

Numerical investigation of pulsed flow in aortic aneurism with and without stent by lattice Boltzmann method

Farouk Mezali, Saâdia Benmamar, Mohamed Saber Hamouda, and Mohamed Aghiles Zakaria Mezali

Abstract– Endovascular treatment of aortic aneurysms with an endoprosthesis is proposed for the prevention of the aneurysms' rupture risk and clogging of blood vessels. This paper aims to study the impact of stents with different porosities, placed in fusiform aortic aneurysms, on the hemodynamic parameters: velocity and shear rate of the flow inside the aneurysms, using pulsed boundary conditions derived from real measurements. The numerical investigations are performed with the Lattice Boltzmann method. The blood is treated as a Newtonian fluid and then as a non-Newtonian fluid with the Carreau-Yasuda and Cross models. The reduction obtained by a single stent is modest and unfavorable to thrombotic occlusion. Overlapping of several stents is proposed to promote occlusion. The results found show good agreement with medical theory.

Keywords– Lattice Boltzmann, Fusiform aortic aneurysm, stent, pulsed flow, Computational Fluid Dynamics.

I. INTRODUCTION

Cardiovascular diseases represent the leading cause of mortality worldwide. They are responsible for 36 million deaths annually and almost 70% of total deaths worldwide [1]. Among these diseases are aneurysms.

An aneurysm is a vascular disorder related to the wall weakening of a vessel, resulting in a small sac of various shapes (fusiform, saccular). Under some conditions, the wall of this sac or aneurysm may rupture and become harmful, even fatal [2, 3]. There are two techniques for treating it: surgical, which consists of carrying out an operation on the patient to directly approach the aneurysmal zone and replace it with a synthetic tube called a prosthesis, and other surgical forms, such as clipping, coating, etc[4, 5].

The second technique is endovascular treatment, which involves inserting a stent (an endoprosthesis) inside the aneurysmal aorta through the femoral arteries. The endovascular stent graft technique is mainly used for the treatment of abdominal aortic aneurysms (AAAs). Stent graft techniques have the advantage of being minimally invasive and present a lower risk of mortality and complications than surgery in the short term. On the other hand, it has the disadvantage of producing poorer results over the long term and necessitating lifetime monitoring [3].

Flow diverter stents are relatively new techniques in the treatment of aneurysms, particularly intracranial aneurysms. However, this technique is sometimes used in the treatment of aortic aneurysms. Regarding the aneurysm's volume reduction and thrombosis, as well as the patency of the lateral branches and the prevention of rupture, it has demonstrated satisfactory results. These results must be investigated further, to provide more complete and extensive guidance [6].

This kind of aneurysm is also treated by the uncovered stent. The common basic idea behind these last two techniques is the reduction of blood flow in the aneurysmal sac to stabilize it and, therefore, reduce the risk of its rupture [7].

This work is devoted to understanding and evaluating the hemodynamics of abdominal aortic aneurysm (AAA) with and without stents through a hemodynamic analysis carried out using the Lattice Boltzmann method (LBM). To reach this goal, the real pulsating flow is used as boundary condition input. The basic idea behind the LBM is to use discrete velocity models to obtain an approximate solution for an incompressible Navier Stokes [8, 9]. Non-Newtonian behavior is taken into account in the flow modeling through the Carreau-Yasuda and Cross models. Two types of analysis are carried out: qualitative and quantitative. The qualitative analysis is made on the shape and form of the results based on the velocity and the shear rate. Quantitative analysis is performed on "velocity" and "shear rate" within the aneurysm sac before and after stent placement.

II. LATTICE-BOLTZMANN (LB) MODEL

In this paragraph, the main aspects of the used model are explained. The fluid is represented by the lattice-Bhatnagar-Gross-Krook model (LBGK). This model is defined by the propagation-collision equation as follows:

$$f_i(r + \Delta t \mathbf{v}_i, t + \Delta t) = \left(\frac{1}{\tau}\right) f_i^0(r, t) \left(1 - \left(\frac{1}{\tau}\right)\right) \quad (1)$$

Where $f_i(r, t)$ is the probability distribution function. This function gives the probability that a fluid particle of velocity $\mathbf{v}_i$ enters at the point of coordinate r of the lattice at time t . The probable allowable velocity $\mathbf{v}_i$ depends on the lattice structure. In our case, one thing a cell of the type D2Q9 (Fig.1).

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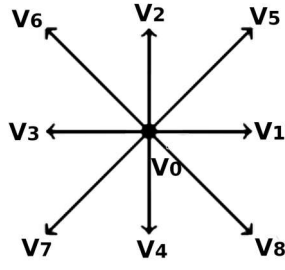


Fig. 1: Discrete velocities of the D2Q9 lattice model.

The index i varies from 0 to 8, where 8 is the number of lattice link. By convention $v_0 = 0$, and f_0 represents the distribution of particle density at rest.

The density and velocity of the fluid at the macroscopic scale are given as follows:

$$\rho = \sum_{i=0}^8 m_i f_i \quad (2)$$

$$\rho \mathbf{u} = \sum_{i=0}^8 m_i f_i \mathbf{v}_i \quad (3)$$

The time step Δt of the simulation is taken equal to 1 [9-11].

The number τ is the relaxation time and $f_i^{(0)}$ are the functions of local equilibrium. Their expressions are:

$$f_i^{(0)} = \rho \frac{C_4}{C_2} \left(1 + \frac{v_{ia} u_a}{c_s^2} + \frac{1}{2} \left(\frac{v_{ia} u_a}{c_s^2} \right)^2 - \frac{u^2}{2c_s^2} \right), \quad (4)$$

$$f_0^{(0)} = \rho \left(1 - \frac{C_0 C_4}{C_2^2} \right) \left(1 - \frac{u^2}{2c_s^2} \right)$$

Generally, the square speed of sound is chosen like $c_s^2 = v_l^2 \left(\frac{C_4}{C_2} \right)$ where $v_l = \frac{\Delta x}{\Delta t}$.

The quantities m_i are the lattice weight, their values are associated with the lattice directions and depend on its dimension: $m_{1-4} = \frac{1}{9}$ and $m_{5-8} = \frac{1}{36}$.

C_0 , C_2 and C_4 are constants and depend on the structure of the lattice. All these constants, follow from the isotropy condition [11, 12]. In the case of D2Q9, they are equal to: $C_0 = 20$, $C_2 = 12$, $C_4 = 4$.

