre d’Orientation à Gradient de Descente (GDOF) utilisé dans les Systèmes Référentiels d’Attitudes et Directions (AHRS): et résume des résultats de publications qui ont mené une comparaison de ce filtre avec d’autres filtres bien connus.

Mots clés : AHRS, GDOF, Quaternions, représentation d’Euler, Matrice de rotation, EKF, CF.

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1 Motivation

The present document is fulfilled in the scope of a final year graduation project, where a comparison between available Orientation Filter Algorithms for Attitude and Heading Reference System (AHRS) applications on low cost Inertial Measurement Units (IMU) and Gradient Descent Orientation Filter (GDOF) was needed, to justify the filter choice. Thus, papers available online were consulted and resumed in order to obtain a quick insight about different filters, their strong sides and weaknesses compared to each other.

The reader won't find in this document detailed filters mathematical derivations as they can be found on the authors papers. The documents of reference used for our purpose are given at the bibliography.

Nomenclature

EKF = Extended Kalman Filter

QKF = Quaternion-based Kalman Filter

GDOF = Gradient Descent Orientation Filter

ECF = Explicit Complementary Filter

CF = Complementary Filter

AHRS = Attitude and Heading Reference System.

2 AHRS Orientation Filters

Attitude estimation involves a two-part process [1]:

1) estimation of a vehicle's orientation from body measurements and known reference observations.

2) filtering of noisy measurements.

The second part is achieved by combining the measurements with models, which in itself can be done a number of different ways.

The first step, where an attitude estimate is obtained from body measurements to feed a filter (or an observer), ends up in one of many known representations, e.g., Euler angles, quaternions, Euler angle-axis representation, rotation matrix, etc. For the filtering process there is also a very large number of alternatives, depending on the models and representations of the attitude. Kinematic models, which resort basically to three-axis rate gyros, are exact.

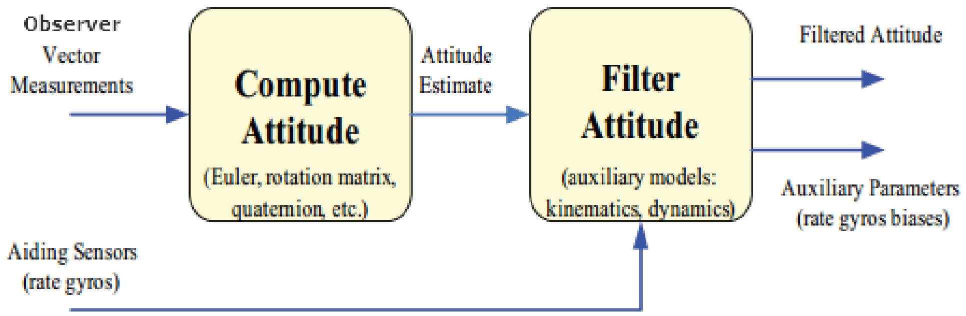

Figure 2.1: Classic attitude estimation

Fig 2.1 depicts a traditional attitude estimation solution. As it is possible to observe, vector measurements such as the gravitational and magnetic fields are first used to compute a representation of the attitude of the vehicle. Afterwards, the attitude filter evolves according to the its representation and resorting to kinematic or dynamic attitude models.

The signal output of low-cost IMU systems is characterized by low resolution signals subject to high noise levels as well as general time-varying bias terms [2]. Therefore, raw signals must be processed to reconstruct smoothed attitude estimates and bias-corrected angular velocity measurements through suitable sensor fusion algorithms. In fact, suitable exploitation of acceleration measurements can avoid drift caused by numerical integration of gyroscopic measurements.

However, it is well-known that use of only these two source of information cannot correct the drift of the estimated heading, thus an additional sensor is needed, i.e., a tri-axial magnetometer, which allows to obtain a correct heading estimation.

consider a vehicle equipped with an Inertial Measurement Unit, which contains three triads of orthogonally mounted rate gyros, accelerometers, and magnetometers. The magnetometers provide the magnetic field in bodyfixed coordinates. This quantity is locally constant in inertial coordinates and it is therefore a feasible vector observation for attitude estimation.

