RÉPUBLIQUE ALGÉRIENNE DÉMOCRATIQUE ET POPULAIRE

MINISTÈRE DE L'ENSEIGNEMENT SUPÉRIEUR ET DE LA RECHERCHE SCIENTIFIQUE

ECOLE NATIONALE POLYTECHNIQUE

EN COTUTELLE AVEC L'UNIVERSITÉ D'ORLÉANS

DEPARTMENT : **ÉLECTRONIQUE**

END-OF-STUDY PROJECT DISSERTATION
FOR OBTAINING THE STATE ENGINEER'S DEGREE IN ELECTRONICS

Direction of arrival estimation and tracking

Presented publicly the 26/06/2023 by:

Chafaa Lyes ABBASSEN

Jury composition:

President	**Prof. Mourad ADNANE**	**ENP**
Examinator	**Dr. Nesrine BOUADJENEK**	**ENP**
Supervisors	**Prof. Adel BELOUCHRANI**	**ENP**
	Prof. Karim ABED-MERAIM	**University of Orléans**

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Mémoire de projet de fin d'études pour l'obtention du diplôme d'ingénieur d'état en Électronique

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Estimation et poursuite des directions d'arrivée

Présenté et soutenu publiquement le 26/06/2023 par :

ABBASSEN Chafaa Lyes

Composition du jury :

Président	**Prof. Mourad ADNANE**	**ENP**
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	Prof. Karim ABED-MERAIM	**Université d'Orléans**

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Dedication

I dedicate my work to my family and loved ones. A special feeling of gratitude to my loving parents.

Acknowledgement

I would like to express my appreciation to all those who have supported this project. I feel incredibly fortunate to have had this adventure, which proved to be a remarkable mix of scientific research and personal development.

First and foremost, I extend my heartfelt appreciation to my supervisors Prof. Karim ABED-MERAIM and Prof. Adel BELOUCHRANI for their precious guidance, expertise and patience throughout this process.

I also owe the PRISME laboratory at the University of Orléans a debt of gratitude for providing me with the research environment and facilities that allowed me to conduct this project.

Finally, I would like to thank the members of my jury for agreeing t o review and evaluate my work.

ملخص

تقدير اتجاهات الوصول في معالجة إشارات المصفوفة هو مجال بحث نشط يمتلك تطبيقات متنوعة في الاتصالات اللاسلكية والرادار والسونار والكلام والملاحة وعلم الزلازل ومجالات أخرى. عندما تكون اتجاهات الوصول متغيرة مع مرور الوقت بسبب وجود مصادر متحركة، تفشل الطرق مثل طريقة تصنيف الإشارات المتعددة و خوارزمية تقدير إعدادات الإشارات عبر تقنيات ثابتة بالدوران في توفير تقديرات صحيحة لأن هذه الأخيرة تفترض إتجاهات ثابتة. يتناول هذا المشروع مشكلة تقدير إتجاهات الوصول المتغيرة مع مرور الوقت. يقدم أربع طرق : الطريقة الأولى تعتمد على مفهوم التردد الفوري ، الطريقتان الثانية و الثالثة توسعان نطاق الطرق الكلاسكية إلى الحالة الغير مستقرة و الطريقة الأخيرة تعتمد على تحليل الفضاء الفرعي ، على التوالي.

الكلمات المفتاحية : معالجة الإشارات ،تحديد إتجاهات الوصول

Résumé

L'estimation des directions d'arrivée (DoA) en traitement d'antennes est un domaine d'actualité avec de nombreux champs d'application tels que les communications sans fil, radar, sonar, traitement de la voix, navigation, séismologie. Lorsque les directions d'arrivée varient dans le temps, à cause du mouvement des sources, les approches telles que les algorithmes MUltiple Signal Classification (MUSIC) et Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) ne parviennent pas à estimer correctement les directions d'arrivée, car ils supposent que ces dernières sont constantes. Ce projet a pour but d'estimer des directions d'arrivée qui varient au cours du temps. Il introduit quatre approches : la première s'appuie sur le concept de fréquence instantanée, la seconde et troisième étendent les méthodes classiques au cas non stationnaire et la dernière est basée sur la décomposition sous-espace, respectivement.

Mots clés : traitement d'antennes, antennes adaptatives, direction d'arrivée.

Abstract

Directions of arrival (DoA) estimation in array signal processing is an active research area with various applications in wireless communications, radar, sonar, speech, navigation, seismology and other fields. When directions of arrival are time-varying, due to moving sources, approaches such as the MUltiple SIgnal Classification (MUSIC) method and the Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) algorithm fail in providing correct estimates, because the latter assume constant DoAs. This project addresses the problem of estimating time-varying directions of arrival. It introduces four approaches: the first one relies on the instantaneous frequency concept, the second and third ones extend the classical methods to the non-stationary case and the last one is based on subspace decomposition, respectively.

Keywords: array processing, direction finding, adaptive antennas, direction of arrival.

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List of Acronyms

DoA Direction of Arrival

MVDR Minimum Variance Distortionless Response

MUSIC MUltiple SIgnal Classification

ESPRIT Estimation of Signal Parameters via Rotational Invariance Techniques

TDoA Time Difference of Arrival

RLS Recursive Least Squares

LMS Least Mean Squares

IF Instantaneous Frequency

TFR Time-Frequency Representation

TFD Time-Frequency Distribution

FRFT Fractional Fourier Transform

OPAST Orthonormal Projection Approximation and Subspace Tracking

Chapter 1

Introduction

1.1 Context and challenge

Electromagnetic fields are often measured by an array of sensors. A sensor array consists of a number of sensors arranged in a particular configuration. In array processing, where an incoming wave is measured by an array, the corresponding signals at different points in space can be processed to extract various types of information including their direction of arrival (DoA), which is the object of our study. DoA methods can also be used to design and adapt the directivity of array antennas in order to accept signals from certain directions only while rejecting other signals that are considered as interference. This is known as spatial filtering.

In what follows, we present the signal model for narrowband arrays. The structure of propagation delays is discussed for a uniform linear array. This data model will be used throughout the project.

Consider the case of a moving narrowband source, in a noise-free environment, emitting plane waves from the time-varying direction $\theta_1(t)$ towards a uniform linear array of M sensors spaced by a distance d (see Figure 1.1).

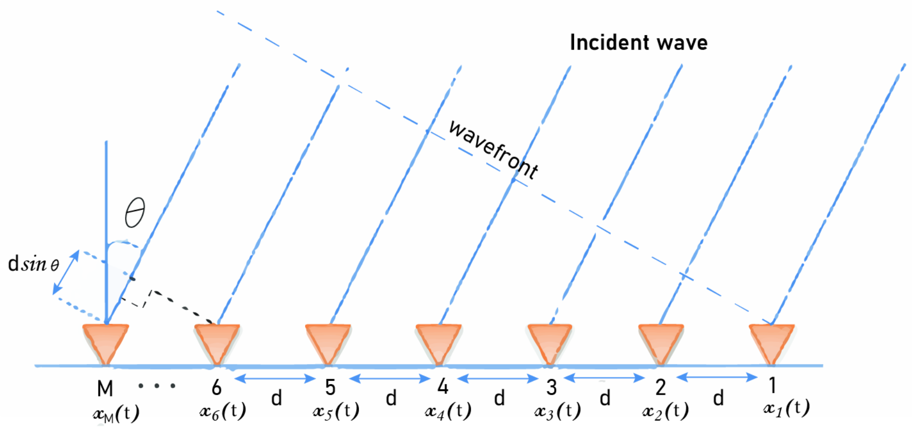

Figure 1.1: Model configuration

Each antenna, except the first one, receives delayed versions of the demodulated transmitted signal $s_1(t)e^{j2\pi f_c t}$, where f_c is the carrier frequency.

By putting the received signals in one vector, called the sensor vector, one obtains:

$$\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_M(t) \end{bmatrix} = \begin{bmatrix} s_1(t) \\ s_1(t-\tau_1(t))e^{-j2\pi f_c \tau_1(t)} \\ \vdots \\ s_1(t-(M-1)\tau_1(t))e^{-j(M-1)2\pi f_c \tau_1(t)} \end{bmatrix}$$

However:

$$\begin{cases} f_c = \frac{c}{\lambda} \\ \tau_1(t) = \frac{d}{c}\sin\theta_1(t) \end{cases}$$

$$\implies \mathbf{x}(t) = \begin{bmatrix} s_1(t) \\ s_1(t-\tau_1(t))e^{-j2\pi\frac{d}{\lambda}\sin\theta_1(t)} \\ \vdots \\ s_1(t-(M-1)\tau_1(t))e^{-j(M-1)2\pi\frac{d}{\lambda}\sin\theta_1(t)} \end{bmatrix}$$

$$\implies \mathbf{x}(t) = \begin{bmatrix} s_1(t) \\ s_1(t-\tau_1(t))e^{-j\phi_1(t)} \\ \vdots \\ s_1(t-(M-1)\tau_1(t))e^{-j(M-1)\phi_1(t)} \end{bmatrix} \tag{1.1}$$

where: $\phi_1(t) = 2\pi\frac{d}{\lambda}\sin\theta_1(t)$ is the electrical angle.

Narrowband signals are signals for which:

$$\tau_{max} = (M-1)\tau_1(t) << \frac{1}{W}$$

where W is the bandwidth of $s_1(t)$

Based on the above property, we can state that:

$$s_1(t-(M-1)\tau_1(t)) \approx s_1(t)$$

Proof: Let W be the bandwidth of $s_1(t)$.

$$s_1(t-(M-1)\tau_1(t)) = \int_{-\frac{W}{2}}^{\frac{W}{2}} S_1(f)e^{j2\pi f(t-(M-1)\tau_1(t))}df$$

$$s_1(t-(M-1)\tau_1(t)) \approx \int_{-\frac{W}{2}}^{\frac{W}{2}} S_1(f)e^{j2\pi ft}df = s_1(t)$$

Equation (1.1) then becomes:

$$\mathbf{x}(t) = \begin{bmatrix} 1 \\ e^{-j\phi_1(t)} \\ \vdots \\ e^{-j(M-1)\phi_1(t)} \end{bmatrix} s_1(t) = \mathbf{a}(\theta_1(t))s_1(t)$$

For the multi-source case, the same procedure is followed:

$$\mathbf{x}(t) = \mathbf{a}(\theta_1(t))s_1(t) + \mathbf{a}(\theta_2(t))s_2(t) + \cdots + \mathbf{a}(\theta_L(t))s_L(t)$$

$$\Longrightarrow \mathbf{x}(t) = A(t)\mathbf{s}(t)$$

where

$$\mathbf{A}(t) = \big[\mathbf{a}(\theta_1(t)), \ldots, \mathbf{a}(\theta_L(t))\big]$$

is the $M \times L$ steering matrix which consists of the vectors

$$\mathbf{a}(\theta_i(t)) = \big[1, e^{-j\pi \sin\theta_i(t)}, \ldots, e^{-j(M-1)\pi \sin\theta_i(t)}\big]^T$$

In the presence of additive noise, one obtains:

$$\mathbf{x}(t) = A(t)\mathbf{s}(t) + \mathbf{n}(t)$$

where $\mathbf{n}(t)$ is the sensor noise vector, which is assumed to be Gaussian with zero-mean and variance σ^2.

1.2 Contributions

The contributions of our work are the suggestion of several methods such as the use of the instantaneous frequency concept, the adaptive versions of both the Minimum Variance Distortionless Response (MVDR) filter and the MUSIC algorithm along with the reduced rank technique (Krylov subspace) in order to estimate and track time-varying DoAs.

1.3 Outline

The outline of the report is as follows:

- **Chapter 2** gives a brief literature review about source localization.
- **Chapter 3** proposes to estimate time-varying DoAs using the instantaneous frequency concept.

- **Chapter 4** gives two other methods (Adaptive MVDR filter and Reduced Rank technique) for tracking moving sources. It also provides details on how the Kalman filter can be used for smoothing the DoAs estimates.
- **Chapter 5** extends the application of the MUSIC algorithm to the non-stationary case.
- **Chapter 6** summarizes the contributions of this project and discusses interesting directions for future research.

Chapter 2

State of the art

2.1 Introduction

The need for direction of arrival (DoA) estimation arises in many engineering applications, such as *radar, wireless communications, astronomy, etc.*
Much of the work that has been done throughout the years focused on estimating the direction of electromagnetic waves impinging on one or more antennas [1].

2.2 Literature review

DoA estimation algorithms can be divided into two basic categories, namely [2]:

1. Classical methods which use beamformers or spatial filters:
   (a) Conventional beamformer (*Bartlett,1948*)
   (b) Minimum Variance Distortionless Response (MVDR) filter (*Capon,1969*)
2. Subspace methods which apply linear algebra principles:
   (a) MUltiple SIgnal Classification (MUSIC) algorithm (*Schmidt,1986*)

(b) Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) algorithm (*Roy & Kailath,1989*)

Note that these algorithms have been developed under the assumption that the sources are stationary in space.

However, in the case where the directions of arrival are changing with time, several methods have been introduced in earlier days, such as:

- (*Dowling et al.1994*) who came up with an adaptive singular values decomposition that enables tracking real time angles [3].
- Or even (*Strobach,1998*) who presented a fast recursive subspace adaptive ESPRIT algorithm [4].

Moreover, and in the recent decades, numerous new approaches have been developed, for instance:

- (*Sakhtari et al.2004*) who combined the time difference of arrival (TDOA) with the recursive least squares (RLS) algorithm for source tracking [5].
- (*Badeau et al.2005*) who proposed a much faster adaptive (on-line) ESPRIT algorithm [6].
- and (*Valizadeh et al.2007*) who introduced a robust adaptive beamformer via LMS-type procedure [7].

2.3 Conclusion

As we can see, only the ESPRIT algorithm has been made adaptive, compared to the MVDR filter and the MUSIC algorithm. That is why the adaptive versions of the latters will be the subject of this project.