One can show that the equation (1), which represents the dynamics of fluid particles on the microscopic scale, can reproduce the Navier-Stokes and continuity equations [9-11]:

$$\partial_t \rho + \rho \nabla \cdot \mathbf{u} = 0 \quad (5)$$

$$\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \nu \nabla^2 \mathbf{u}$$

The pressure is related to the density by $p = c_s^2 \rho$. The cinematic viscosity, expressed in lattice units, is written

$$\nu = \Delta t v_l^2 \frac{C_4}{C_2} \left(\tau - \frac{1}{2} \right)$$

In D2Q9 case, the speed of sound becomes $c_s = \frac{1}{\sqrt{3}} v_l$ and the viscosity

$$\nu = \frac{1}{3} \left(\tau - \frac{1}{2} \right) v_l^2 \Delta t. \quad (6)$$

Previous macroscopic variables (u, p, ν), given by equations of lattice Boltzmann, are in lattice unit (lu). The conversion to physical units is given by a simple transformation: $u_p = \frac{\delta x}{\delta t} u_{lb}$,

$\nu_p = \frac{\delta x^2}{\delta t} \nu_{lb}$ where δx and δt are the space step and the time step respectively.

A. Boundary conditions to the limits

1. Dirichlet condition at the inlet

A Dirichlet boundary condition is posed at the inlet aorta artery, and a subroutine has been introduced inside the elaborate computer code to make the velocity variable at the inlet.

The realization of the Dirichlet condition is carried out by Zou and He condition [13]. In fact, this condition extends the bounce-back conditions to the non-equilibrium part $f_i^{neq} = f_i - f_i^{eq}$ of the distribution function f_i . For Dirichlet boundary condition, the values that the solution (velocity or density) must verify on the solid or fluid boundaries are specified, and subsequently the missing

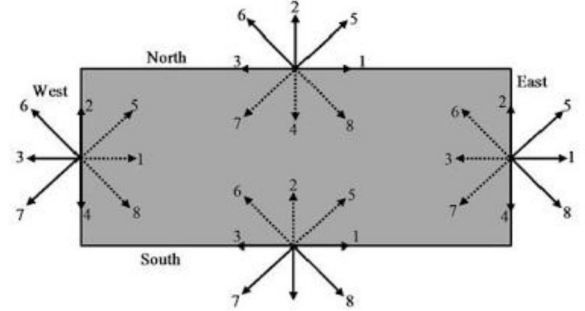


Fig. 2: The East, West, North and South boundaries for the Dirichlet condition.

Consider the case of fluid entering a channel on its western side, as shown in fig. 2. u_x and u_y are specified. Following the propagation phase, the distribution functions $f_0, f_2, f_3, f_4, f_6, f_7$ for the D2Q9 model are known. It is still unknown ρ and f_1, f_5, f_8 which are given by:

$$\rho = \frac{1}{1-u} (f_0 + f_2 + f_4 + 2(f_3 + f_6 + f_7))$$

$$f_1 = f_3 + \frac{2}{3} \rho u$$

$$f_5 = f_7 - \frac{1}{2} (f_2 - f_4) + \frac{1}{2} \rho v$$

$$f_8 = f_6 + \frac{1}{2} (f_2 - f_4) - \frac{1}{2} \rho v + \frac{1}{6} \rho u$$

It is to highlight that by setting the velocity to zero ($u = v = 0$), the conditions of Zou and He are reduced to a simple bounce-back.

2. Wall Conditions (Bounce-Back)

Bounce-back boundary conditions are most often used for straight or non-straight solid walls to achieve the no-slip condition. They are widely used in the simulation of fluid flows with complex geometries. The principle is that when a fluid particle hits a solid wall, its momentum is reversed, in other words, the particle bounces in the opposite direction of its motion (Fig. 2), the mathematical expression which expresses this occurrence is:

$$f_i^{out}(x_b, t + \Delta t) = f_i^{in}(x_b, t) \quad (7)$$

where f_i^{out} denotes the outgoing distribution functions of the solid wall. While f_i^{in} expresses the incoming distribution functions.

The bounce-back condition can be coded by two ways: either all distributions are reflected, or only outgoing distributions are swapped out for their geometric opposites.

The reflection of all the distributions makes it possible to ensure that the adhesion condition is obtained regardless of the geometric configuration. Thus, the code has only one subroutine for all zero-velocity boundaries [14]. In this work, this option was adopted.

3. Outlet boundary conditions

The normal gradient of the distribution functions at the output boundary is zero. After propagation, the unknowns distributions f_3 , f_6 and f_7 are calculated simply : $f_a(Nx, j) = f_a(Nx_{-1}, j)$, ($a = 3, 6, 7 ; j = 1, N_y$), where N_x and N_y are the numbers of the cells meshes in the x and y direction respectively.

B. Initial conditions

One imposes initial velocity macroscopic quantities $u_0 = 0$, which are used to calculate the distributions functions $f_i^0(x, u_0, t = 0)$, which are used in their turns as initial conditions for f_i : $f_i = f_i^0$.

III. NON-NEWTONIAN BLOOD VISCOSITY MODELS

Tu and Deville [15] found that the rheological properties of blood can significantly affect flow phenomena.

For our case study, O'Callaghan [16] showed that in areas of slow flow such as aneurysms and recirculation zones, where blood stagnates for a significant time period, the non-Newtonian nature of blood is important.

For Newtonian fluids, the stress tensor follows Newton's law for viscosity, expressing a linear and explicit stress-strain rate by the relation:

$$\sigma = \mu \dot{\gamma} \quad (8)$$

With,

σ : the shear stress tensor Pa.

$\dot{\gamma}$: is the shear rate tensor s⁻¹.

μ : Newtonian dynamic viscosity, equal to 0.0035 Pa.s.

For non-Newtonian fluids, the dynamic viscosity is a function of the shear rate. The stress tensor is expressed by the relation:

$$\sigma = \mu(\dot{\gamma}) \dot{\gamma}$$

Where $\dot{\gamma} = \sqrt{2 S_{\alpha\beta} S_{\alpha\beta}}$ is the module of the strain rate tensor

with $S_{\alpha\beta} = 1/2 (\partial_\beta u_\alpha + \partial_\alpha u_\beta)$

The tensor $S_{\alpha\beta}$ can be calculated locally without passing

through the operations of derivation, such as:

$$S_{\alpha\beta} = -\frac{1}{2\rho\tau\Delta t} \frac{C_2}{C_1} \sum_i v_{i\alpha} v_{i\beta} (f_i - f_i^{(0)}) \quad (9)$$

In this study, it's proposed to make a comparison between the Carreau-Yasuda model, the Cross model, and the Newtonian model. The last model is taken as the reference. The two models that are used for modeling the non-Newtonian behavior of blood are presented below. There are different models that allow characterization of the behavior of non-Newtonian fluids. They sometimes make it possible to highlight threshold stress, shear-thickening, or shear-thinning behavior. The description given here avoids the molecular mechanisms which govern these laws.