On the other hand, for sufficiently low frequency bandwidths, the gravitational field also dominates the accelerometer measurements. This provides a second vector observation, which is, in general, not parallel to the first. Therefore, it is possible to determine the accurate attitude of the vehicle with an 9-axis IMU.

3 Compared Orientation Filters

Several fusion methods have been proposed in the literature. Crassidis et al [3] have published in 2007 a detailed survey of existing Non-linear orientation filters used for various applications. Mahony et al. (2008) formulate the filtering problem as a deterministic observer posed on the Special Orthogonal group SO(3) termed 'Explicit Complementary Filter (ECF)'. Madgwick et al. (2011) present a computationally efficient orientation algorithm based on optimized gradient descent algorithm (GDOF) designed to support a wearable inertial human motion tracking system for rehabilitation applications.

A large number of different solutions can be found in the literature, so some researchers carried out experimental comparison of the most popular among them, Extended Kalman Filter (EKF) with its different offshoots, GDOF, ECF and Quaternion Kalman Filter (QKF) are nowadays attracting the interest of many low cost AHRS application consumers, each one of them offers some advantages and suffers from drawbacks.

4 Gradient Descent Orientation Filter (GDOF)

GDOF is applicable to IMUs consisting of tri-axis gyroscopes and accelerometers, and magnetic angular rate and gravity (MARG) sensor arrays that also include tri-axis magnetometers. The algorithm takes in consideration magnetic distortion compensation and it uses a quaternion representation, allowing accelerometer and magnetometer data to be used in an analytically derived and optimized gradient descent algorithm to compute the direction of the gyroscope measurement error as a quaternion derivative.

The first, and most prevalent reason of interest to this filter appears to be the low computational load required in the implementation of this orientation filter [2]. This fact proves especially attractive for embedded systems where microcontroller and microprocessor power, although increasingly enhanced, are still behind what may be found in a conventional computer. Lower required computational power translates into lighter packages and smaller footprints opening the integration possibilities to small UAVs and even wearable technology.

In addition with the first attractive factor, the second stems from the ability to obtain higher estimation accuracy at lower sampling rates [4]. This is due to the fact an actual analytical solution has been derived for the descent gradient algorithm as opposed to the linear regression iterations required by the Kalman filter approach.

The final pillar supporting the popularity of the approach is the employment of quaternions in order to avoid “Gimbal Lock” type singularities that are prevalent with Euler angle attitude representation.

5 Comparison Experiments

Recent Comparison experiments were performed either by mathematical models or field manipulations, A. Cavallo et all [2] used a manipulative robotic arm for a comparison between EKF, GDOF and ECF. Precise command signals issued to the hand were the reference observers. The same method was used by Choi et al[5], where in his experiment GDOF performance was compared to Gyro-free QKF and Complementary Filter (CF).

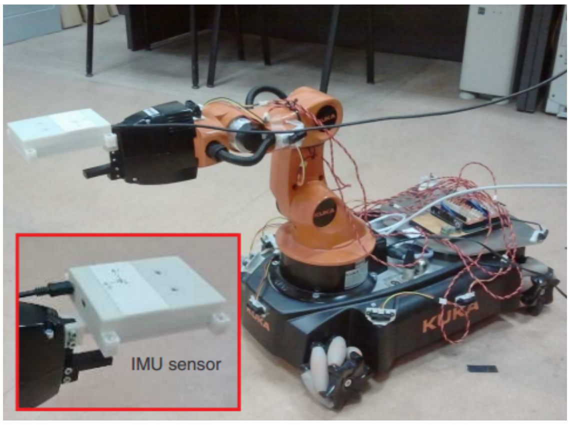

Figure 5.1: KUKA robot used in the experiment for [2]

Figure 5.2: Yaskawa Motoman robotic arm holding the IMU electronics box in [5]

Alternatively, F.Alam et al [6]have performed their comparison based on mathematical models of filters between ECF and GDOF.