Chapter 3

Instantaneous Frequency based estimation of time varying Directions of Arrival

3.1 Introduction

In this chapter, we propose to solve the problem of estimating the time varying DoAs using the instantaneous frequency estimates of the array outputs. In contrast to high resolution approaches such as the MUltiple SIgnal Classification (MUSIC) method [8] and the Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) algorithm [9] which fail in providing correct estimates due to the time-variation of the DoAs, our proposed approach not only provides good estimates, but is also able to solve the underdetermined case where the number of sources is greater than the number of sensors.

This chapter is structured as follow : Section 3.2 provides the signal model under consideration, the proposed algorithm is presented in Section 3.3, Section 3.4 provides the reader with an insight into the FAST IF algorithm and simulation results are presented in Section 3.5, and finally, Section 3.6 concludes the chapter.

3.2 Signal Model

Consider a uniform linear array of M sensors spaced by half a wavelength, which receives plane waves emitted by L moving narrowband sources from time-varying directions $\{\theta_1(t), \ldots, \theta_L(t)\}$.

Let $\mathbf{x}(t) = \left[x_1(t), \ldots, x_M(t)\right]^T$ be the sensor vector which can be modeled as:

$$\mathbf{x}(t) = \mathbf{A}(t)\mathbf{s}(t) + \mathbf{n}(t) \tag{3.1}$$

where

$$\mathbf{A}(t) = \left[\mathbf{a}(\theta_1(t)), \ldots, \mathbf{a}(\theta_L(t))\right] \tag{3.2}$$

is the $M \times L$ steering matrix which consists of the vectors

$$\mathbf{a}(\theta_i(t)) = \left[1, e^{-j\pi \sin\theta_i(t)}, \ldots, e^{-j(M-1)\pi \sin\theta_i(t)}\right]^T \tag{3.3}$$

and

$$\mathbf{s}(t) = [s_1(t), \cdots, s_L(t)]^T \tag{3.4}$$

is an L-dimensional vector containing the source waveforms. $\mathbf{n}(t)$ is the sensor noise vector, which is assumed to be Gaussian with zero-mean and variance σ^2.

In the sequel, we propose to estimate the time variation of the Directions of Arrival $\theta_i(t), i = 1, \cdots, L$ through the estimates of the instantaneous frequencies [10] of the sensor outputs $x_i(t), i = \cdots, M$.

3.3 Proposed Algorithm

3.3.1 One source case

Let us first consider the case of $L = 1$ source in a noise-free environment, it follows from equation (3.1), that:

$$\mathbf{x}(t) = \mathbf{a}(\theta_1(t))s_1(t) \tag{3.5}$$

and by considering the source signal $s_1(t)$ as analytic, i.e. $s_1(t) = b_1(t)e^{j\phi_{s_1}(t)}$, together with equation (3.3), we obtain the following observation vector:

$$\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_M(t) \end{bmatrix} = b_1(t) \begin{bmatrix} e^{j\phi_{s_1}(t)} \\ e^{j(-\phi_1(t)+\phi_{s_1}(t))} \\ \vdots \\ e^{j(-(M-1)\phi_1(t)+\phi_{s_1}(t))} \end{bmatrix} \tag{3.6}$$

where

$$\phi_1(t) = \pi \sin\theta_1(t) \tag{3.7}$$

denotes the *electrical angle* of the 1^{st} source.

It appears clearly from equation (3.6) that the instantaneous phase of the i^{th} sensor signal is given by

$$\phi_{x_i}(t) = -(i-1)\phi_1(t) + \phi_{s_1}(t)$$

and its instantaneous frequency is given by :

$$f_{x_i}(t) = \frac{1}{2\pi}\frac{d\phi_{x_i}(t)}{dt} = -(i-1)\frac{1}{2\pi}\frac{d\phi_1(t)}{dt} + f_{s_1}(t) \tag{3.8}$$

Accordingly, one obtains the following vector whose entries are the instantaneous frequencies of the M sensor signals:

$$\begin{bmatrix} f_{x_1}(t) \\ f_{x_2}(t) \\ \vdots \\ f_{x_M}(t) \end{bmatrix} = \begin{bmatrix} f_{s_1}(t) \\ -\frac{1}{2\pi}\frac{d\phi_1(t)}{dt} + f_{s_1}(t) \\ \vdots \\ -(M-1)\frac{1}{2\pi}\frac{d\phi_1(t)}{dt} + f_{s_1}(t) \end{bmatrix} \tag{3.9}$$

By subtracting the instantaneous frequency of the first sensor signal from the M-1 other sensor signals, one gets the following new vector :

$$\begin{bmatrix} f_{x_2}(t) - f_{x_1}(t) \\ f_{x_3}(t) - f_{x_1}(t) \\ \vdots \\ f_{x_M}(t) - f_{x_1}(t) \end{bmatrix} = \begin{bmatrix} -\frac{1}{2\pi}\frac{d\phi_1(t)}{dt} \\ -2\frac{1}{2\pi}\frac{d\phi_1(t)}{dt} \\ \vdots \\ -(M-1)\frac{1}{2\pi}\frac{d\phi_1(t)}{dt} \end{bmatrix} \tag{3.10}$$

$$= -\frac{1}{2\pi}\frac{d\phi_1(t)}{dt} \begin{bmatrix} 1 \\ 2 \\ \vdots \\ M-1 \end{bmatrix} \tag{3.11}$$

In order to extract the first derivative of the electrical angle $\phi_1(t)$, we multiply both sides of (3.10) by $\begin{bmatrix} 1 & 2 & \dots & M-1 \end{bmatrix}$, which leads to:

$$\begin{bmatrix} 1 & 2 & \dots & M-1 \end{bmatrix} \begin{bmatrix} f_{x_2}(t) - f_{x_1}(t) \\ f_{x_3}(t) - f_{x_1}(t) \\ \vdots \\ f_{x_M}(t) - f_{x_1}(t) \end{bmatrix} = -\frac{1}{2\pi}\frac{d\phi_1(t)}{dt}\sum_{i=1}^{M-1} i^2$$

Finally, the *"angular instantaneous frequency"* is then estimated as:

$$\frac{1}{2\pi}\frac{d\phi_1(t)}{dt} = -\frac{\sum_{i=1}^{M-1} i(f_{x_{i+1}}(t) - f_{x_1}(t))}{\sum_{i=1}^{M-1} i^2}$$

$$= -\frac{6\sum_{i=1}^{M-1} i(f_{x_{i+1}}(t) - f_{x_1}(t))}{M(M-1)(2M-1)} \tag{3.12}$$

By applying to the estimated angular instantaneous frequency of equation (3.12) an integrator filter whose transfer function is given by:

$$H(z) = \frac{T_s}{2}\frac{1+z^{-1}}{1-z^{-1}}$$

and where T_s is the sampling period, one obtains an estimate of the electrical angle $\hat{\phi}_1(t)$ up to 2π constant. Since the electrical angle and the direction of arrival of the source are related by (3.7), one can also estimate $\hat{\theta}_1(t)$.

3.3.2 Multiple sources case

In the case of several sources, one has to estimate first the instantaneous frequency of each source at each sensor and then apply the above procedure for each source separately. In this project, we use as estimation technique of the instantaneous frequencies of a multi-component signal the **FAST IF** algorithm of reference [11].

3.4 Insight into the FAST IF algorithm

In various research fields such as vibration analysis, speech recognition, signals are non-stationary and have multi components, that is why a time-frequency representation (**TFR**) is necessary since the frequency content of such signals is time varying.

The present section introduces the algorithm of reference [11] which provides a TFR without computing a time-frequency distribution (**TFD**) [10].

Consider having a signal with **K** components:

$$s(t) = \sum_{k=1}^{K} a_k(t) e^{j2\pi \int_{-\infty}^{t} f_k(\tau) d\tau}$$

where $a_k(t)$ and $f_k(t)$ represent the instantaneous amplitude and the instantaneous frequency of the k^{th} component.

The FAST IF algorithm suggests estimating the IF in an iterative way according to the following steps:

1. First, it finds the time instant of the highest signal energy t_0, i.e.:

$$t_0 = \underset{t}{argmax}\, s_e(t)$$

where

$$s_e(t) = \int_{t-\Delta t}^{t+\Delta t} |s(\tau)|^2 d\tau$$

$s_e(t)$ computes the signal energy in the time interval $t - \Delta t < t < t + \Delta t$. Where the parameter Δt determines the width of the window to estimate the local energy of the signal. In our study, $\Delta t = 63$

2. Then it finds out the location of maximum frequency and optimum rotation order of analysis window f_0 and α_0, respectively:

$$(f_0, \alpha_0) = \underset{f,\alpha_l}{argmax}\, S_{\alpha_l}(f)$$

where

$$S_{\alpha_l}(f) = \int_{-\infty}^{+\infty} s(t) w_{\alpha_l}(t - t_0) e^{-j2\pi f t} dt \qquad (3.13)$$

is the Fourier transform of the signal multiplied by a window shifted by t_0

and

$$w_{\alpha_l}(t) = \frac{e^{j\alpha_l/2}}{\sqrt{jsin(\alpha_l)}} \int_{-\infty}^{+\infty} e^{-\frac{\mu}{2\sigma^2}} e^{j\pi(\mu^2+t^2)cos(\alpha_l - 2t\mu)/sin\alpha_l} d\mu$$

α_l is the rotation order of the fractional Fourier transform and is given by $\alpha_l = \frac{l}{L}$ where the parameter L determines the number of quantization levels (see Appendix A.1 for more details about the fractional Fourier transform).

3. The IF of the strongest component at time-instant t_0 is $\hat{f}_1(t_0) = f_0$.

4. Then we estimate the IF for $t > t_0$ iteratively as follows :

   (a) We first initialize $\hat{t} = t_0$. Then we increment it with step $\frac{1}{f_s}$, i.e. $\hat{t} = \hat{t} + \frac{1}{f_s}$ (f_s being the sampling frequency). And each time, we estimate the IF at $\hat{t}$ by searching the maximum energy in the neighborhood of f_0, i.e in the interval $f_0 - \Delta f < f < f_0 + \Delta f$ using 3 different fractional windows: one with $\alpha_0 - \Delta\alpha_0$ and one with α_0 and finally one with $\alpha_0 + \Delta\alpha_0$ ($\Delta\alpha_0$ is set to be $\Delta\alpha_0 = \frac{1}{2L+1}$ and $\Delta f = 2$ since the frequency search is limited to 5 bins in our study)

$$(f_0, \alpha_0) = \underset{f,\alpha_l}{argmax} \left| \int_{-\infty}^{+\infty} s(t) w_{\alpha_l}(t - \hat{t}) e^{-j2\pi f t} dt \right|$$

   (b) The estimated IF at $\hat{t}$ is considered to be $\hat{f}_1(\hat{t}) = f_0$.

   (c) We repeat the iterations until the energy is less than 0.1 times the energy of the highest energy TF point.

5. Once the IF of the strongest component is estimated for $t > t_0$, the same process is applied to estimate the IF for $t < t_0$ (the increment will slightly change and becomes $\hat{t} = \hat{t} - \frac{1}{f_s}$).

6. After estimating the IF "$\hat{f}_1(t)$" corresponding to the strongest component, we remove the latter from the mixture signal according to the following process:

   (a) First consider the signal:

$$y(t) = s(t)e^{-j2\pi \int_{-\infty}^{t} \hat{f}_1(\tau)d\tau}$$

$$y(t) = \left(\sum_{k=1}^{K} a_k(t)e^{j2\pi \int_{-\infty}^{t} f_k(\tau)d\tau} \right) e^{-j2\pi \int_{-\infty}^{t} \hat{f}_1(\tau)d\tau}$$

   which can be written as:

$$y(t) = a_1(t)e^{j2\pi(\int_{-\infty}^{t} f_1(\tau)-\hat{f}_1(\tau)d\tau)} \quad + \\ \left(\sum_{k=2}^{K} a_k(t)e^{j2\pi \int_{-\infty}^{t} f_k(\tau)d\tau} \right) e^{-j2\pi \int_{-\infty}^{t} \hat{f}_1(\tau)d\tau}$$

   (b) Under the assumption $\hat{f}_1(t) - f_1(t) \approx 0$, $y(t)$ becomes:

$$y(t) = a_1(t) + \left(\sum_{k=2}^{K} a_k(t)e^{j2\pi \int_{-\infty}^{t} f_k(\tau)d\tau} \right) e^{-j2\pi \int_{-\infty}^{t} \hat{f}_1(\tau)d\tau}$$

   (c) Then we apply on $y(t)$ a low pass filter in order to get the estimated instantaneous amplitude of the strongest component, i.e. $\hat{a}_1(t)$.

   (d) Having $\hat{a}_1(t)$, we subtract the strongest component "$\hat{s}_1(t)$" from the original signal:

$$s(t) = s(t) - \hat{s}_1(t)$$

   with

$$\hat{s}_1(t) = \hat{a}_1(t)e^{j2\pi \int_{-\infty}^{t} \hat{f}_1(\tau)d\tau}$$

7. The same process is repeated for the next strongest component in the remaining mixture signal and so on untill the energy of the extracted component is less than 0.05 times the energy of the strongest component.

3.5 Simulation results

The performance of the proposed DoA estimation algorithm are assessed using synthetic signals.

3.5.1 One source case

Let us first consider $L = 1$ source impinging, in the absence of noise, on an array of $M = 4$ sensors and whose direction of arrival varies linearly, i.e. $\theta_1(t) = \alpha t$ where α is set to equal 0.8.