A. Carreau-Yasuda model

In Carreau-type models (Cross, Carreau-Yasuda, Cross), λ is the relaxation time constant with the unit of second. Dynamic viscosity for C-Y model is given by

$$\mu = \mu_\infty + \frac{\mu_0 - \mu_\infty}{(1 + (\lambda\dot{\gamma})^a)^{n-1}} \quad (10)$$

With $\mu_0 = 0.16 \text{ Pa.s}$, $\mu_\infty = 0.0035 \text{ Pa.s}$, $\lambda = 8.2 \text{ s}$, $n = 0.2128$, $a = 0.64$ [17].

At high shear rates, well above 100 s^{-1} , the viscosity of Carreau's model is very close to that of Newton. The Carreau-Yasuda model is the type of the Carreau model that tends to Newtonian rheology at low shear rates.

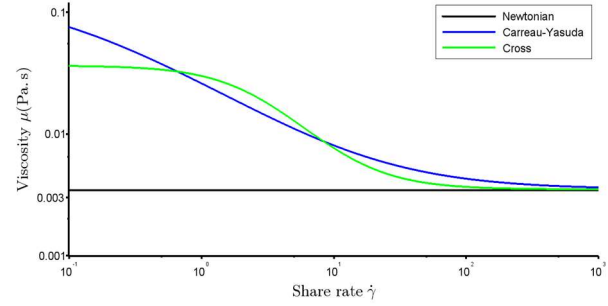


Fig.3: Log-log plot of C-Y, Cross and Newtonien model viscosity.

B. Cross model

Also known as the Bird-Carreau model, is the simplified form of the Carreau model, which may be a good candidate for modeling blood flow in large arteries due to the shorter viscosity range generated by the model [17].

$$\mu = \mu_\infty + \frac{\mu_0 - \mu_\infty}{1 + (\lambda\dot{\gamma})^a} \quad (11)$$

With $\mu_0 = 0.0364 \text{ Pa.s}$, $\mu_\infty = 0.0035 \text{ Pa.s}$, $\lambda = 0.38 \text{ s}$, $a = 1.45$. The fig. 3 presents the plot of dynamic viscosity versus shear rate for the three models in a log-log scale reference.

IV. SIMULATION PARAMETERS

A. Geometric parameters

The case of fusiform abdominal aortic aneurysm (AAA) is treated in this section (Fig. 4). The geometry and dimensions are taken from a real image of an AAA [18]. This aneurysm is shown in fig. 5.

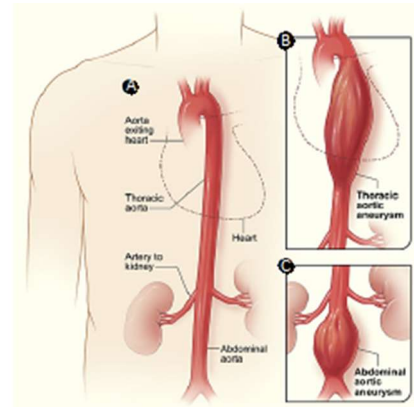


Fig. 4: A: Normal aorta, B: Thoracic aortic aneurysm, C: Abdominal aortic aneurysm [19].

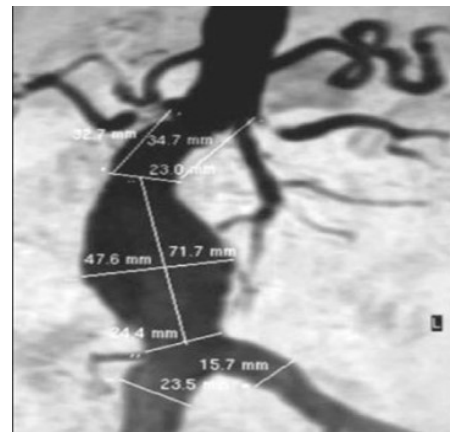


Fig. 5: Abdominal aortic aneurysm [18].

The characteristics of this aneurysm are:

Length $l = 71.7\text{mm}$, height $h = 47.6\text{mm}$, vessel length $L = 126\text{mm}$, inlet diameter $D_e = 23\text{mm}$, outlet diameter $D_s = 24.4\text{mm}$.

Due to the complexity of arterial geometry, an ideal simplified geometry has been used to represent the actual geometry above. The geometry size of our simulation model is 274×120 (lattice unit), with a channel width of 274 (lattice unit).

The AAA geometry used is shown in fig. 6 below and their dimensions (in lattice) are detailed in table I.

Due to the dimensions of aneurysms, two types of stent are used, one with 4 struts and the other with 6 struts. A strut is set of metal parts that form the stent, separated from each other by spaces. The struts used in simulations have a square shape, length $l_s = 40$ (lattice unit) and thickness $e_s = 4$ (lattice unit) for the case of stent with 4 struts. And for case of the stent of 6 struts, a length is $l_s = 25$ (lattice unit), thickness $e_s = 4$ (lattice unit). The parameters of the stents used for each case are shown in table II.

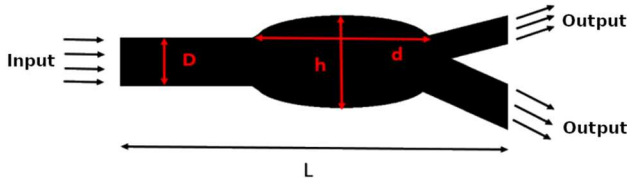


Fig. 6: Simplified geometry of the abdominal aorta

Table. I

Abdominal aortic aneurysm dimensions in (lu)

Aneurysm Parameters	D	d	h	L
Size (lu)	93	311	184	274

Table. II

Parameters of the stents for the simulation.

Parameters(lu)			
Stent type	l_s	Espacement	Porosit� %
4 struts	40	44	50
6 struts	25	38	61

The porosity was calculated with the following relationship:

$$P = \frac{d - (n * l_s)}{d}$$

With :

P: porosity;

d: the aneurysm's length (lu);

l_s : the strut's width (lu);

n: strut's number (lu).

The three geometries used in the simulations are shown below:

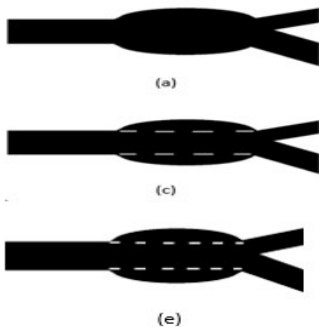


Fig. 7: AAA without stent b) AAA with 4-strut stent- c) AAA with 6-strut stent.