6 Considerations for Representing Rotations

Rotation representation palys an important role in filter performance, A **rotation matrix** is a 3x3 matrix whose multiplication with a vector rotates the vector while preserving its length. A rotation matrix may also be referred to as a direction cosine matrix, because the elements of this matrix are the cosines of the unsigned angles between the body-fixed axes and the world axes [7].

$$R_{123} = R_1 R_2 R_3$$

$$R_1(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha) & \sin(\alpha) \\ 0 & -\sin(\alpha) & \cos(\alpha) \end{bmatrix}$$

$$R_2(\alpha) = \begin{bmatrix} \cos(\alpha) & 0 & -\sin(\alpha) \\ 0 & 1 & 0 \\ \sin(\alpha) & 0 & \cos(\alpha) \end{bmatrix}$$

$$R_3(\alpha) = \begin{bmatrix} \cos(\alpha) & \sin(\alpha) & 0 \\ -\sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

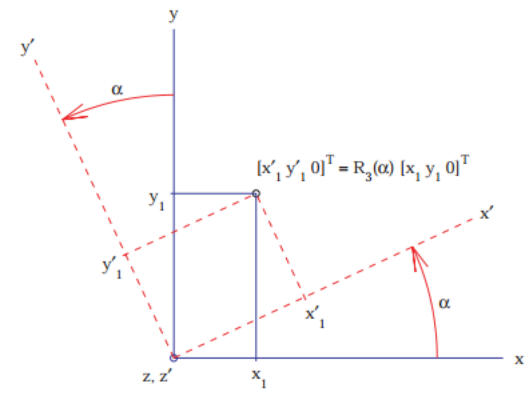

Figure 6.1: A sample coordinate rotation about the z-axis by an angle α [7].

Euler Angles are three coordinate rotations in sequence which can describe any rotationand are sometimes called *Cardan* angles,They are commonly used in aerospace engineering and computer graphics. Despite the lack of consensus on the issue, these angles are also commonly referred to simply as Euler angles in the aeronautics field, in which φ ,θ *and* ϕ are known respectively as roll, pitch, and yaw, or, equivalently, bank, attitude, and heading [7] .

The function that maps an Euler angle vector to its corresponding rotation matrix,
R_{ijk} : $\Re^3 \rightarrow \mathrm{SO}(3)$, is:

$R_{ijk}(\varphi\ ,\theta;\phi)$
$= R_i(\varphi)R_j(\theta)R_k(\phi)$

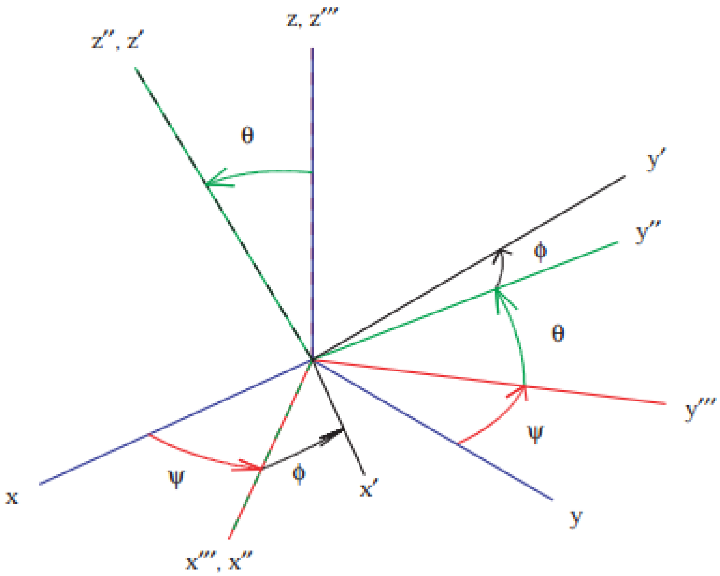

Figure 6.2: Euler Angle Sequence [7]

The singularities found in the various Euler angle representations are said to arise from gimbal lock. Gimbal lock may be understood in several different ways. Intuitively, it arises from the indistinguishability of changes in the first and third Euler angles when the second Euler angle is at some critical value.

In the study of the gyroscopic motion of a spinning rigid body, the Euler angles φ ,θ *and* ϕ are known respectively as spin, nutation, and precession. Take, for example, the case of the sequence above, when the nutation angle is zero, changes in the spin angle are the same as changes in the precession angle.