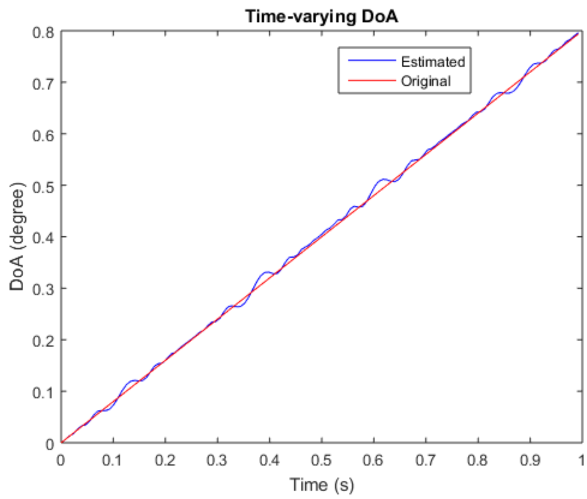

Figure 3.1: Estimated DoA vs Original DoA (1 source and 4 sensors)

Interpretation of result: Fig.3.1 displays the time variation of the estimated DoA (in blue) versus the original DoA (in red). Indeed, the blue curve fits the red one, meaning that the linearly time varying DoA was well estimated.

3.5.2 Multiple sources case

The DoA estimation algorithm can be extended to the multi source scenario with the following experimental setup:

- Number of sensors (M=10).

- Quadratic and cubic variations of the two sources, $\theta_1(t) = 10t^2$ and $\theta_2(t) = 15t^3$, respectively.
- Additive noise with a signal-to-noise ratio $SNR = 20dB$.

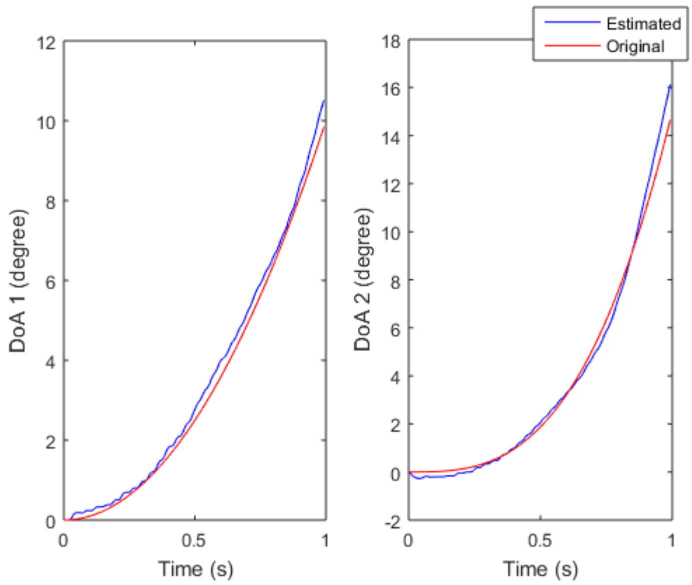

Figure 3.2: 2 sources and 10 sensors with $SNR = 20dB$

Interpretation of results: Fig.3.2 contains two plots which show quadratic and cubic variations of the DoAs, respectively. The obtained results show that the proposed approach allows estimating quadratic angular time variation that is related to angular acceleration in practice and also a hypothetical cubic angular time variation.

3.5.3 Underdetermined case

Herein, we consider the case of 2 sensors and 3 sources with an $SNR = 20dB$.

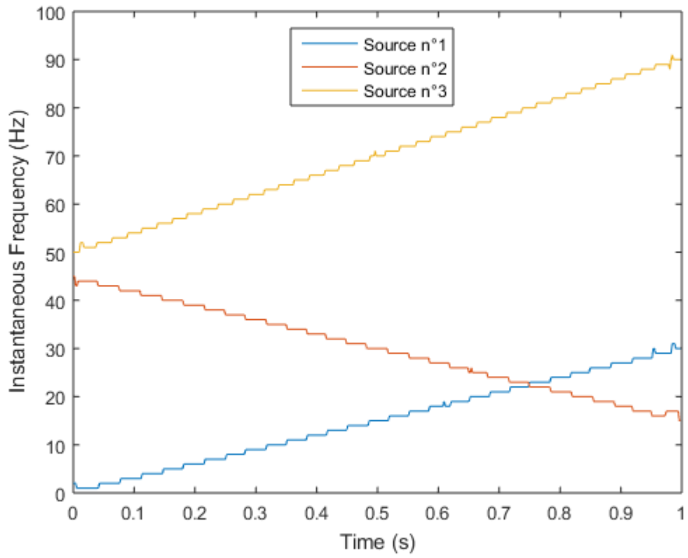

Figure 3.3: IF estimates of the 1^{st} sensor signal

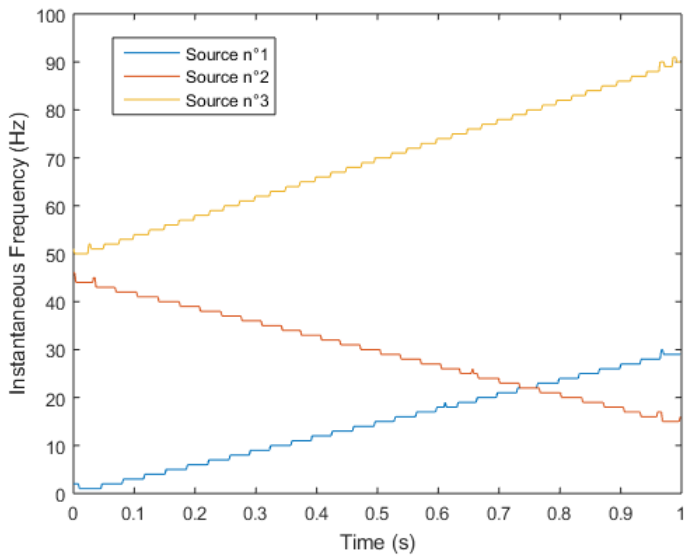

Figure 3.4: IF estimates of the 2^{nd} sensor signal

Interpretation of results: Figures 3.3 and 3.4 display the estimates of the instantaneous frequencies of the 1^{st} sensor signal and the 2^{nd} sensor signal, respectively, while using the FAST IF algorithm. Evidently, the FAST IF algorithm successfully estimates the instantaneous frequencies of the sensors signals.

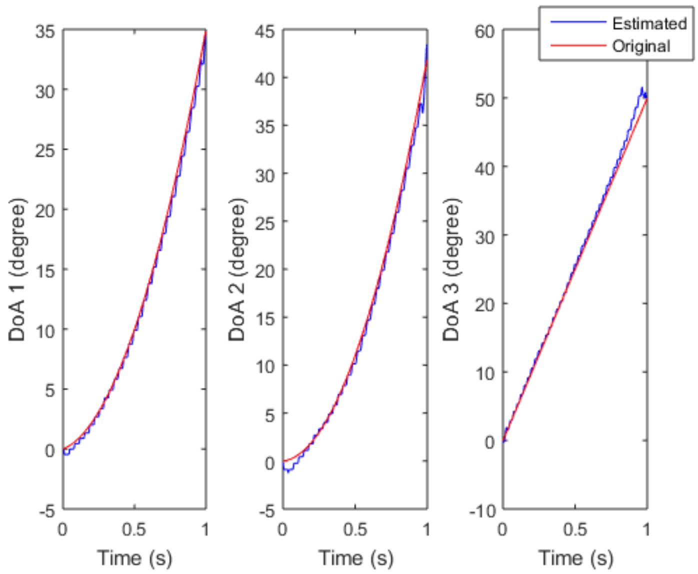

Figure 3.5: 3 sources and 2 sensors with $SNR = 20dB$

Interpretation of results: Fig.3.5 contains three plots, one for each source and where each estimated DoA has been plotted along with the original DoA. The figure highlights the ability of our suggested method to address the underdetermined case where 3 time varying directions of arrival have been estimated using only 2 sensors and with an $SNR = 20dB$.

3.6 Conclusion

In this chapter, we introduced a novel method in order to estimate time-varying DoAs. Indeed, feeding the estimates of the array outputs' instantaneous frequencies to an integrator filter allows estimating any angular time variation, whether it is linear or hyperbolic. Furthermore, the advantage of the proposed approach is its ability to solve the underdetermined case. The implemented method is a batch technique, i.e. it has to *"wait"* until it has all the data. An extension of the proposed approach to a tracking one depends only on the use of an efficient online instantaneous frequency estimation algorithm. On the other hand, the subsequent chapters introduce tracking approaches, i.e. methods that estimate the time-varying DoAs in an adaptive manner, making the tracking possible.

Chapter 4

Estimation of time varying Directions of Arrival using the adaptive version of the MVDR filter and the Reduced Rank approach

4.1 Introduction

Two other methods for estimating time-varying DoAs are given subsequently in this chapter. The first one is an extension of Capon's approach [12] to the non-stationary case in order to estimate and track the time-varying DoAs adaptively. And the second one is the Reduced-Rank adaptive filtering using Krylov subspace.

The chapter is structured as follows : in Section 4.2, the data model is recalled. Section 4.3 introduces the reader to Capon's method in the stationary case, followed by its extension to the non-stationary case in Section 4.4. In Section 4.5, the reduced rank method based on Krylov subspace is presented. Section 4.6 details how the Kalman filter can be used for smoothing the DoAs estimates provided by the proposed adaptive Capon algorithm and the reduced rank method. Simulation results are presented in Section 4.7, and finally, Section 4.8 concludes the chapter.

4.2 Data Model

As in chapter 3, we consider a uniform linear array of M sensors spaced by half a wavelength, which receives plane waves emitted by L moving narrowband sources from time-varying directions $\{\theta_1(t), \ldots, \theta_L(t)\}$.

Let $\mathbf{x}(t) = \left[x_1(t), \ldots, x_M(t)\right]^T$ be the sensor vector which can be modeled as:

$$\mathbf{x}(t) = \mathbf{A}(t)\mathbf{s}(t) + \mathbf{n}(t) \tag{4.1}$$

where

$$\mathbf{A}(t) = \left[\mathbf{a}(\theta_1(t)), \ldots, \mathbf{a}(\theta_L(t))\right] \tag{4.2}$$

is the $M \times L$ steering matrix which consists of the vectors

$$\mathbf{a}(\theta_i(t)) = \left[1, e^{-j\pi \sin\theta_i(t)}, \ldots, e^{-j(M-1)\pi \sin\theta_i(t)}\right]^T \tag{4.3}$$

and

$$\mathbf{s}(t) = [s_1(t), \cdots, s_L(t)]^T \tag{4.4}$$

is an L-dimensional vector containing the source waveforms. $\mathbf{n}(t)$ is the sensor noise vector, which is assumed to be Gaussian with zero-mean and variance σ^2.

4.3 Insight into Capon's method

Capon's beamformer or **M**inimum **V**ariance **D**istortionless **R**esponse (**MVDR**) filter was first introduced in the stationary case. It consists of minimizing, with a constraint, the contributions of signals not of interest SNOI (i.e signals coming from directions different from the direction of interest θ_1) and noise by minimizing the output power of the beamformer.

This problem can be stated mathematically as a constrained minimization problem :

$$\begin{cases} argmin_{\mathbf{w}} \, P(\mathbf{w}) = argmin_{\mathbf{w}} \, \mathbf{w}^H R_x \mathbf{w} \\ \mathbf{w}^H \mathbf{a}(\theta_1) = 1 \end{cases} \tag{4.5}$$

where:

- $R_x = E[\mathbf{x}(t)\mathbf{x}(t)^H]$ is the covariance matrix of the sensor vector.
- $\mathbf{a}(\theta_1(t)) = \left[1, e^{-j\pi \sin\theta_1(t)}, \ldots, e^{-j(M-1)\pi \sin\theta_1(t)}\right]^T$ is the steering vector of the source of interest.

The solution of the above optimization problem [13] is given by:

$$\mathbf{w}_{Capon}(\theta) = \frac{R_x^{-1}\mathbf{a}(\theta)}{\mathbf{a}(\theta)^H R_x^{-1} \mathbf{a}(\theta)} \tag{4.6}$$

which is known as Capon's filter.

Since the MVDR filter was first proposed in the stationary case, the data model is similar to the one in Section 4.2 but with a slight change since the DoAs are constant. Thus Equation (4.1) just becomes:

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \tag{4.7}$$

$$\mathbf{x}(t) = \mathbf{a}(\theta_1)s_1(t) + \mathbf{a}(\theta_2)s_2(t) + \cdots + \mathbf{a}(\theta_L)s_L(t) + \mathbf{n}(t) \tag{4.8}$$

which can be rewritten as :

$$\mathbf{x}(t) = \mathbf{a}(\theta_1)s_1(t) + \mathbf{J}(t) \tag{4.9}$$

where $\mathbf{J}(t)$ is M-dimensional column vector containing the SNOI and the noise.

The output power of the beamformer is given by:

$$P(\mathbf{w}) = E[|y(t)|^2] = \sigma_{s_1}^2 |\mathbf{w}^H \mathbf{a}(\theta_1)|^2 + \mathbf{w}^H R_J \mathbf{w} \tag{4.10}$$

where $R_J = E[\mathbf{J}(t)\mathbf{J}(t)^H]$ is the covariance matrix of $\mathbf{J}(t)$

Since the beamformer gain is constrained to be 1 in the direction θ_1, i.e $\mathbf{w}^H\mathbf{a}(\theta_1) = 1$, minimizing the output power minimizes indeed the contributions of the SNOI and noise.

Using the Lagrange multipliers, Equation (4.10) is minimized and its optimal solution is [2]

$$\mathbf{w} = \frac{R_x^{-1}\mathbf{a}(\theta_1)}{\mathbf{a}(\theta_1)^H R_x^{-1}\mathbf{a}(\theta_1)} \tag{4.11}$$

This procedure can be applied for each possible angle θ and provides Capon's filter (4.6).

By replacing (4.6) in the output power expression, one obtains:

$$P_{Capon}(\theta) = \frac{1}{\mathbf{a}(\theta)^H R_x^{-1}\mathbf{a}(\theta)} \tag{4.12}$$

The angles for which (4.12) has peaks represent the estimates of the directions of arrival.

4.4 Adaptive Capon's method

This section introduces the adaptive version of the MVDR filter, which is an extension of Capon's approach to the non-stationary case. The mono-source case is considered first, then the multi-sources one is discussed.