B. Flow Parameters

To render the simulation more realistic, a pulsating flow is used, and therefore the effect of heartbeats is no longer neglected. The elasticity of the wall is not taken into account. The average of one heartbeat cycle was taken as 0.8 seconds since the contraction phase of the heart (systole) lasts about 0.25 seconds. The relaxation phase (filling phase) is called diastole. Its duration is about 0.55 seconds, so the sum of the two phases is 0.8 seconds.

To fit the pulse velocity into our code, the blood flow rate graphs corresponding to the study area (the aorta) are taken from a real experiment, as shown in fig. 8 and 9 below. Then, dividing by the surface, the desired velocity graph is obtained.

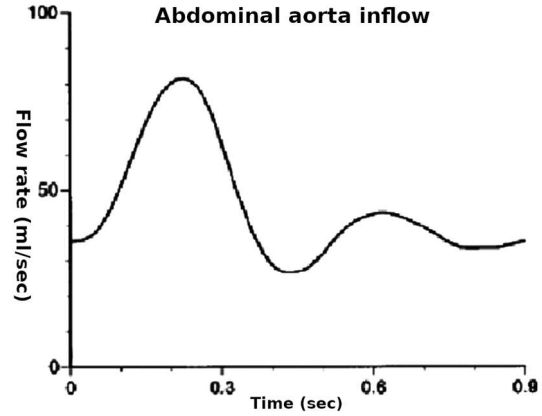


Fig.8: Blood flow in the abdominal aorta [20].

After this step, we used the Origin software to find the speed functions which are of polynomial type of the tenth degree after we provided the maximum points of the graphs of real speeds. The approximate mathematical function of blood velocity is:

$$V(t) = 0.091 + 0.013t - 1.191t^2 + 53.981t^3 + 115.608t^4 - 3959.907t^5 + 19484.015t^6 - 44659.211t^7 + 54599.737t^8 - 34553.635t^9 + 8929.843t^{10}$$

This function is valid for a time less than 0.8s, so it had to be made periodic with a period of 0.8s.

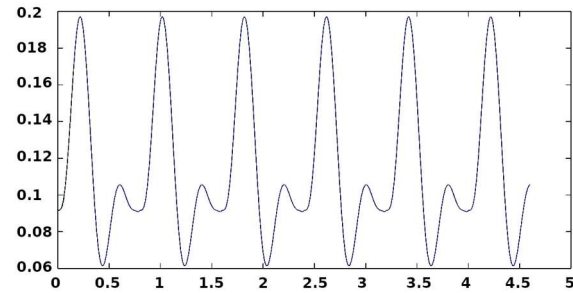


Fig. 9: Periodic pulse velocity curve in the abdominal aorta.

For blood rheology, blood is treated as a Newtonian fluid and also as a non-Newtonian fluid by assuming that the viscosity is constant concerning the shear rate in the case of Newtonian behavior. For the case of non-Newtonian behavior of blood, the equations (10)-(11) of Cross and Carreau-Yasuda are used to model blood flow.

I. RESULTS AND DISCUSSIONS

In this section, the results of the simulations obtained by the developed code are presented. First, a qualitative analysis of the velocity and shear rate of the blood is made, followed by a quantitative analysis of the systolic and diastolic velocity generated by the heart.

The visualization of the results is carried out by the free software ParaView, which is created from the VTK libraries (Visualization Tool Kit). The software is cross-platform and intended for scientific and interactive data visualization. Then, to confirm our results, we compared them to the results provided by the reference software in the Computational fluid dynamics (CFD) field "Ansys Fluent". Blood flow in the abdominal aorta is calculated from the Navier-Stokes equations (5) solved using Fluent 19.0 by the finite volume method. In this simulation, the blood is considered as a Newtonian fluid with a viscosity of 0.003 Pa.s and a density of 1060 kg/cm^3 . As Fluent does not deal with variable velocities boundary conditions directly, it was necessary to write an algorithm in the C++ language that contains the velocity equations and introduce them into the initial conditions before launching the simulation.

A. Comparison with ANSYS-FLUENT results

In this part, the analyze of the results of two simulations is performed. The first is done using the elaborate LBM code. The second simulation is made with the commercial software ANSYS-FLUENT based on the Navier-Stokes equations (5) solved by the finite volume method. This simulation allowed us to validate our model by comparing our results with the results of ANSYS-FLUENT.

A qualitative analysis of the two simulations is performed by introducing the same input conditions. From the comparison of the results in fig. 10 above, it's shown that the distribution of the velocity is similar in both solvers, with a small difference inside the aneurysm, which shows that they are in acceptable agreement.

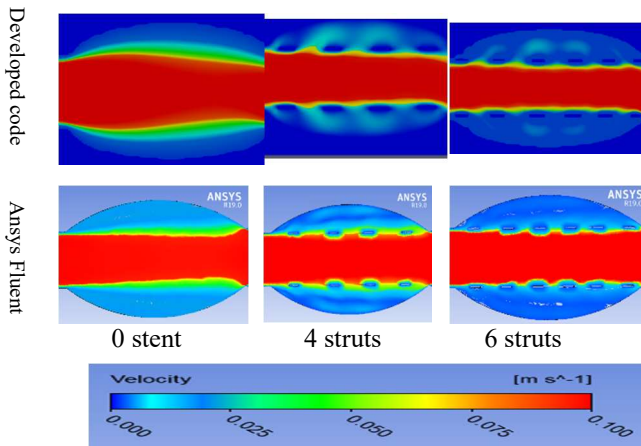


Fig. 10: The comparison between Flow velocity for the two solvers: developed code and ANSYS for fusiform aneurysm.

B. Qualitative analysis

The qualitative analysis is made based on the study of the variation of the velocity and the shear rate for the two studies' cases.

1. Velocity field: systolic

The relatively high velocities are found at the entrance and exit of the aneurysm. And the lowest velocity is observed at the level of the aneurysmal sac.

A comparison between the different models in the case of systolic velocity is given in fig.11. It is the greatest velocity that crosses the abdominal aorta.

Table III contains the maximum velocity values obtained inside the sacs for the three rheological models.