The quaternion representation of attitude orientation in three dimensional space may be given by a four dimensional complex number. The conceptual motivation to this approach is to represent any arbitrary rotation of a frame relative to frame as a rotation of angle around a specific axis defined in the frame. A graphical representation is presented in Fig 6.3

The quaternion representation of the above frame rotation may be given as:

$${}^{A}_{B}\hat{q} = [q_1 \quad q_2 \quad q_3 \quad q_4]$$
$$= [\cos\frac{\theta}{2} \quad -r_x \cos\frac{\theta}{2} \quad -r_y \cos\frac{\theta}{2} \quad -r_z \cos\frac{\theta}{2}]$$

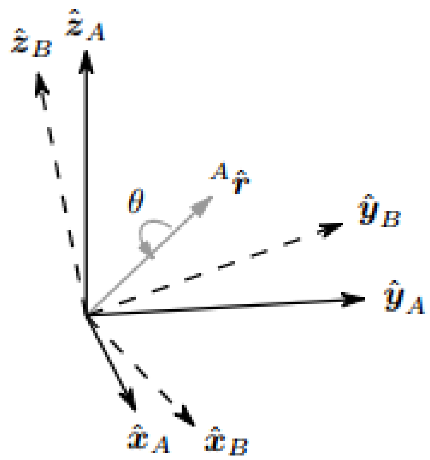

Figure 6.3: The orientation of frame B is achieved by a rotation, from alignment with frame A , of angle θ around the axis ${}^{A}\hat{r}$ [7]

Because of their simplicity, elegant and lightweight mathematical operations, with lack of any singularities, quaternions are a very popular representation for encoding the attitude of a rigid body. This includes applications in which quaternions are included as state variables in an optimization for existing algorithms in other representations [3].

7 Performance Evaluation

Algorithms evaluation involves many aspects, like running them in comparison to a known attitude, evaluating execution time or even implementation easiness compared to each other.

7.1 Estimated attitude vs true attitude

In the paper of **Cavallo et al**[2], the codes where ran at 500 Hz for two robot trajectories. In the first (slow) trajectory, an average speed of 18 deg/s is applied to robot joints, while, in the second (fast) trajectory, the average speed is raised to 45 deg/s.

To quantify the algorithms performance, the static and dynamic RMSE (root-mean-square-error) still in terms of Euler angles have been computed and reported in Tab 1.

Tab 1. Static and dynamic RMSE[2].

Euler angles [°]	EKF	Madgwick	Mahony
Roll (static)	0.04	0.03	0.02
Pitch (static)	0.01	0.05	0.05
Yaw (static)	0.30	1.92	1.85
Roll (slow)	4.71	4.85	5.07
Pitch (slow)	1.91	2.65	2.89
Yaw (slow)	5.19	5.13	5.67
Roll (fast)	6.55	6.51	6.69
Pitch (fast)	2.83	3.34	2.85
Yaw (fast)	6.71	7.07	6.92

The results of the experiments show that in the slow trajectory, the three algorithms provide comparable results in terms of accuracy. In terms of RMSE, the proposed EKF algorithm provides a more accurate estimation in both static and dynamic conditions while GDOF and ECF showed comparable results.

To test the capability of the algorithms to work under severe disturbance conditions a specific experiment has been carried out by intentionally actuating the robot gripper with a periodic signal so as to generate an electromagnetic disturbance which mainly affects the magnetometer.

Tab 2. Dynamic RMSE with noisy measurements[2].

Euler angles [°]	EKF	Madgwick	Mahony
Roll	5.05	5.54	5.87
Pitch	3.24	3.93	4.53
Yaw	5.93	6.27	6.66

Tab 2 shows that EKF copes better with noisy environments and GDOF performing better than ECF.