4.4.1 Mono-source case

Consider the case where $L = 1$ source radiates plane waves towards a uniform linear array of M sensors spaced by half a wavelength from a time-varying direction.

Assuming the slow variation of the DoA, the gradient ascent optimization method has been used in this adaptive version.

We first begin by initializing our algorithm using a batch of length N_0 where we assumed stationarity and ergodicity in order to estimate the initial covariance matrix as :

$$\hat{R}_0 = \frac{1}{N_0}\sum_{n=1}^{N_0} \mathbf{x}(n)\mathbf{x}(n)^H$$

We then estimated the initial value of the DoA by searching for the peak of (4.12), i.e:

$$\hat{\theta}_0 = \underset{\theta}{argmax} \frac{1}{\mathbf{a}(\theta)^H \hat{R}_0^{-1}\mathbf{a}(\theta)}$$

Secondly, we estimate the covariance matrix in an adaptive manner as:

$$\hat{R}(n) = \beta\hat{R}(n-1) + (1-\beta)\mathbf{x}(n)\mathbf{x}(n)^H \quad (4.13)$$

with $0 < \beta < 1$ being the forgetting factor.

and used Schur inversion lemma [14]:

$$\hat{R}(n)^{-1} = \frac{1}{\beta}\hat{R}(n-1)^{-1} - \frac{(\frac{\hat{R}(n-1)^{-1}}{\beta}\sqrt{1-\beta}\mathbf{x}(n))(\frac{\hat{R}(n-1)^{-1}}{\beta}\sqrt{1-\beta}\mathbf{x}(n))^H}{1+\sqrt{1-\beta}\mathbf{x}(n)^H(\frac{\hat{R}(n-1)^{-1}}{\beta}\sqrt{1-\beta}\mathbf{x}(n))}$$

since we need the inverse in (4.12).

Finally, we fed $\hat{\theta}_0$ to our adaptive gradient ascent algorithm:

$$\hat{\theta}_n = \hat{\theta}_{n-1} + \mu\nabla P_{Capon}(\hat{\theta}_{n-1})$$

which will search for the values of θ that maximize the objective function (4.12) and where:

- μ is the learning rate.
- $\nabla P_{Capon}(\theta) = -(\mathbf{a}(\theta)^H R_x^{-1}\mathbf{a}(\theta))^{-2}(\frac{d\mathbf{a}(\theta)^H}{d\theta} R_x^{-1}\mathbf{a}(\theta) + \mathbf{a}(\theta)^H R_x^{-1}\frac{d\mathbf{a}(\theta)}{d\theta})$ is the gradient of the objective function (4.12).

4.4.2 Multi-source case

We basically applied the aforementioned procedure to each source separately, i.e. we first estimated the initial values of the DoAs using Capon's method. Then, we fed them individually to our adaptive gradient ascent algorithm as shown in the figure below:

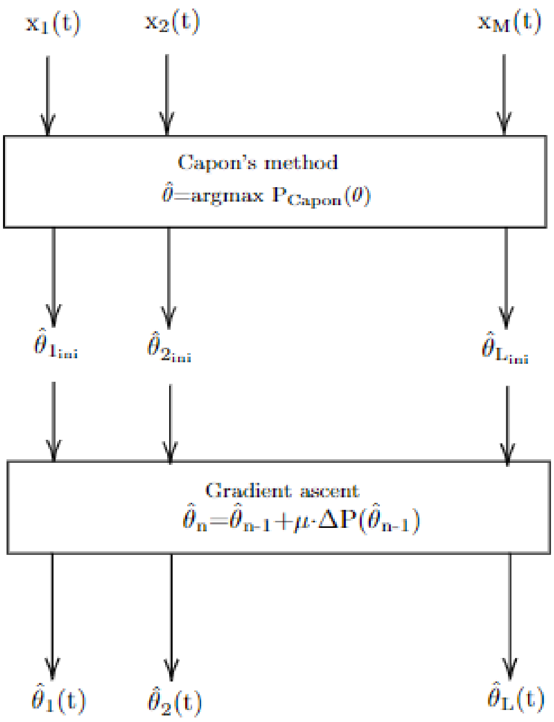

Figure 4.1: Bloc diagram of adaptive Capon

A well-known problem in DoA estimation is when two or more sources have incident angles that are below the spatial resolution of the processing algorithm and where the latter is defined as the ability to distinguish these sources. Naturally, the crossing sources problem falls under the aforementioned category.

A way to tackle this problem would be the use of a Kalman filter.

4.5 Reduced rank method based on Krylov subspace

4.5.1 Motivation

This method belongs to the class of dimension reduction method, and what motivated the use of the latter is the fact that it reduces the computation complexity.

4.5.2 Principle

In this subsection, the principle of the reduced rank approach is presented. Since the method is based on Krylov subspace [15], let us first define the latter.

Definition: Given a square matrix A and a nonzero vector $\mathbf{v}$, the D^{th} Krylov subspace associated with the pair $(A, \mathbf{v})$ is defined as:

$$K^D(A, \mathbf{v}) = span(\mathbf{v}, A\mathbf{v}, A^2\mathbf{v}, \ldots, A^{D-1}\mathbf{v})$$

In this project, we deal with $K^D(R_x, \mathbf{a}(\theta))$ where:

- $R_x = E[\mathbf{x}(t)\mathbf{x}(t)^H]$ is the covariance matrix of the sensor vector.
- $\mathbf{a}(\theta_1(t)) = \left[1, e^{-j\pi \sin\theta_1(t)}, \ldots, e^{-j(M-1)\pi \sin\theta_1(t)}\right]^T$ is the steering vector of the source of interest.

Capon's method (see Section 4.3) consists of minimizing the output power of the beamformer under a specific constraint :

$$\begin{cases} argmin_{\mathbf{w}}\, \mathbf{w}^H R_x \mathbf{w} \\ \mathbf{w}^H \mathbf{a}(\theta) = 1 \end{cases}$$

The reduced rank approach suggests that the beamformer $\mathbf{w}$ can be written as:

$$\mathbf{w} = Q\tilde{\mathbf{w}} \tag{4.14}$$

where:

- Q is an $M \times r$ projection matrix defined as:

$$Q = \begin{bmatrix} \mathbf{a}(\theta) & R_x\mathbf{a}(\theta) & R_x^2\mathbf{a}(\theta) & \dots & R_x^{r-1}\mathbf{a}(\theta) \end{bmatrix}$$

- $\tilde{\mathbf{w}}$ is an r-dimensional vector with $r < M$

The minimization problem then becomes:

$$\begin{cases} argmin_{\tilde{\mathbf{w}}}\, \tilde{\mathbf{w}}^H R_z \tilde{\mathbf{w}} \\ \tilde{\mathbf{w}}^H \tilde{\mathbf{a}}(\theta) = 1 \end{cases} \tag{4.15}$$

where:

- $\tilde{\mathbf{a}}(\theta) = Q^H\mathbf{a}(\theta)$ is the projected r-dimensional steering vector
- $R_z = E[\mathbf{z}(t)\mathbf{z}(t)^H] = Q^H R_x Q$ is the covariance matrix of the projection of the sensor vector $\mathbf{z}(t) = Q^H\mathbf{x}(t)$

Proof: We have the following minimization problem:

$$\begin{cases} argmin_{\mathbf{w}}\, \mathbf{w}^H R_x \mathbf{w} \\ \mathbf{w}^H \mathbf{a}(\theta) = 1 \end{cases}$$

However $\mathbf{w} = Q\tilde{\mathbf{w}}$

$$\implies \begin{cases} argmin_{\tilde{\mathbf{w}}}\, \tilde{\mathbf{w}}^H Q^H R_x Q \tilde{\mathbf{w}} \\ \tilde{\mathbf{w}}^H Q^H \mathbf{a}(\theta) = 1 \end{cases}$$

$$\implies \begin{cases} argmin_{\tilde{\mathbf{w}}}\, \tilde{\mathbf{w}}^H R_z \tilde{\mathbf{w}} \\ \tilde{\mathbf{w}}^H \tilde{\mathbf{a}}(\theta) = 1 \end{cases}$$

Interestingly, instead of looking for the M-dimensional vector $\mathbf{w}$, we now search for an r-dimensional vector $\tilde{\mathbf{w}}$ with $r < M$.

In this project, r is set to equal the number of sources.

Similarly to Capon's method (see Section 4.3), the angles which maximize:

$$P_{RR}(\theta) = \frac{1}{\tilde{\mathbf{a}}(\theta)^H R_z^{-1} \tilde{\mathbf{a}}(\theta)} \tag{4.16}$$

represent the estimates of the DoAs.

4.5.3 Implementation

The family of Reduced Rank methods were developed for solving large systems, that is why the multi-source case will only be addressed.

The Reduced Rank algorithm is similar to the adaptive MVDR filter with slight changes.

Consider L sources radiating plane waves towards a uniform linear array of M sensors spaced by half a wavelength from time-varying directions.

In order to initialize the Reduced Rank algorithm, we use the same batch technique as in the Adaptive Capon's method (see Section 4.4), i.e.

$$\hat{\theta}_{k_{ini}} = \underset{\theta}{argmax} \frac{1}{\mathbf{a}(\theta)^H \hat{R}_0^{-1} \mathbf{a}(\theta)} \quad \text{for} \quad k = 1, \ldots, L$$

The batch technique is also used to initialize the projection matrix and the covariance matrix of the projected sensor vector:

$$\begin{cases} Q_{k_{ini}} = \left[\mathbf{a}(\hat{\theta}_{k_{ini}}) \quad \hat{R}_0 \mathbf{a}(\hat{\theta}_{k_{ini}}) \quad \hat{R}_0^2 \mathbf{a}(\hat{\theta}_{k_{ini}}) \quad \ldots \quad \hat{R}_0^{r-1} \mathbf{a}(\hat{\theta}_{k_{ini}})\right] \\ \hat{R}_{z_{k_{ini}}} = Q_{k_{ini}}^H \hat{R}_0 Q_{k_{ini}} \end{cases}$$

Secondly, we estimate the covariance matrix of the projected sensor vector and the projection matrix adaptively:

$$\begin{cases} \mathbf{z}_k(n) = Q_k^H(n-1)\mathbf{x}(n) \\ \hat{R}_{\mathbf{z}_k}(n) = \beta \hat{R}_{\mathbf{z}_k}(n-1) + (1-\beta)\mathbf{z}_k(n)\mathbf{z}_k(n)^H \\ Q_k(n) = \begin{bmatrix} \mathbf{q}_1(n) & \mathbf{q}_2(n) & \mathbf{q}_3(n) \end{bmatrix} \\ \mathbf{q}_1(n) = \mathbf{a}(\hat{\theta}_k(n)) \quad \text{where } \hat{\theta}_k(n) \text{ is the } k^{th} \text{ DoA estimated by} \\ \text{the Reduced Rank algorithm at time instant } n \\ \mathbf{q}_2(n) = \beta \mathbf{q}_2(n-1) + (1-\beta)\mathbf{x}(n)\mathbf{x}(n)^H \mathbf{q}_1(n) \\ \mathbf{q}_3(n) = \beta \mathbf{q}_3(n-1) + (1-\beta)\mathbf{x}(n)\mathbf{x}(n)^H \mathbf{q}_2(n) \end{cases}$$

Finally, and like the adaptive MVDR filter, we use the gradient ascent optimization method:

$$\hat{\theta}_k^{(n)} = \hat{\theta}_k^{(n-1)} + \mu \nabla P_{RR}(\hat{\theta}_k^{(n-1)})$$

which will search for the values of θ that maximize the cost function (4.16)
and where :

- μ is the learning rate.
- $\nabla P_{RR}(\theta) = -(\tilde{\mathbf{a}}(\theta)^H R_z^{-1} \tilde{\mathbf{a}}(\theta))^{-2} (\frac{d\tilde{\mathbf{a}}(\theta)^H}{d\theta} R_z^{-1} \tilde{\mathbf{a}}(\theta) + \tilde{\mathbf{a}}(\theta)^H R_z^{-1} \frac{d\tilde{\mathbf{a}}(\theta)}{d\theta})$ is the gradient of the cost function (4.16).

4.6 Smoothing using Kalman filter

This section gives the details on how the Kalman filter is implemented for both smoothing and solving the crossing sources problem.

Let us consider the following state vector:

$$\mathbf{y}_k(t) = \begin{bmatrix} \theta_k(t) \\ \dot{\theta}_k(t) \end{bmatrix}$$

where $\theta_k(t)$ is the k^{th} DoA and $\dot{\theta}_k(t)$ is its velocity.

The dynamic of the motion, as well as the observations, are modeled, respectively, as follows [16]:

$$\begin{cases} \mathbf{y}_k(t+1) = F\mathbf{y}_k(t) + \psi_k(t) \\ \hat{\theta}_k(t) = H\mathbf{y}_k(t) + \mathbf{v}_k(t) \end{cases}$$

where:

- $F = \begin{bmatrix} 1 & T_s \\ 0 & 1 \end{bmatrix}$ with T_s being the sampling period.
- $H = \begin{bmatrix} 1 & 0 \end{bmatrix}$.
- $\psi_k(t)$ is the noise model supposed to be Gaussian with zero mean and covariance matrix:

$$Q(t) = E[\psi_k(t)\psi_k(t)^H] = \begin{bmatrix} \frac{T_s^4}{4} & \frac{T_s^3}{2} \\ \frac{T_s^3}{2} & T_s^2 \end{bmatrix} \sigma_\psi^2$$

Proof: By applying a limited development to both the i^{th} DoA and its velocity, one obtains:

$$\begin{cases} \theta_k(t+1) = \theta_k(t) + T_s\dot{\theta}_k(t) + \frac{T_s^2}{2}\ddot{\theta}_k(t) + \dots \\ \dot{\theta}_k(t+1) = \dot{\theta}_k(t) + T_s\ddot{\theta}_k(t) + \dots \end{cases}$$

After truncating the limited development to the second order, we obtain:

$$\mathbf{y}_k(t+1) = F\mathbf{y}_k(t) + \psi_k(t)$$

where:

$$\psi_k(t) = \begin{bmatrix} \frac{T_s^2}{2} \\ T_s \end{bmatrix} \ddot{\theta}_k(t)$$

$$\implies Q(t) = E[\psi_k(t)\psi_k(t)^H] = \begin{bmatrix} \frac{T_s^2}{2} \\ T_s \end{bmatrix} \begin{bmatrix} \frac{T_s^2}{2} & T_s \end{bmatrix} E[|\ddot{\theta}_k(t)|^2]$$

$$\Longrightarrow Q(t) = \begin{bmatrix} \frac{T_s^4}{4} & \frac{T_s^3}{2} \\ \frac{T_s^3}{2} & T_s^2 \end{bmatrix} \sigma_\psi^2$$

- $\mathbf{v}_k(t)$ is the observation noise assumed to be Gaussian with zero mean and variance σ_v^2.