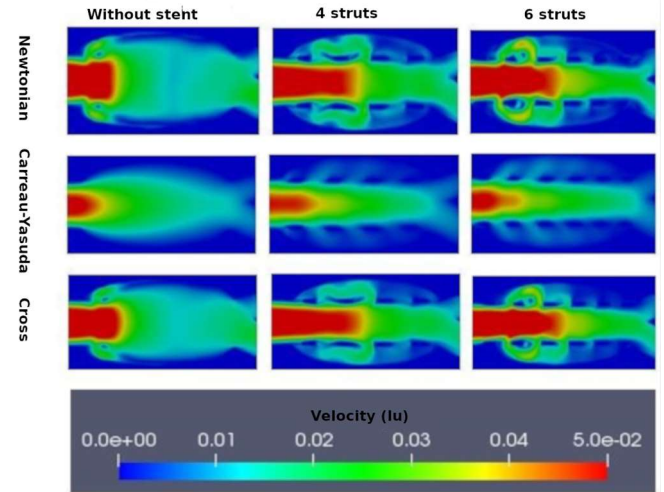


Fig. 11: Representation of the velocity field with different porosities for the three rheological models (systolic velocity)

Table. III

Maximum values of the velocity (lu) obtained in the control line (inside the sacs) for the three rheological models

	0 stent	4 struts	6 struts
Newt	0.0173	0.0138	0.0126
C-Y	0.0142	0.0118	0.0109
Cross	0.0166	0.0131	0.0119

Newtonian model

The maximum velocity is around 0.0173 (lu) for aneurysm without a stent, it decreases afterward to 0.0138 (lu) in the model with four struts and also decreases to reach the value of 0.0126 (lu) for the six struts model, this is due to the decrease in porosity caused by the stent struts at the aneurysm entrance.

Carreau-Yasuda model

This model is different from the other models, the only similar thing is that we have a reduction in the velocity obtained inside the aneurysmal sacs after the placement of a stent. The maximum velocity in the control line inside the sacs decreases from 0.0142 (lu) in the without stent model to 0.0109 (lu) in the six-strut model.

Cross model

The same interpretation is made as that on the Newtonian model. Velocity decreases in aneurysmal sacs after the placement of a stent in the aneurysm. The maximum velocity for this model decreases from 0.0166 (lu) for the aneurysm without a stent until reaching a value of 0.0119 (lu) in the aneurysm with six struts. These values are smaller than those recorded in the Newtonian model.

Velocity in the middle of the artery

To see the influence of the location of the stents on the flow, we checked the velocities in the middle of the artery for the three rheological models (

Table. IV).

This model is different from the other two models, the only thing that is similar is that we have a velocity reduction

Table. IV

Maximum velocity values (lu) obtained in the middle of the stent for the 3 rheological models

	0 stent	4 struts	6 struts
Newt	0.04889	0.0687	0.0864
C-Y	0.0381	0.0562	0.0627
Cross	0.0467	0.0651	0.0783

After visualizing the flow of the three models with and without a stent, we notice a significant increase in flow velocity after the placement of a stent, more specifically in the middle of the stent, indicating that the stent placement promotes flow toward the fusiform aneurysm's outlet.

2. Velocity field: diastolic

The velocity results for the smallest velocity in the abdominal aorta is presented

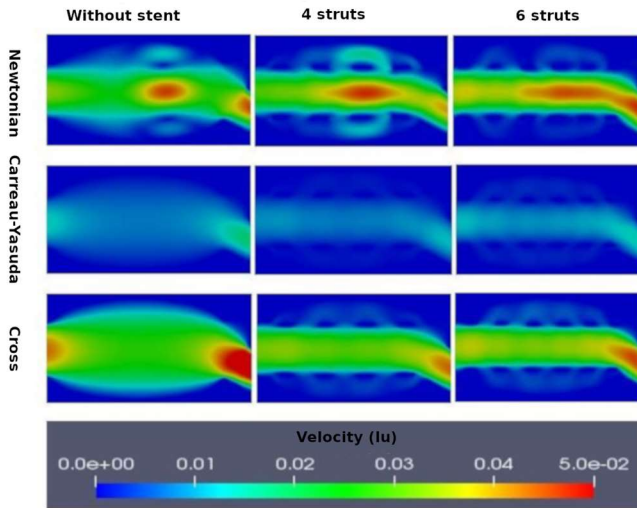


Fig. 12: Representation of the velocity field with different porosities for the three rheological models (diastolic velocity).

Table. V

Maximum velocity values obtained in the control line (inside the sacs) for the three rheological models.

	0 stent	4 struts	6 struts
Newt	0.00189	0.00107	0.00068
C-Y	0.00091	0.00056	0.00042
Cross	0.00174	0.00102	0.00056

Newtonian model

The maximum velocity is around 0.00189 (lu) for a without stent aneurysm, it decreases afterwards to 0.00107 (lu) in the model with four struts and also decreases to reach the value 0.0006 (lu) for the model with six struts. This is due to the decrease in porosity caused by the stent struts at the entrance to the aneurysm.

Carreau-Yasuda model

obtained inside the aneurysmal sacs after the placement of a stent. The maximum velocity in the control line inside the sacs decreases from 0.00091 (lu) in the without stent model to 0.00042 (lu) in the six-strut model.

Cross model

Same interpretation as that on the Newtonian model is made. Velocity decreases in aneurysmal sacs after placement of a stent in the aneurysm. The maximum velocity for this model decreases from 0.00174 (lu) for the aneurysm without stent until reaching a value of 0.00056 (lu) in the aneurysm with six struts. These values are smaller than those recorded in the Newtonian model.

Velocity in the middle of the artery

To see the influence of the location of the stents on the flow, we checked the velocities in the middle of the artery for the three rheological models. Table. VI gives the maximum values of velocity (lu) obtained in the middle of the stent.

Table. VI

Maximum velocity values (lu) obtained in the middle of the stent for the three rheological models

	0 stent	4 struts	6 struts
Newt	0.00910	0.01020	0.01080
C-Y	0.00199	0.00210	0.00220
Cross	0.00840	0.00980	0.01020

Upon visualization of the flow of the three models with and without a stent, we notice a considerable increase in the velocity of the flow after the placement of a stent, more precisely in the middle of the stent. This means that the placement of the stent promotes flow towards the exit of the fusiform aneurysm.

3. Shear rate: systolic

Fig. 13 show a presentation of the shear rate between the different models with and without stent in the case of systolic velocity.

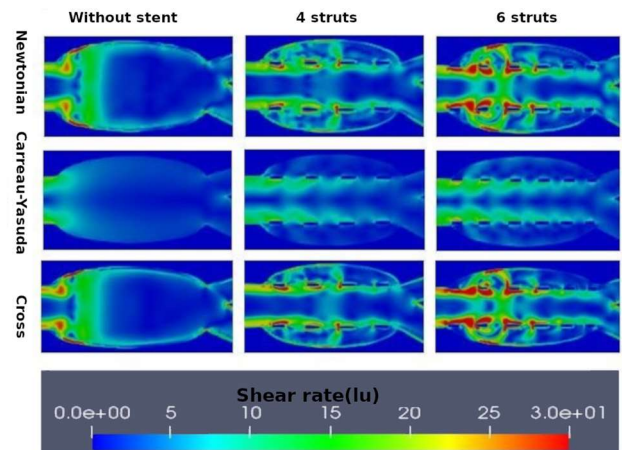


Fig. 13: Representation of the shear rate for the three rheological models (systolic velocity).