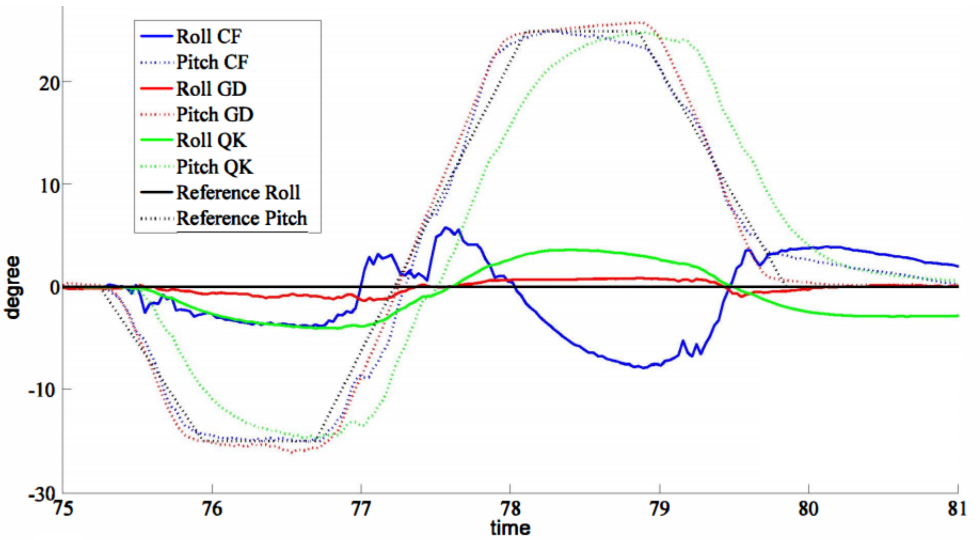

Figure 7.1: Experimental results from the three algorithms CF, GDOF and QKF[2].

For **Choi et al** [5], The robotic arm was programmed to perform a sequence of motions so as to pitch down 15° then pitch up 25° and since the motion sequence is pure pitch motion, all the roll motion is considered as error. Results are shown in Fig 7.1.

Results show that the Gradient Decent algorithm performs most accurately of these three methods. Although, the estimation result is slightly greater than the true attitude, it follows the shape of the motion. Moreover, the GDOF has the most accurate roll estimation during the pure pitch motion.

For Quaternion-based Kalman Filter, the estimation is the smoothest of these algorithms. However, it has a small delay from the reference value which as predicted by theory and has significant roll estimation error during the pure pitch motion and has significant roll estimation error during the pure pitch motion.

As the exact attitude is given by Yaskawa Motomoman arm robot, the absolute error of each algorithm can be measured. Results were given as shown in the table below:

Tab 3.Absolute error of the three algorithms[5].

Absolute Error	Complementary Filter	Gradient Decent	Quaternion Kalman
Roll-Mean	3.28	0.50	2.28
Roll-SD	2.00	0.35	1.26
Pitch-Mean	1.10	1.06	3.85
Pitch-SD	1.02	0.97	2.71

a. SD means standard deviation

Complementary Filter and GDOF Algorithms produce accurate estimation of the pitch motion. However, the Complementary Filter has a significant error in the roll motion estimate.

F.Alam [6] has shown on the other hand that for certain parameter choices the GDOF and ECF perform at identical manners. MPU-6050 IMU (Inertial Measurement Unit) was used after that to generate data for different scenarios (at 100 Hz) and the aforementioned two algorithms were applied for roll and pitch estimation. Different scenarios were simulated and the orientation in quaternion were computed which were converted to Euler angles representation for comparison purpose.

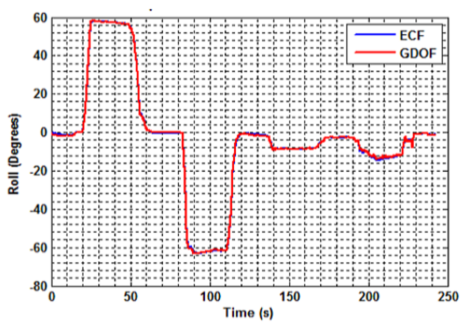

Figure 7.2: Roll angle estimation[6].

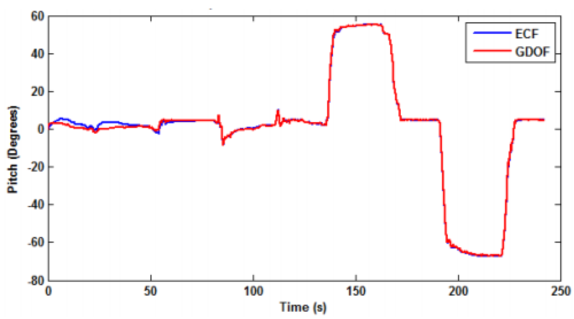

Figure 7.3: Pitch angle estimation [6]

7.2 Comparison on the computational burden

Computational efficiency is a major parameter to take in consideration for orientation filters, as most of them will be used for decision considerations in larger systems where actuators occupy a long part of processing time and lightweight filters expedite real time execution.