The Kalman filter proceeds in two steps : first a prediction then an update.

4.6.1 Prediction step

From the data available at time t, i.e. the DoAs estimated by the adaptive Capon's algorithm, we can make a prediction of the state vectors:

$$\hat{\mathbf{y}}_k(t+1|t) = F\hat{\mathbf{y}}_k(t|t) \tag{4.17}$$

4.6.2 Update step

After obtaining the new observations, the DoA estimates are improved using:

$$\begin{cases} P(t+1) = F[I - K(t)H]P(t)F^T + Q(t) \\ K(t+1) = P(t+1)H[HP(t+1)H^T + \sigma_v^2]^{-1} \\ \hat{\mathbf{y}}_k(t+1|t+1) = \hat{\mathbf{y}}_k(t+1|t) + K(t+1)[\hat{\theta}_k(t+1) - HF\hat{\mathbf{y}}_k(t|t)] \end{cases}$$

where:

- $P(t) = E[(\hat{\mathbf{y}}_k(t|t) - \mathbf{y}_k(t))(\hat{\mathbf{y}}_k(t|t) - \mathbf{y}_k(t))^H]$ is the covariance matrix of the estimated state vector at time instant t.

- $K(t)$ is the Kalman filter gain at time instant t.

In order to initialize the Kalman filter, we use the two points technique:

$$\begin{cases} P_{ini} = \begin{bmatrix} 1 & \frac{1}{T_s} \\ \frac{1}{T_s} & \frac{1}{T_s^2} \end{bmatrix} \sigma_v^2 \\ K_{ini} = P_{ini} H [H P_{ini} H^T + \sigma_v^2]^{-1} \\ \hat{\mathbf{y}}_{kini} = \hat{\mathbf{y}}_k(N_0|N_0) = \begin{bmatrix} \hat{\theta}_k(N_0) \\ \frac{\hat{\theta}_k(N_0+1)-\hat{\theta}_k(N_0)}{T_s} \end{bmatrix} \end{cases}$$

where $\hat{\theta}_k(N_0+1)$ and $\hat{\theta}_k(N_0)$ are the DoA estimates of the k^{th} source provided by the proposed adaptive Capon algorithm at time instants $t = N_0 + 1$ and $t = N_0$, respectively.

4.7 Simulation results

4.7.1 Mono-source case

Consider $L = 1$ source impinging on an array of $M = 4$ sensors in the presence of an additive noise with a signal-to-noise ratio $SNR = 20dB$ and whose DoA's variation is quadratic.

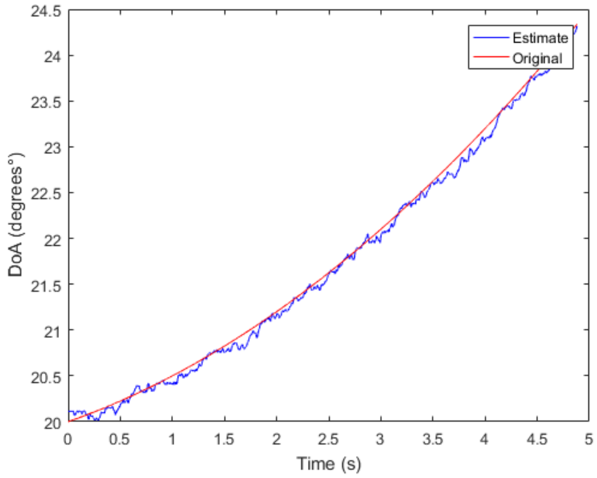

Figure 4.2: Adaptive Capon (1 source and 4 sensors with SNR = 20dB)

Interpretation of result: Fig.4.2 displays the time variation of the estimated DoA (blue solid line) versus the original DoA (red solid line). As shown, the estimate has variations due to the learning rate.

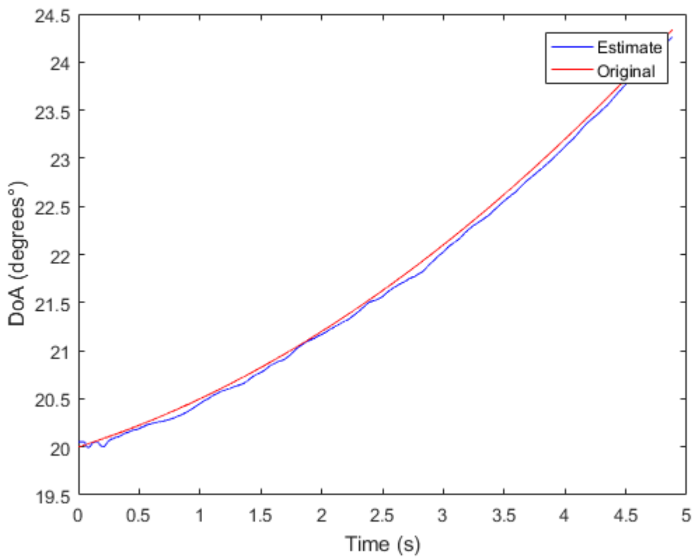

Figure 4.3: Smoothed Adaptive Capon (1 source and 4 sensors with SNR = 20dB)

Interpretation of result: Fig.4.3 contains on the same plot the original DoA (red solid line) and the DoA estimate provided by the adaptive Capon algorithm which has been smoothed using a Kalman filter [17] (blue solid line). Indeed, the variations due to the learning rate were reduced.

4.7.2 Multi-source case

Non-crossing case

The proposed adaptive Capon algorithm is also applicable in the multi-source scenario. For example, let us have $L = 2$ sources, $M = 4$ sensors and $SNR = 20dB$ and where the sources don't cross each other.

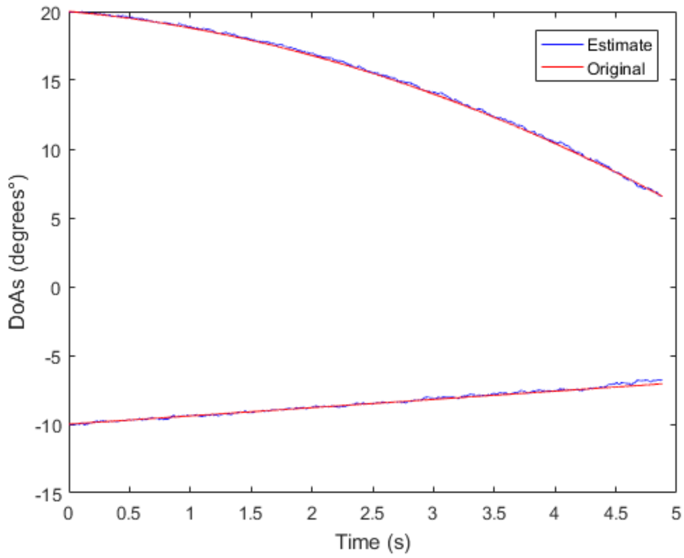

Figure 4.4: Adaptive Capon (2 non-crossing sources and 4 sensors with SNR = 20dB)

Interpretation of results: In Fig.4.4, the blue solid lines represent the DoAs' estimates provided by the adaptive Capon algorithm and the red solid lines represent the original DoAs. As illustrated, the proposed algorithm provides good estimates.
Let us also assess the performance of the Reduced Rank method with the following experimental setup:

- Number of sources ($L = 2$) which don't cross.
- Number of sensors ($M = 10$).
- Additive noise with a signal-to-noise ration $SNR = 20dB$.

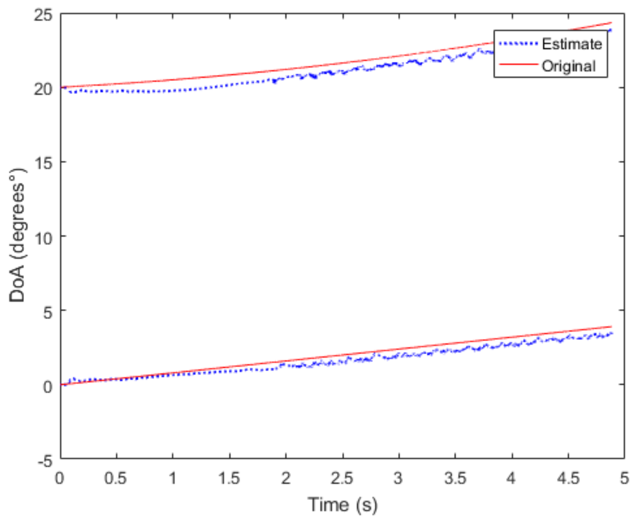

Figure 4.5: Reduced Rank method (2 non-crossing sources and 10 sensors with SNR = 20dB)

Interpretation of results: In Fig.4.5, the blue dotted lines represent the DoAs' estimates provided by the Reduced Rank approach and the red solid lines represent the original DoAs. As shown, the Reduced Rank method provides estimates that are close to the original DoAs.

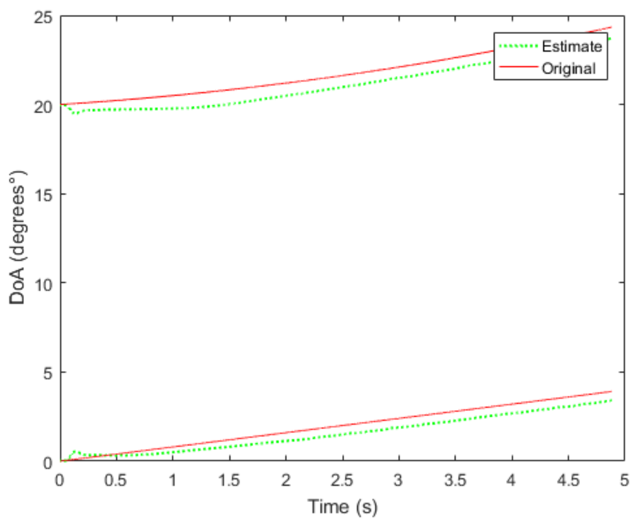

Figure 4.6: Smoothed Reduced Rank method (2 non-crossing sources and 10 sensors with SNR = 20dB)

Interpretation of results: Fig.4.6, contains on the same plot the original DoAs (red solid lines) and the DoAs' estimates provided by the Reduced Rank method which have been smoothed using a Kalman filter (see Section 4.6) (green dotted lines). Indeed, the variations due to the learning rate were reduced, yet the green curves still don't fit the red curves, we do observe a bias.

Crossing case

Herein, we kept the same configuration ($L = 2$ sources, $M = 4$ sensors and $SNR = 20dB$) but where the two sources cross each other.

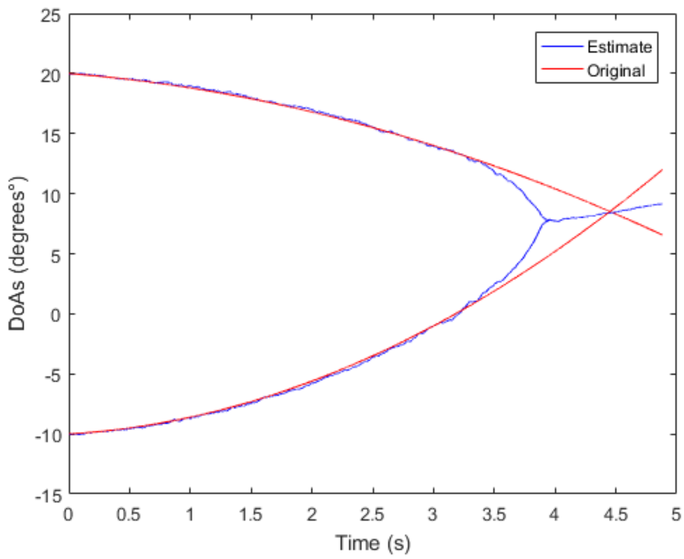

Figure 4.7: Adaptive Capon (2 crossing sources and 4 sensors with SNR = 20dB)

Interpretation of results: Fig.4.7 displays the time variation of the estimated DoAs (blue solid lines) versus the original DoAs (red solid lines). As demonstrated in the figure, the algorithm fails in providing correct estimates after a while. This is related to the fact that the difference between the two DoAs becomes less than the MVDR filter resolution making the distinction impossible.

This problem can be partially solved using a Kalman filter as shown in Fig.4.8.