The Table. VII below contains the maximum values of the shear rate in the sac of the aneurysm for the three rheological models and for the systolic velocity.

Table. VII

Maximum values of the shear rate (lu) obtained in the sac of the aneurysm for the three rheological models (systolic velocity).

	0 stent	4 struts	6 struts
Newt	24.780	20.410	15.330
C-Y	19.835	16.224	12.187
Cross	24.499	19.246	15.124

Newtonian model

The shear stress values in the parent vessel are greater than the values inside. An increase in shear stress is observed after placement of the stent. The maximum shear stress in the control line is 24.78 (lu) for the model without a stent and increases until reaching a value of 15.33 after the placement of a stent with six struts.

This model is different compared with other models. This model gives small values of shear stress inside the aneurysmal sac compared to other models. Its maximum shear stress is 19.835 (lu) for the non-stent model and 12.187 (lu) for the six struts model.

Cross model

This model provides almost the same results as those of the Newtonian model; there is a decrease in shear stress inside the aneurysmal sac after the placement of a stent is observed. The decrease is from 24.499 (lu) for the non-stent model down to 15.124 (lu) for the six-strut model.

4. Shear rate: diastolic

Fig. 14 gives the representation of the shear rate for the three rheological models for the smallest velocity in the abdominal aorta. The Table. VIII below gives the maximum values of the shear rate in the aneurysmal sacs for the three rheological models and for the diastolic velocity.

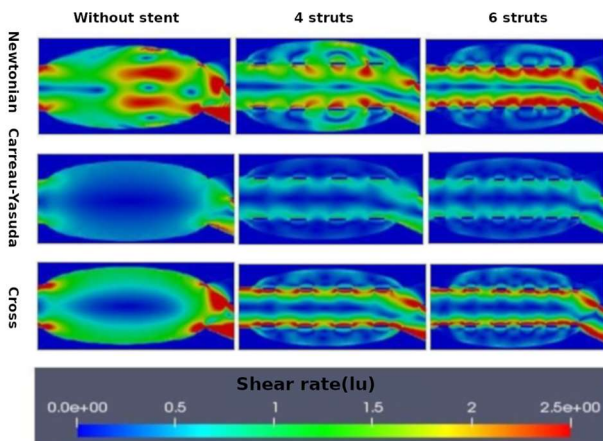


Fig. 14: Representation of the shear rate for the three rheological models (diastolic velocity)

Table. VIII

Maximum values of the shear rate (lu) obtained in the sacs of the aneurysm for the three rheological models (diastolic velocity).

	0 stent	4 struts	6 struts
Newt	1.610	1.140	0.980
C-Y	0.920	0.690	0.650
Cross	1.399	0.920	0.870

Newtonian model

The values of shear stress in the parent vessel are greater than the values inside. Following stent placement, there is an increase in shear stress. The maximum shear stress in the control line is 1.61 (lu) in the absence of a stent and increases to 1.02 after the placement of a six strut stent.

Carreau-Yasuda Model

This model is different compared to the other models as for all the previous visualizations on the velocity fields. This model gives small values of the shear stress inside the aneurysmal sac compared to the other models. The maximum shear stress is 1.12 (lu) for the non-stent model and 0.76 (lu) for the six strut model.

Cross model

This model provides almost the same results as the Newtonian model. An increase in shear stress is observed within the aneurysmal sac after placement of a stent. The increase is from 1.499 (lu) for the aneurysm without stent up to 0.94 (lu) for the stent with six struts.

C. Quantitative analysis

To perform a quantitative analysis on the percent change in velocity and shear rate related to stent effect and efficiency, the velocities for the largest and smallest heartbeats were taken to test the role of the stent in different blood flow regimes. To do this analysis, we must look for:

- The rate of change of velocity for the largest heartbeat V_{rg} ,
- The rate of change of velocity for the smallest heartbeat V_{rp} ,
- The rate of change in shear rate for the largest heartbeat Tc_{rg}
- The rate of change of the shear rate for the smallest heartbeat Tc_{rp} .

Such as :

$$V_r = \frac{V^{ns} - V^s}{V^{ns}} \times 100$$

$$Tc_r = \frac{Tc^{ns} - Tc^s}{Tc^{ns}} \times 100$$

With, V^s : the velocity for an aneurysm with stent, V^{ns} : the velocity for an aneurysm studied without stent, Tc^{ns} : the shear rate for an aneurysm without stent and Tc^s : the shear rate for an aneurysm with stent.

Percentage changes in velocity and shear rate are calculated at different points in the aneurysm to see the impact of the stent on blood flow in different parts of the aneurysm. This quantitative analysis on the aneurysm is carried out in order to know the role and the effectiveness of stents.

The quantitative analysis is made for three points of the fusiform aneurysm (at the entrance, in the middle and at the exit of the aneurysm).

1. In the entrance of aneurysmal sac

Model	stent	Porosity (%)	V_{rg} (%)	V_{rp} (%)	Tc_{rg} (%)	Tc_{rp} (%)
Newt	0 stent	100.00	0.00	0.00	0.00	0.00
	4 struts	70.00	5.61	12.30	6.61	4.46
	6 struts	59.00	10.29	18.01	11.18	8.93
C-Y	0 stent	100.00	0.00	0.00	0.00	0.00
	4 struts	70.00	16.95	11.20	18.66	10.90
	6 struts	59.00	20.08	13.79	21.82	16.00
Cross	0 stent	100.00	0.00	0.00	0.00	0.00
	4 struts	70.00	7.77	2.54	5.52	17.36
	6 struts	59.00	12.03	9.60	10.48	22.40

2. In the output of aneurysmal sac

Model	stent	Porosity (%)	V_{rg} (%)	V_{rp} (%)	Tc_{rg} (%)	Tc_{rp} (%)
Newt	0 stent	100.00	0.00	0.00	0.00	0.00
	4 struts	70.00	11.57	8.30	17.57	9.91
	6 struts	59.00	16.80	8.00	22.50	14.69
C-Y	0 stent	100.00	0.00	0.00	0.00	0.00
	4 struts	70.00	5.26	4.58	9.13	6.19
	6 struts	59.00	7.63	6.66	25.82	17.85
Cross	0 stent	100.00	0.00	0.00	0.00	0.00
	4 struts	70.00	17.50	11.38	5.11	2.8
	6 struts	59.00	10.89	14.02	6.92	5.31