Cavallo et al[2] implemented the three previously studied algorithms in Matlab/Simulink environment on an Intel I7 quad-core processor at 1.6 GHz. Matlab functions have been used to estimate the execution time of a single cycle that includes the gyroscope, accelerometer and magnetometer measurement and the attitude estimation. Table 4 reports the average time required to compute one estimation cycle in both Matlab/Simulink environment and embedded system implementation on an ARM-Cortex M4-based evaluation board.

Table 4. Computational burden estimation[2].

Algorithm	Matlab/Simulink [ms]	Embedded System [ms]
EKF	0.1	2.7
Madgwick	0.017	0.15
Mahony	0.014	0.11

superior performance of the EKF can be attributed to the availability of a tunable parameter for each sensor measurement, which is paid in terms of a higher execution time. On the other hand, quaternion representation helped the GDOF filter to remain relatively more precise than the CF while being computationally efficient.

8 Conclusion

In this summary of reports we showed briefly reasons to consider using the GDOF as a new interesting orientation filter with fairly good competition with the EKF and better performance than the CF. We have seen also the cause of recent interest in this filter by embedded applications developers and consumers, as it presents an efficient and lightweight algorithm suitable for implementation on low cost and low computation capacity hardware.

References

[1] P. Batista, C. Silvestre, P. Oliveira, and B. Cardeira, "Low-cost attitude and heading reference system: filter design and experimental evaluation", in *2010 IEEE International Conference on Robotics and Automation (ICRA)*, May 2010, pp. 2624–2629. DOI: 10.1109/ROBOT.2010.5509537.

[2] A. Cavallo, A. Cirillo, P. Cirillo, G. De Maria, P. Falco, C. Natale, and S. Pirozzi, "Experimental comparison of sensor fusion algorithms for attitude estimation", in *19th World Congress of the International Federation of Automatic Control, Cape Town, South Africa*, 2014. [Online]. Available: http://www.nt.ntnu.no/users/skoge/prost/proceedings/ifac2014/media/files/1173.pdf (visited on 06/16/2015).

[3] J. L. Crassidis, F. L. Markley, and Y. Cheng, "Survey of nonlinear attitude estimation methods", *Journal of Guidance, Control, and Dynamics*, vol. 30, no. 1, pp. 12–28, 2007. [Online]. Available: http://arc.aiaa.org/doi/abs/10.2514/1.22452 (visited on 06/17/2015).

[4] S. O. Madgwick, "An efficient orientation filter for inertial and inertial/magnetic sensor arrays", *Report x-io and University of Bristol (UK)*, 2010. [Online]. Available: http://sharenet-wii-motion-trac.googlecode.com/files/An_efficient_orientation_filter_for_inertial_and_inertialmagnetic_sensor_arrays.pdf (visited on 06/14/2015).

[5] M. Choi, R. Porter, and B. Shirinzadeh, "Comparison of attitude determination methodologies with low cost inertial measurement unit for autonomous aerial vehicle", in *2013 IEEE/ASME International Conference on Advanced Intelligent Mechatronics (AIM)*, Jul. 2013, pp. 1349–1354. DOI: 10.1109/AIM.2013.6584282.

[6] F. Alam, Z. ZhaiHe, and H. JiaJia, "A comparative analysis of orientation estimation filters using MEMS based IMU", presented at the 2nd International Conference on Research in Science, Engineering and Technology (ICRSET'2014), March 21-22, 2014 Dubai (UAE). [Online]. Available: http://iieng.org/siteadmin/upload/7697E0314552.pdf (visited on 06/16/2015).

[7] J. Diebel, "Representing attitude: euler angles, unit quaternions, and rotation vectors", *Matrix*, vol. 58, pp. 15–16, 2006.