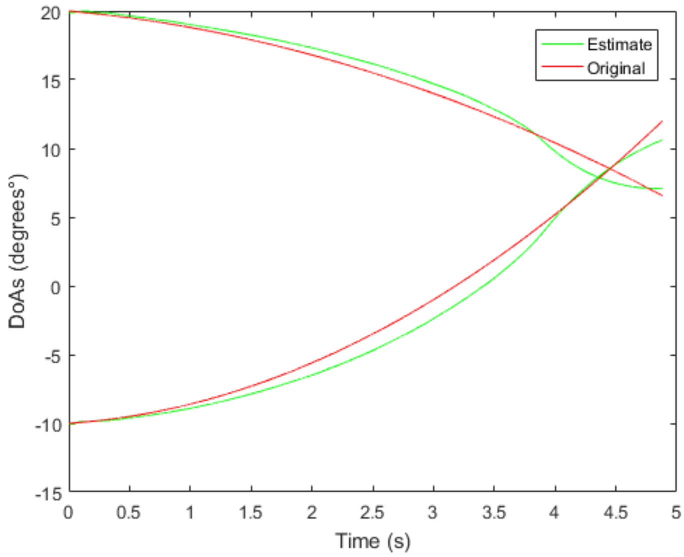

Figure 4.8: Smoothed Adaptive Capon (2 crossing sources and 4 sensors with SNR = 20dB)

Interpretation of results: In Fig.4.8, the green solid lines represent the DoAs' estimates provided by the adaptive Capon algorithm which were smoothed by the Kalman filter, and the red solid lines represent the original DoAs. As illustrated, the green curves approach the red curves and almost fit them.

In what follows, we added a third source to the previous configuration regarding the Reduced Rank method, i.e. $L = 3$ sources, $M = 10$ sensors and $SNR = 20dB$, but where two of the three sources cross each other.

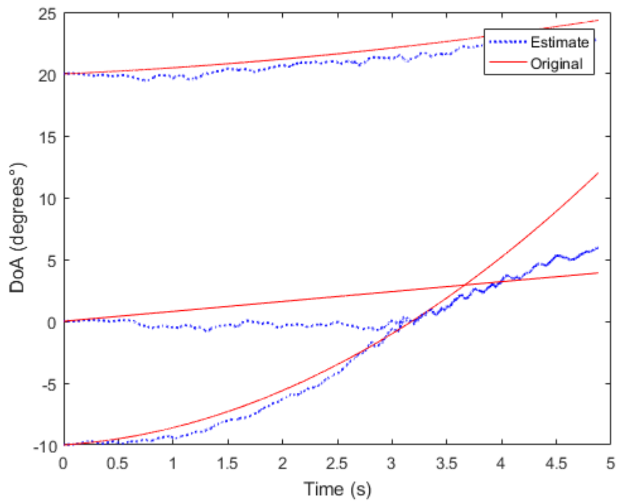

Figure 4.9: Reduced Rank method (2 crossing sources among 3 and 10 sensors with SNR = 20dB)

Interpretation of results: Fig.4.9 displays the time variation of the estimated DoAs using the Reduced Rank algorithm (blue dotted lines) versus the original ones (red solid lines). As illustrated, the algorithm tracks the first source with some variation related to the learning rate. Meanwhile, it fails in providing correct estimates for the remaining sources due to resolution problem.

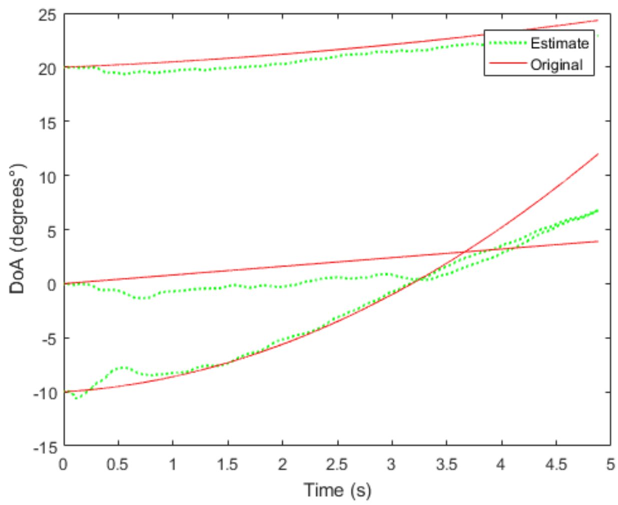

Figure 4.10: Smoothed Reduced Rank method (2 crossing sources among 3 and 10 sensors with SNR = 20dB)

Interpretation of results: Fig.4.10 contains the smoothed estimates of the DoAs (green dotted lines) and the original DoAs (red solid lines) on the same plot. In this context, we tried to reduce the fluctuations of the estimates of the DoAs and also solve the resolution problem using a Kalman filter (see Section 4.6). We succeeded regarding the first source, as its green curve approaches the red curve, but failed in doing so for the other sources.

4.8 Conclusion

In this chapter, we have presented the adaptive version of the MVDR filter and the Reduced Rank method which were both cascaded with a Kalman filter in order to track moving targets. Furthermore, the advantage of the adaptive MVDR filter is its higher resolution compared to the conventional beamformer. Regarding the Reduced Rank technique, its advantage resides in the fact that it reduces the computation complexity. Nevertheless, the shortcoming of the first approach is its numerical complexity, since it doesn't perform a dimension reduction unlike the second method. As for the Reduced Rank method, its disadvantage would be the fact that it restricts the degrees of freedom of the spatial filter which it is looking for, since it imposes a certain form on the latter (see Equation 4.14).

Chapter 5

Estimation of time varying Directions of Arrival using the adaptive version of the MUSIC algorithm

5.1 Introduction

This chapter focuses on the adaptive version of the MUltiple SIgnal Classification (MUSIC) algorithm [8] which extends the application of the latter to a non-stationary environment.

It is structured as follows: In Section 5.2, the data model is recalled again. Section 5.3 provides the reader with details about the MUSIC algorithm, followed by its extension to the non-stationary case in Section 5.4. Simulation results are presented in Section 5.5, and finally, Section 5.6 concludes the chapter.

5.2 Data Model

Consider again a uniform linear array of M sensors spaced by half a wavelength, which receives plane waves emitted by L moving narrowband sources from time-varying directions $\{\theta_1(t), \ldots, \theta_L(t)\}$.

Let $\mathbf{x}(t)=\left[x_1(t), \ldots, x_M(t)\right]^T$ be the sensor vector which can be modeled as:

$$\mathbf{x}(t) = \mathbf{A}(t)\mathbf{s}(t) + \mathbf{n}(t) \tag{5.1}$$

where

$$\mathbf{A}(t) = \left[\mathbf{a}(\theta_1(t)), \ldots, \mathbf{a}(\theta_L(t))\right] \tag{5.2}$$

is the $M \times L$ steering matrix which consists of the vectors

$$\mathbf{a}(\theta_i(t)) = \left[1, e^{-j\pi \sin\theta_i(t)}, \ldots, e^{-j(M-1)\pi \sin\theta_i(t)}\right]^T \tag{5.3}$$

and

$$\mathbf{s}(t) = [s_1(t), \cdots, s_L(t)]^T \tag{5.4}$$

is an L-dimensional vector containing the source waveforms. $\mathbf{n}(t)$ is the sensor noise vector, which is assumed to be Gaussian with zero-mean and variance σ^2.

5.3 Insight into the MUSIC algorithm

The MUltiple SIgnal Classification algorithm [8] was first introduced in the stationary case. It is a subspace-based method which exploits the eigenstructure of the covariance matrix:

$$R_x = E[\mathbf{x}(t)\mathbf{x}(t)^H] \tag{5.5}$$

$$R_x = UDU^H = \begin{bmatrix} U_s & U_n \end{bmatrix} \begin{bmatrix} \Lambda_s & 0 \\ 0 & \sigma^2 I \end{bmatrix} \begin{bmatrix} U_s^H \\ U_n^H \end{bmatrix} \tag{5.6}$$

where:

- U_s is an $M \times L$ matrix which contains L eigenvectors corresponding to the L greatest (major) eigenvalues.
- U_n is an $M \times (M-L)$ matrix which contains $M-L$ eigenvectors corresponding to the minor eigenvalues.
- Λ_s is an $L \times L$ diagonal matrix whose diagonal elements are the " *signal* " eigenvalues.

- $\sigma^2 I$ is an $(M-L)\times(M-L)$ diagonal matrix whose diagonal elements are the " *noise* " eigenvalues.

The MUSIC algorithm estimates the DoAs by searching through the set of all possible steering vectors and find those that are orthogonal to the noise subspace (the subspace spanned by the minor eigenvectors), i.e.:

$$U_n^H \mathbf{a}(\theta) = 0 \qquad \text{for} \quad \theta = \theta_1, \ldots, \theta_L \tag{5.7}$$

In practice, $\mathbf{a}(\theta)$ will not be exactly orthogonal to the noise subspace due to errors in estimating U_n. Nevertheless, the MUSIC spectrum

$$P_{MUSIC}(\theta) = \frac{1}{\mathbf{a}(\theta)^H U_n U_n^H \mathbf{a}(\theta)} \tag{5.8}$$

presents a peak when θ is equal to the DoA of one of the incoming signals.

5.4 Adaptive MUSIC

This section introduces the adaptive version of the MUSIC algorithm. The mono-source case is considered first, then the multi-sources one is discussed.

5.4.1 Mono-source case

Consider the case where $L = 1$ source radiates plane waves towards a uniform linear array of M sensors spaced by half a wavelength from a time-varying direction.

Assuming the slow variation of the DoA, the gradient ascent optimization method has been used in this adaptive version.

We first begin by initializing our algorithm using a batch of length

N_0 where we assume stationarity and ergodicity in order to estimate the initial covariance matrix as:

$$\hat{R}_0 = \frac{1}{N_0} \sum_{n=1}^{N_0} \mathbf{x}(n)\mathbf{x}(n)^H$$

We then estimated the initial value of the DoA as:

$$\hat{\theta}_0 = \underset{\theta}{argmax}\, J(\theta) = \underset{\theta}{argmax} \quad \mathbf{a}(\theta)^H U_s U_s^H \mathbf{a}(\theta) \tag{5.9}$$

Proof: In the MUSIC algorithm, the DoAs are estimated by:

$$\underset{\theta}{min} \quad \mathbf{a}(\theta)^H U_n U_n^H \mathbf{a}(\theta)$$
$$= \underset{\theta}{min} \quad \mathbf{a}(\theta)^H (I - \Pi_s)\mathbf{a}(\theta)$$

where Π_s is the projector of the signal subspace defined as:

$$\Pi_s = U_s U_s^H \tag{5.10}$$

After a workout of the previous equation, we obtain:

$$\underset{\theta}{min} \quad ||\mathbf{a}(\theta)||^2 - \mathbf{a}(\theta)^H U_s U_s^H \mathbf{a}(\theta)$$

which is equivalent to:

$$\underset{\theta}{max} \quad \mathbf{a}(\theta)^H U_s U_s^H \mathbf{a}(\theta)$$

Secondly, we tracked the signal subspace using the OPAST algorithm [18] (see Appendix A.2 for more details).

Finally, we fed $\hat{\theta}_0$ to our adaptive gradient ascent algorithm:

$$\hat{\theta}_n = \hat{\theta}_{n-1} + \mu \cdot \nabla J(\hat{\theta}_{n-1})$$

which will search for the values of θ that maximize the objective function $J(\theta)$ and where:

- μ is the learning rate.
- $\nabla J(\theta) = \frac{d\mathbf{a}(\theta)^H}{d\theta} U_s U_s^H \mathbf{a}(\theta) + \mathbf{a}(\theta)^H U_s U_s^H \frac{d\mathbf{a}(\theta)}{d\theta}$ is the gradient of $J(\theta)$.

5.4.2 Multi-source case

We basically applied the aforementioned procedure to each source separately, i.e. we first estimated the initial values of the DoAs using the MUSIC algorithm. Then, we fed them individually to our adaptive gradient ascent algorithm as shown in the figure below:

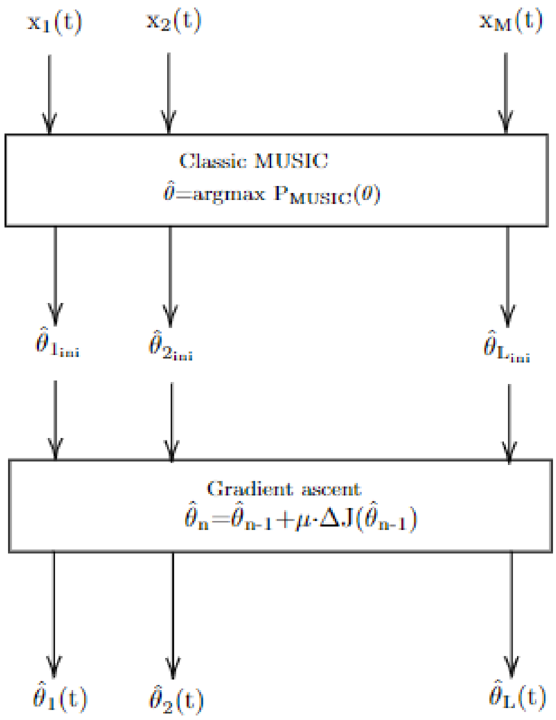

Figure 5.1: Bloc diagram of adaptive MUSIC

5.5 Simulation results

5.5.1 Mono-source case

Consider $L = 1$ source impinging on an array of $M = 4$ sensors in the presence of an additive noise with a signal-to-noise ratio $SNR = 20dB$ and whose DoA's variation is quadratic.

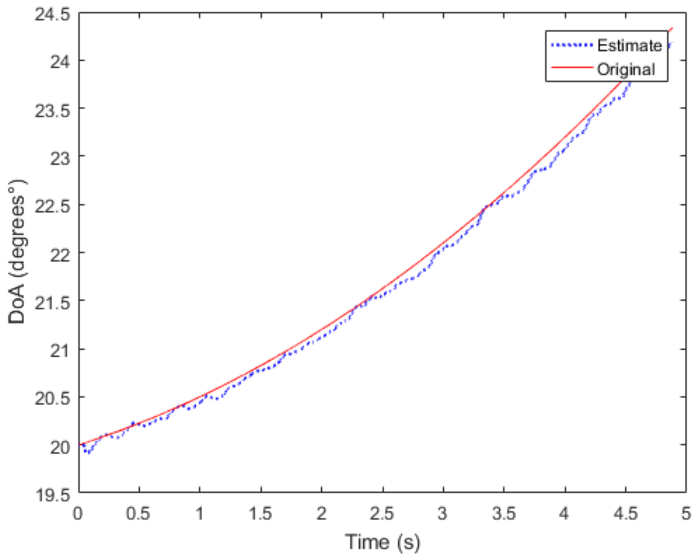

Figure 5.2: Adaptive MUSIC (1 source and 4 sensors with SNR = 20dB)

Interpretation of result: Fig.5.2 displays the time variation of the estimated DoA (blue dotted line) versus the original DoA (red solid line). Indeed, the blue curve approaches the red one, meaning that the quadratic DoA was well estimated.