3. In the middle of the aneurysmal sac

Model	stent	Porosity (%)	V_{rg} (%)	V_{rp} (%)	Tc_{rg} (%)	Tc_{rp} (%)
Newt	0 stent	100.00	0.00	0.00	0.00	0.00
	4 struts	70.00	5.78	36.36	13.22	14.44
	6 struts	59.00	10.93	19.09	19.09	18.88
C-Y	0 stent	100.00	0.00	0.00	0.00	0.00
	4 struts	70.00	6.25	37.93	25.00	16.25
	6 struts	59.00	8.12	46.89	31.90	20.18
Cross	0 stent	100.00	0.00	0.00	0.00	0.00
	4 struts	70.00	2.08	25.31	29.16	25.81
	6 struts	59.00	3.14	26.46	33.54	38.36

It's noted that there a reduction in the flow velocity and the shear rate in the aneurysmal sacs after the placement of a stent and that it differs from one point to another, and it's also noted a slight increase at the entrance and exit of the aneurysm. The percentage changes in velocity and shear rate are not influenced by the type of pulsation or by the rheological model, but the number of struts (or porosity) influences the results for the three models, where we see that the rate of change for the cases of the stents with six struts is higher than that with four struts, except for a few cases. These results are comparable to the results obtained by [7].

The reduction in hemodynamics obtained remains modest, with a maximum of 40%, which remains probably unfavorable for

thrombotic occlusion. The use of multiple overlapping stents is recommended to produce occlusion [7, 21].

II. CONCLUSION

In this work, a numerical investigation was conducted by the Lattice Boltzmann method using a D2Q9 discretization and three types of boundary conditions: bounce-back at the walls; Dirichlet at the entrance; Von Neumann at the exit of our computer code; and visualization on the ParaView software using a pulsed flow.

The results show many sub-cases such as several stents, systolic and diastolic velocity, velocity and shear rate. It is interesting to demonstrate the reduction effect of the stents for each case and for each variable (velocity and shear rate). This detail may seem repetitive but it provides information of a different nature concerning the effect of the stent for the treatment of AAA. Indeed, velocity alone is not enough as an index for monitoring the generation of thrombosis. The shear rate must be added since this index is proposed as a key variable for the appearance and development of thrombosis [22-24].

According to the results found on the case study of the fusiform abdominal aortic aneurysm, it's noted that:

- For all three rheological models, the velocity decreases after the placement of a stent. The decrease in velocity and shear rate for the fusiform aortic aneurysm observed is less significant compared to the case of the saccular intracranial aneurysm. This explains the formation of thrombosis for cerebral aneurysms compared to abdominal aortic aneurysms.
- The velocity field results given by the Carreau-Yasuda model are different compared to the other two models, and the Newtonian model presents the greatest velocity values for the two cases of aneurysms treated.
- The velocity values for the Newtonian and Cross models are very close.
- For all three rheological models, the velocity increases in the middle of the fusiform aneurysm after the placement of a stent. The decrease in the porosity of the stents slows the flow in the aneurysmal sacs and increases the flow in the middle of the artery.
- The effect of stents is more clearly visualized in the case of systolic velocity than in the case of diastolic velocity.
- The shear rate values are much higher in the case of the abdominal aortic aneurysm than in the anterior cerebral aneurysm, which coincides with the medical theory that the phenomenon of thrombosis is rare in aortic aneurysms (a high value of the shear rate indicates a low probability of thrombosis formation [23]).
- The purpose of placing stents in the case of cerebral saccular aneurysms is to promote the formation of thrombosis inside the aneurysmal sac. The purpose of placement of stents in the case of abdominal aortic aneurysm is to promote blood flow to the outside of the aneurysm in the case of abdominal aortic aneurysm and to decrease the pressure on the aneurysmal sac.
- The effect of stents is more visualized in the case of systolic velocity than diastolic velocity.
- The increase in flow velocity entering and exiting the fusiform aneurysm is much greater than that of the saccular aneurysm after stent placement, confirming that the stent is usable to promote horizontal flow in the case of fusiform aneurysms. However, if a thrombotic occlusion is desired, several stents must be superimposed.
- The pulsation of the heart has no influence on the quantitative analysis of the flow.

The main obstacle to the development of this work is the lack of relevant experimental studies in order to validate the findings given by numerical simulations.

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Performance Analysis of a Novel Multiple Access Spatial Modulation Architecture based on STBC-OFDM and CDMA over Nakagami- m fading Channel.

Fadila BERRAHMA, Khalida GHANEM, and Hicham BOUSBIA-SALAH

Abstract—One of the schemes which are likely to be adopted in the forthcoming wireless communication standards is the so-called spatial modulation (SM). However, the wireless channels pertaining to such standards will mostly be frequency selective because of the involved high data rates, and SM technique alone can not overcome the resulting signal distortion. In this context, orthogonal frequency division multiplexing (OFDM) has been used as an efficient solution to combat this impediment, and space-time block coding (STBC)-OFDM has followed as an innovative approach merging the viable features of OFDM and the reliability enhancement capabilities of STBC. Obviously, SM solutions in the presence of multiple users communicating in frequency selective channels should be targeted. To fulfill such a need, a new SM-based architecture incorporating STBC-OFDM and CDMA is proposed in this work and is shown to exhibit a performance which is superior to state-of-the-art alternatives. Furthermore, its sensitivity to the aforementioned adverse parameters is studied in Nakagami channel.

Keywords—SM, STBC, OFDM, CDMA, Nakagami.

NOMENCLATURE

SM	Spatial Modulation.
OFDM	Orthogonal Frequency Division Multiplexing.
STBC	Space-time Block Coding.
CDMA	Code Division Multiple Access.
MIMO	Multiple Input Multiple Output.

I. INTRODUCTION

It has long been believed that equipping the wireless communication systems with multiple input and output (MIMO) antennas was sufficient to ensure the communication requirements for a long period. However, because of the important revolutions wireless industry has gone through and the ever-increasing number of subscribers, along with the need for very high data rates, new challenging concepts and solutions have to be designed, such as massive MIMO [1]. Massive MIMO systems involve the use of a very high number of antennas, more particularly at the transmit side, each with its associated radio-frequency (RF) chain, thereby implying a non-practical hardware complexity. Spatial modulation (SM) is a recently proposed scheme which promises a low complexity transmitter

and receiver design, without decreasing the performance of massive MIMO block transceiver. This complexity reduction is achieved by adopting a simple mapping paradigm. SM exploits the spatial information as an additional source to convey the information. In order to increase the data reliability at the reception side, STBC techniques are by now well-established among the research community [2]. The improvement of the quality of the reception has been envisaged by combining STBC and SM schemes. The two STBC-coded words and the two antenna indices are used to identify the source information. Motivated by this principle, the performance of SM-STBC architectures has been investigated in the literature only in quasi-static Rayleigh channel. Indeed, no system has been used to support SM-STBC-OFDM paradigm in multi-user system [3]. OFDM and its variants have been widely adopted. It is also worthy to recall that Nakagami model is one of the most adopted to reflect different channel conditions, since it encompasses a wide range of models, from the pure Rayleigh to the one approaching deterministic behavior. We propose a novel architecture which supports 5G and beyond communications standard, by incorporating STBC-OFDM structure in SM-CDMA design. The performance of the resulting SM-STBC-OFDM-CDMA system is investigated in Nakagami channel fading, more particularly in the presence of imperfect channel estimates.