5.5.2 Multi-source case

Non-crossing case

Adaptive MUSIC can also be applied in the case where we have multiple sources. For instance, let us have $L = 2$ sources, $M = 10$ sensors and $SNR = 10dB$ and where the sources don't cross each other.

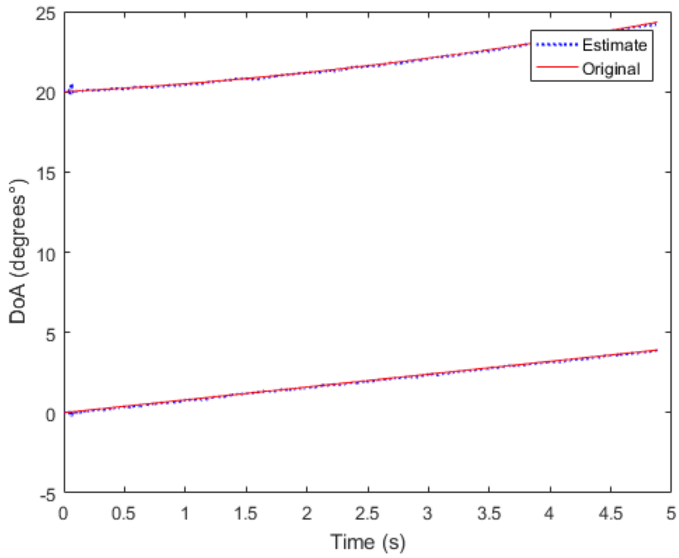

Figure 5.3: Adaptive MUSIC (2 non-crossing sources and 10 sensors with SNR = 10dB)

Interpretation of results: Fig.5.3 contains the plots of the estimated DoAs (blue dotted lines) along with the original DoAs (red solid lines) as a reference. The algorithm provides good estimates, as illustrated.

Crossing case

Herein, we consider the case of $M = 10$ sensors, $L = 3$ sources with an $SNR = 10dB$ and where two of the sources cross each other.

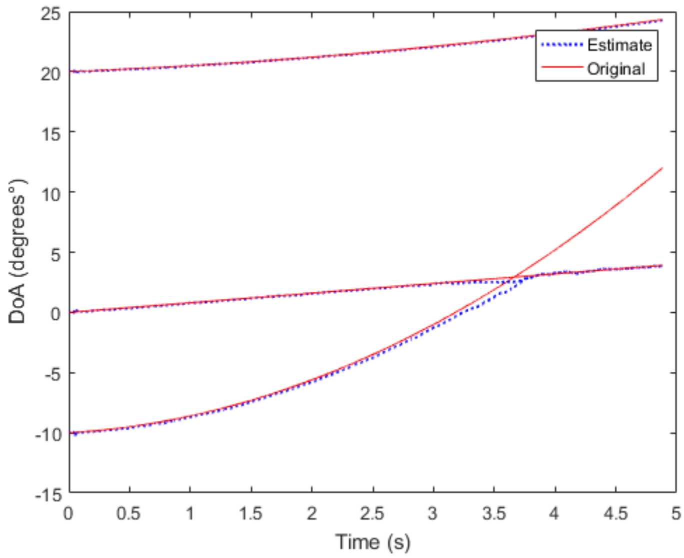

Figure 5.4: Adaptive MUSIC (2 crossing sources among 3 and 10 sensors with SNR = 20dB)

Interpretation of results: Fig.5.4 displays the time variation of the estimated DoAs (blue dotted lines) versus the original ones (red solid lines). As shown, the algorithm keeps a good track of the first source throughout all the duration of observation. Nevertheless, it fails in providing correct estimates for the second and third sources after the crossing happens. This is due to the fact that at the crossing point, the steering vectors of the 2^{nd} and 3^{rd} sources, which span the signal subspace, have angles that are below MUSIC's resolution, making this subspace deficient. On the other hand, the first source does not suffer from the aforementioned problem (since it doesn't cross with any other sources), that is why the tracking in its direction keeps going.

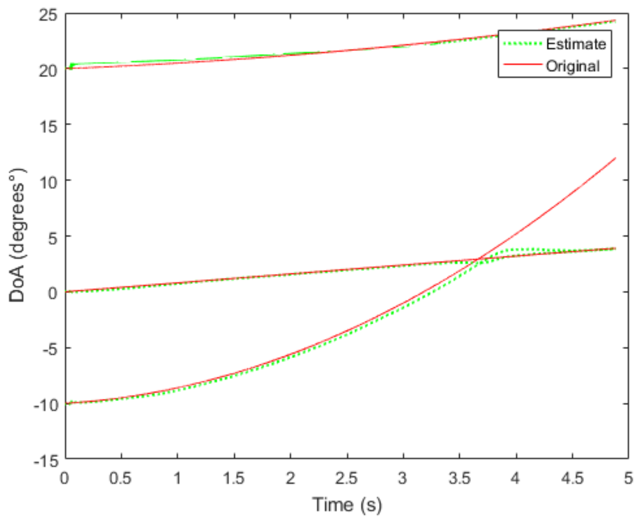

Figure 5.5: Smoothed Adaptive MUSIC (2 crossing sources among 3 and 10 sensors with SNR = 20dB)

Interpretation of results: Fig.5.5 contains the smoothed estimates of the DoAs (green dotted lines) and the original DoAs (red solid lines) on the same plot. In this context, we tried to solve the above problem using a Kalman filter (see Section 4.5), but it was unsuccessful. This is due to the fact that the filter acts only on the estimates of the DoAs and does not bring a correction to the estimated signal subspace.

5.6 Conclusion

In this chapter, we have presented the adaptive version of the MUSIC algorithm which was cascaded with a Kalman filter in order to track moving targets. Moreover, the advantage of this method is its high resolution. Nonetheless, its downfall is none other than its incapability to resolve the underdetermined case, since both the signal and noise subspaces cannot be generated if the number of sensors is less than the number of sources, which is not the case of the Instantaneous Frequency based DoA estimation technique of Chapter 3.

Chapter 6

Conclusion

In this project, we addressed the problem of estimating time-varying directions of arrival. For this purpose, we introduced four approaches.

The first one uses the instantaneous frequency estimates of the array outputs. Its advantage is its ability to solve the under-determined case, besides not suffering from the crossing sources problem. Its first version is a batch technique. It could be made adaptive by using online instantaneous frequency estimation algorithm to cope with the tracking issue. This first contribution gave rise to the publication of a conference article at the 16^{th} edition of the International Symposium on Signals, Circuits and Systems [19].

The material in Chapter 4 proposed two additional tracking methods, the adaptive version of the MVDR filter on one hand and the Reduced Rank technique for computational complexity reduction on the other hand. This chapter also gave details on how the Kalman filter can be implemented in order to smooth the DoAs estimates.

Finally, Chapter 5 focused on extending the MUSIC algorithm to the non-stationary case. Nevertheless, the latter suffers from the crossing sources problem, even after applying the Kalman filter.

6.1 Interesting directions for future research

- Extending the instantaneous frequency based estimation of time-varying DoAs algorithm to a tracking one using an efficient online instantaneous frequency estimator.

- Developing an algorithm that corrects the adaptively estimated signal subspace.

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Appendix A

A.1 Fractional Fourier Transform

A.1.1 Motivation

Several joint time-frequency distributions have been developed throughout the years in order to extract the characteristics of non-stationary signals, i.e. signals whose frequency content varies with time.

Some of the representations, such as the Wigner-Ville distribution, suffers from the cross-terms which may hide some of the auto-terms. In general, the auto-terms might be oriented in a direction on some angle in the time-frequency plane, in which case the axes of maximum signal width do not correspond to time or frequency. These rotated axes, corresponding to the maximum signal width, are referred to as the principal axes. Rotation of time-frequency representations has shown a better reduction of cross-terms without too severely degrading the auto-terms than the corresponding, original time-frequency representations. [20]

A.1.2 Definition and interpretation

The fractional Fourier transform (**FRFT**), defined by equation (3.13), is a generalization of the Fourier transform with an adjustable parameter α which is interpreted as a rotation by an angle α in the time-frequency plane.

In time-frequency representations, one normally uses a plane with two orthogonal axes corresponding to time and frequency respectively. If we consider a signal $x(t)$ represented along the time axis and its Fourier transform $X(f)$ represented along the frequency axis, we can view the Fourier transform as a change in the representation of the signal corresponding to a counterclockwise axis rotation of $\frac{\pi}{2}$ rad. In this context, the FRFT has been introduced to allow a rotation by an angle α that is not necessarily a multiple of $\frac{\pi}{2}$, or in other terms a representation of the signal along an axis u making an angle α with the time axis. [21]

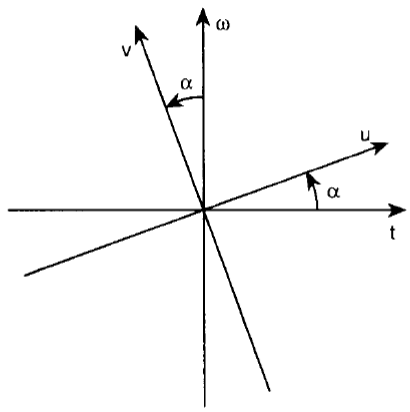

Figure A.1: Time-frequency plane and a set of coordinates (u, v) rotated by an angle α relative to the original coordinates (t, w)

A.2 OPAST

A.2.1 Motivation

Subspace estimation occupies an important place in various signal processing applications, such as system identification, data compression and in our case direction finding.

The Orthonormal Projection Approximation and Subspace Tracking (OPAST) is an algorithm that estimates and tracks the signal subspace.

A.2.2 Definition and interpretation

The OPAST algorithm provides a solution to the following constrained optimization problem :

$$\begin{cases} min_W \; J(W) = min_W \; E[||\mathbf{x}(t) - W(t)W(t)^H\mathbf{x}(t)||^2] \\ W(t)^H W(t) = I \end{cases}$$

where:

- $\mathbf{x}(t)$ is the sensor vector (see Section 3.2)
- $W(t)$ is an $M \times L$ matrix, where L is the number of sources.

Its iterative solution, whose columns span the signal subspace, is given by:

$$W(i) = W(i-1) + \mathbf{p}'(i)\mathbf{q}(i)^H$$

The OPAST algorithm is summarized in the table below:

Algorithm 1 OPAST

$W(i) = W(i-1) + \mathbf{p}'(i)\mathbf{q}(i)^H$
$\mathbf{q}(i) = \frac{1}{\beta}Z(i-1)\mathbf{y}(i)$
$\mathbf{y}(i) = W(i-1)^H\mathbf{x}(i)$
$\gamma(i) = \frac{1}{1+\mathbf{y}(i)^H\mathbf{q}(i)}$
$Z(i) = \frac{1}{\beta}Z(i-1) - \gamma(i)\mathbf{q}(i)\mathbf{q}(i)^H$
$\mathbf{p}(i) = \gamma(i)(\mathbf{x}(i) - W(i-1)\mathbf{y}(i))$
$\tau(i) = \frac{1}{||\mathbf{q}(i)||^2}(\frac{1}{\sqrt{1+||\mathbf{p}(i)||^2||\mathbf{q}(i)||^2}} - 1)$
$\mathbf{p}'(i) = \tau(i)W(i-1)\mathbf{q}(i) + (1+\tau(i)||\mathbf{q}(i)||^2)\mathbf{p}(i)$

- β is the forgetting factor
- $Z(i) \stackrel{\text{def}}{=} (W(i-1)^H R_x(i) W(i-1))^{-1}$

Publication

C. L. Abbassen, A. Belouchrani, and K. Abed-Meraim, “In- stantaneous frequency based estimation of time varying di- rections of arrival,” in International Symposium on Signals, Circuits and Systems, ISSCS 2023, Iasi, Romania, July 13- 14, 2023

Instantaneous Frequency based estimation of time varying Directions of Arrival

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***Abstract*— This paper addresses the problem of estimating time-varying directions of arrival. It demonstrates how the concept of instantaneous frequency can be employed for this purpose. The proposed approach can localize more sources than the number of available sensors. It also theoretically allows the estimation of any angular time variation. Herein, we consider linear and hyperbolic time variations, which, in practice, take into account velocity and acceleration, respectively. Numerical experiments are conducted to validate the effectiveness of the proposed method.**

***Index Terms*— Directions of arrival, linear time variation, hyperbolic time variation, instantaneous frequency, angular instantaneous frequency.**

I. Introduction

Directions of arrival (DoA) estimation in array signal processing is an active research area with applications in wireless communications, radar, sonar, speech, navigation, seismology and other fields.

When directions of arrival are time-varying, either due to moving sources, or moving sensors, or even both, high resolution approaches such as the MUltiple SIgnal Classification (MUSIC) method [1], the Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) algorithm [2] or either the subspace approaches in the time frequency domain [3], [4] fail in providing correct estimates as the latter assume constant DoAs. If the time variation is rather slow, algorithms such as Kalman filter can be used [5] to track the DoAs.

In this paper, we propose to solve the problem of estimating the time varying DoAs by resorting to the instantaneous frequencies estimates of the array outputs. In contrast to the above aforementioned techniques, the proposed approach allows solving the underdetermined case where the number of sources is greater than the number of sensors. In addition, the proposed techniques can estimate any angular time variation, even hypothetical one as for example a cubic time variation. Our numerical experiments consider also linear and hyperbolic time variations, which, in practice, take into account velocity and acceleration, respectively. The conducted simulations show that the performance of the proposed approach rely on the efficiency of the used instantaneous frequency estimation technique.

The paper is organized as follows: Section II provides the signal model under consideration, the proposed solution is presented in Section III, simulation results are presented in Section IV, and finally, Section V concludes the paper.

II. Signal Model

Consider a uniform linear array of M sensors spaced by half a wavelength, which receives plane waves emitted by L moving narrowband sources from time-varying directions $\{\theta_1(t), \ldots, \theta_L(t)\}$.

Let $\mathbf{x}(t)=\left[x_1(t), \ldots, x_M(t)\right]^T$ be the sensor vector which can be modelled as:

$$\mathbf{x}(t) = \mathbf{A}(t)\mathbf{s}(t) + \mathbf{n}(t) \quad (1)$$

where

$$\mathbf{A}(t) = \left[\mathbf{a}(\theta_1(t)), \ldots, \mathbf{a}(\theta_L(t))\right] \quad (2)$$

is the $M \times L$ steering matrix which consists of the vectors

$$\mathbf{a}(\theta_i(t)) = \left[1, e^{-j\pi \sin\theta_i(t)}, \ldots, e^{-j(M-1)\pi \sin\theta_i(t)}\right]^T \quad (3)$$

and

$$\mathbf{s}(t) = [s_1(t), \cdots, s_L(t)]^T \quad (4)$$

is an L-dimensional vector containing the source waveforms. $\mathbf{n}(t)$ is the sensor noise vector, which is assumed to be Gaussian with zero-mean and variance σ^2.

In the sequel, we propose to estimate the time variation of the Directions of Arrival $\theta_i(t), i = 1, \cdots, L$ through the estimates of the instantaneous frequencies [6] of the sensor outputs $x_i(t), i = \cdots, M$.

III. Proposed Solution

Let us first consider the case of $L = 1$ source in a noise-free environment, it follows from equation (1), that:

$$\mathbf{x}(t) = \mathbf{a}(\theta_1(t))s_1(t) \tag{5}$$

And by considering the source signal $s_1(t)$ as analytic, i.e. $s_1(t) = b_1(t)e^{j\phi_{s_1}(t)}$, together with equation (3), we obtain the following observation vector:

$$\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_M(t) \end{bmatrix} = b_1(t) \begin{bmatrix} e^{j\phi_{s_1}(t)} \\ e^{j(-\phi_1(t)+\phi_{s_1}(t))} \\ \vdots \\ e^{j(-(M-1)\phi_1(t)+\phi_{s_1}(t))} \end{bmatrix} \tag{6}$$

Where

$$\phi_1(t) = \pi \sin \theta_1(t) \tag{7}$$

denotes the *electrical angle* of the source under consideration. It appears clearly from equation (6) that the instantaneous phase of the i^{th} sensor signal is given by

$$\phi_{x_i}(t) = -(i-1)\phi_1(t) + \phi_{s_1}(t)$$

And its instantaneous frequency is given by :

$$f_{x_i}(t) = -(i-1)\frac{1}{2\pi}\frac{d\phi_1(t)}{dt} + f_{s_1}(t) \tag{8}$$

Accordingly, one obtains the following vector whose entries are the instantaneous frequencies of the M sensor signals:

$$\begin{bmatrix} f_{x_1}(t) \\ f_{x_2}(t) \\ \vdots \\ f_{x_M}(t) \end{bmatrix} = \begin{bmatrix} f_{s_1}(t) \\ -\frac{1}{2\pi}\frac{d\phi_1(t)}{dt} + f_{s_1}(t) \\ \vdots \\ -(M-1)\frac{1}{2\pi}\frac{d\phi_1(t)}{dt} + f_{s_1}(t) \end{bmatrix} \tag{9}$$

By subtracting the instantaneous frequency of the first sensor signal from the M-1 other sensor signals, one gets the following new vector:

$$\begin{bmatrix} f_{x_2}(t) - f_{x_1}(t) \\ f_{x_3}(t) - f_{x_1}(t) \\ \vdots \\ f_{x_M}(t) - f_{x_1}(t) \end{bmatrix} = \begin{bmatrix} -\frac{1}{2\pi}\frac{d\phi_1(t)}{dt} \\ -2\frac{1}{2\pi}\frac{d\phi_1(t)}{dt} \\ \vdots \\ -(M-1)\frac{1}{2\pi}\frac{d\phi_1(t)}{dt} \end{bmatrix} \tag{10}$$

$$= -\frac{1}{2\pi}\frac{d\phi_1(t)}{dt}\begin{bmatrix} 1 \\ 2 \\ \vdots \\ M-1 \end{bmatrix} \tag{11}$$

In order to extract the first derivative of the electrical angle $\phi_1(t)$, we multiply both sides of (11) by $\begin{bmatrix} 1 & 2 & \dots & M-1 \end{bmatrix}$, which leads to:

$$\begin{bmatrix} 1 & 2 & \dots & M-1 \end{bmatrix} \begin{bmatrix} f_{x_2}(t) - f_{x_1}(t) \\ f_{x_3}(t) - f_{x_1}(t) \\ \vdots \\ f_{x_M}(t) - f_{x_1}(t) \end{bmatrix} = -\frac{1}{2\pi}\frac{d\phi_1(t)}{dt}\sum_{i=1}^{M-1} i^2$$

Finally, the *"angular instantaneous frequency"* is then estimated as:

$$\begin{aligned} \frac{1}{2\pi}\frac{d\phi_1(t)}{dt} &= -\frac{\sum_{i=1}^{M-1} i(f_{x_{i+1}}(t) - f_{x_1}(t))}{\sum_{i=1}^{M-1} i^2} \\ &= -\frac{6\sum_{i=1}^{M-1} i(f_{x_{i+1}}(t) - f_{x_1}(t))}{M(M-1)(2M-1)} \end{aligned} \tag{12}$$

By applying to the estimated angular instantaneous frequency of equation (12) an integrator filter whose z transfer function is given by:

$$H(z) = \frac{T_s}{2}\frac{1+z^{-1}}{1-z^{-1}}$$

where T_s is the sampling period, one obtains an estimate of the electrical angle $\hat{\phi}_1(t)$ up to 2π constant. Since the electrical angle and the direction of arrival of the source are related by (7), one can also estimate $\hat{\theta}_1(t)$.

In the case of several sources, one has to estimate first the instantaneous frequency of each source at each sensor and then apply the above procedure for each source separately. In this paper, we use the Fast IF algorithm of reference [7] as estimation technique of the instantaneous frequencies of the multi-component signal.

In the simulation section, it is shown that the proposed procedure solves the DoA estimation problem in the case of more sources than the number of available sensors and allows estimating any angular time variation, even hypothetical one.

IV. Simulations and Results

The performance of the proposed algorithm are assessed using synthetic signals. For this purpose, we first consider $L = 1$ source impinging, in the absence of noise, on an array of $M = 4$ sensors and whose direction of arrival varies linearly, i.e. $\theta_1(t) = \alpha t$, where α was set to equal 0.8. The time variation of the estimated DoA versus the original one is shown in Fig.1.

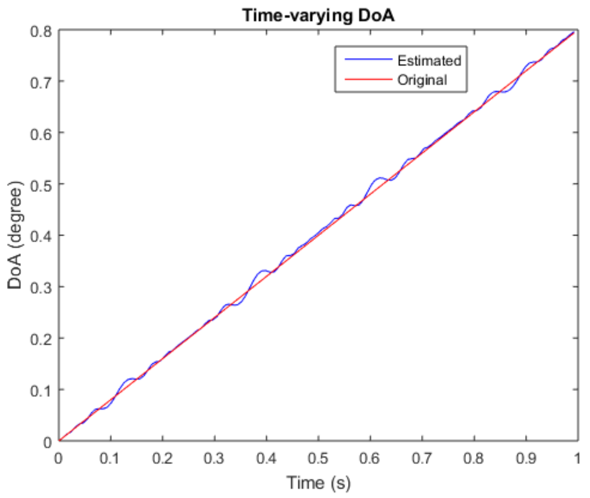

Fig. 1. Estimated DoA vs Original DoA (1 source and 4 sensores)

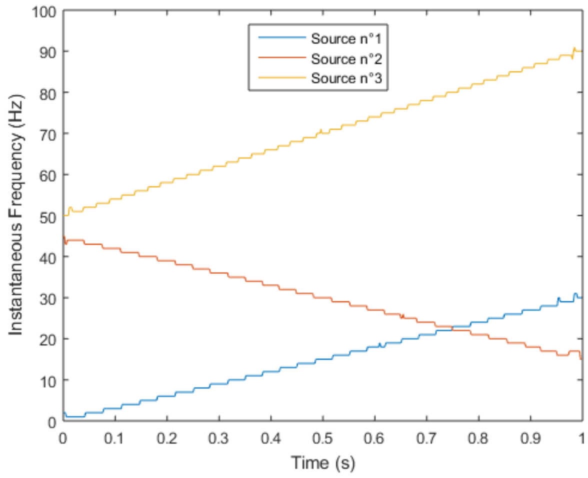

Fig. 3. IF estimates of the 1^{st} sensor signal

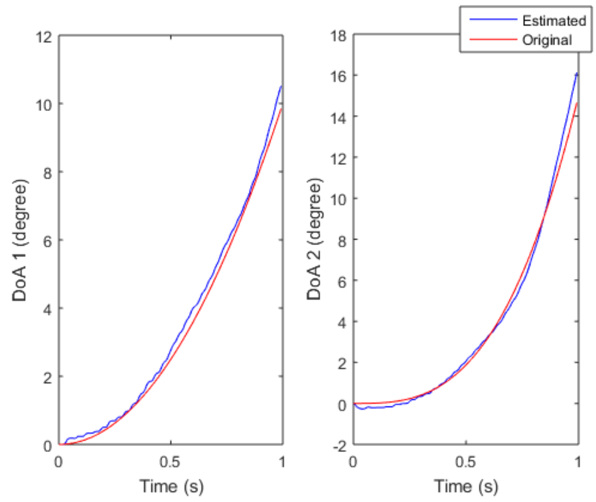

Fig. 2. 2 sources and 10 sensors with $SNR = 20dB$

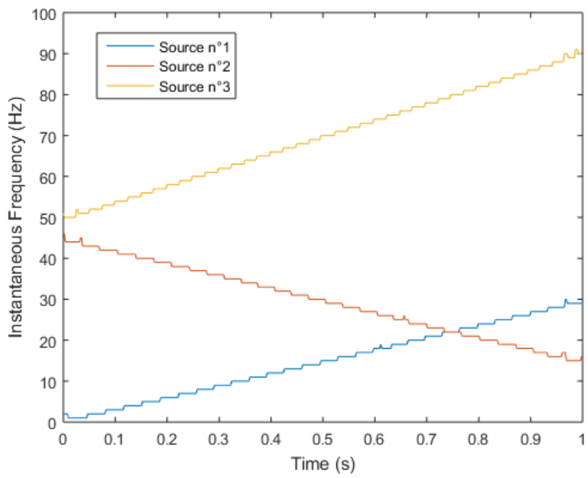

Fig. 4. IF estimates of the 2^{nd} sensor signal

Fig.2 illustrates the extension to the multi source scenario with the following experimental setup:

- Number of sensors (M=10)
- Quadratic and cubic variations of the two sources, $\theta_1(t) = 10t^2$ and $\theta_2(t) = 15t^3$, respectively.
- Additive noise with a signal-to-noise ratio $SNR = 20dB$.

The obtained results show that the proposed approach allows estimating quadratic angular time variation that is related to angular acceleration in practice and also a hypothetical cubic angular time variation.

More sources than the number of available sensors:

Herein, we consider the case of 2 sensors and 3 sources with an $SNR = 20dB$. Figures 3 and 4 display the estimates of the instantaneous frequencies of the 1^{st} sensor signal and the 2^{nd} sensor signal, respectively, while using the Fast IF algorithm [7].

Fig. 5 shows the ability of our suggested method to address the underdetermined case where 3 time varying directions of arrival have been estimated using only 2 sensors.

V. Conclusion

A new approach for estimating time-varying DoAs was proposed. The latter uses the estimates of the array outputs' instantaneous frequencies, followed by an integrator filter. Indeed, simulation results show that the aforementioned combination allows estimating any angular time variation, whether it is linear or hyperbolic and even hypothetical variation such as a cubic one. The proposed method allows solving the underdetermined

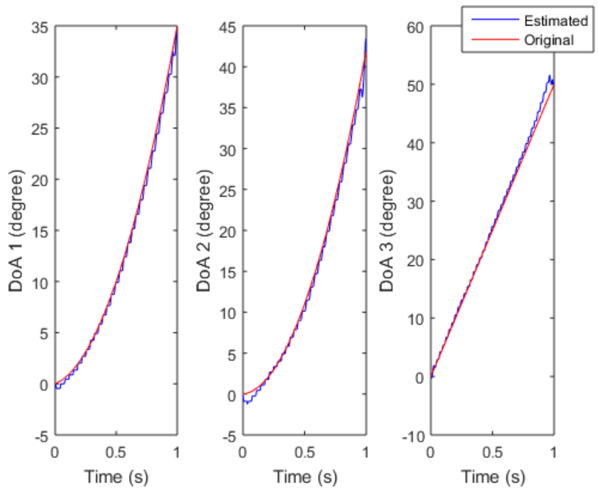

Fig. 5. 3 sources and 2 sensors with $SNR = 20dB$

case, where the number of sources is greater than the number of sensors. The proposed DoA estimation highly depends on the efficiency of the chosen instantaneous frequency estimation technique. In this paper, we have used the Fast IF algorithm [7] for such task. This first version of the proposed DoA estimation approach is a batch technique. An extension of the proposed approach to a tracking one depends only on the use of an efficient online instantaneous frequency estimation algorithm.

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