II. SM-STBC SCHEME WITH OFDM AND CDMA MULTIPLE ACCESS

A single-cell multiuser uplink MIMO model is considered in this work, where K users are served by the N_t antennas of the base station (BS).

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At the transmitter side, the input data sequence corre-

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Table. I
MAPPING TABLE OF SM-OFDM-STBC.

j	I/bits	Matrix	j	I/bits	Matrix
1	0000	$\begin{pmatrix} 1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 \end{pmatrix}^T$	3	1000	$\begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & -1 & 1 & 0 \end{pmatrix}^T$
	0001	$\begin{pmatrix} 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix}^T$		1001	$\begin{pmatrix} 0 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix}^T$
	0010	$\begin{pmatrix} -1 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 \end{pmatrix}^T$		1010	$\begin{pmatrix} 0 & -1 & 1 & 0 \\ 0 & -1 & -1 & 0 \end{pmatrix}^T$
	0011	$\begin{pmatrix} -1 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{pmatrix}^T$		1011	$\begin{pmatrix} 0 & -1 & -1 & 0 \\ 0 & 1 & -1 & 0 \end{pmatrix}^T$
2	0100	$\begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 \end{pmatrix}^T$	4	1100	$\begin{pmatrix} 1 & 0 & 0 & 1 \\ -1 & 0 & 0 & 1 \end{pmatrix}^T$
	0101	$\begin{pmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{pmatrix}^T$		1101	$\begin{pmatrix} 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \end{pmatrix}^T$
	0110	$\begin{pmatrix} 0 & 0 & -1 & 1 \\ 0 & 0 & -1 & -1 \end{pmatrix}^T$		1110	$\begin{pmatrix} -1 & 0 & 0 & 1 \\ -1 & 0 & 0 & -1 \end{pmatrix}^T$
	0111	$\begin{pmatrix} 0 & 0 & -1 & -1 \\ 0 & 0 & 1 & -1 \end{pmatrix}^T$		1111	$\begin{pmatrix} -1 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 \end{pmatrix}^T$

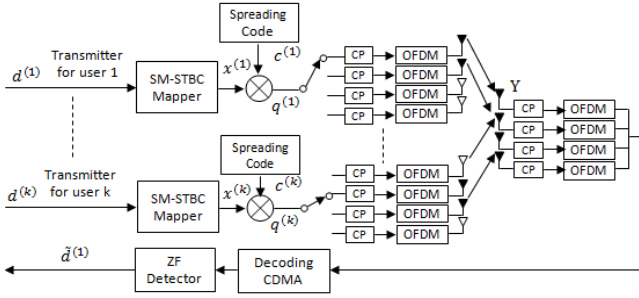


Fig. 1: SM-STBC Scheme with OFDM and CDMA transceiver block

sponding to the k^{th} user, is first encoded by a convolutional encoder, which consists of a concatenation of error correction, preceding an interleaving $\pi^{(k)}$ module. Afterwards, data are divided into blocks of b_{SM} bits, where b_{SM} is given as:

$$b_{SM} = \log_2 N_t + 2 \times \log_2 M \quad (1)$$

where M is the signal modulation order. The first $\log_2 N_t$ bits will select a pair of the transmit antennas to emit the modulated symbols over the two STBC consecutive time slots, while the last $2 \times \log_2 M$ bits are consecutively fed to the modulator to generate two modulated symbols x_1 and x_2 .

In the SM-STBC architecture, the information is carried by the indices of the transmit antennas. We assume that $N_t = N_r = 4$, where N_t and N_r are the number of transmit and receive antennas. Let us consider the k^{th} user's encoding procedure. By using SM-STBC mapping, two complex information symbols (x_1, x_2) , are conveyed from

two transmit antennas, in two symbol intervals, in an orthogonal way by the codeword $\mathbf{X}_j^k$, which identifies one component of the mapping matrix represented by the codeword $\mathbf{X} = \{\mathbf{X}_1^k, \mathbf{X}_2^k, \mathbf{X}_3^k, \mathbf{X}_4^k\}$:

$$\{\mathbf{X}_1^k, \mathbf{X}_2^k\} = \left\{ \begin{pmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ x_1 & -x_2^* \\ x_2 & x_1^* \end{pmatrix} \right\} \quad (2)$$

$$\{\mathbf{X}_3^k, \mathbf{X}_4^k\} = \left\{ \begin{pmatrix} 0 & 0 \\ x_1 & -x_2^* \\ x_2 & x_1^* \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} x_1 & -x_2^* \\ 0 & 0 \\ 0 & 0 \\ x_2 & x_1^* \end{pmatrix} \right\} \quad (3)$$

The subscript j represents the TX-antennas index that identifies the pair of activated antennas, and rows and columns of the mapping matrix in Table I, correspond to the transmit antennas and the two time slots of the STBC coding, respectively.

Example: Consider $N_t = 4$, $M = 2$ (binary phase shift keying) and Alamouti scheme for STBC. The transmitted block from a user should includes 4 bits according to (1). The possible transmission matrices are listed in Table I, where j refers to the pair of activated antennas, and rows and columns of each component of the mapping matrix correspond to the transmit antennas and the two time slots of the STBC coding, respectively. Assume that the bit block to be transmitted from a user is $[0, 1, 1, 1]$. The first two bits 01 indicates the second codeword $\mathbf{X}_2$, and 11 being the transmitted symbol pair $\{x_1, x_2\} = \{1, 1\}$

that are transmitted from the third and fourth antennas respectively. Mapping this to SM-STBC symbol results in:

$$\mathbf{X}_2 = \begin{pmatrix} 0 & 0 & -1 & -1 \\ 0 & 0 & 1 & -1 \end{pmatrix}^T \quad (4)$$

The signal $\mathbf{q}^{(k)}$ results from the Hadamard product between the transmitted signal array $\mathbf{x}_j^k$, generated from the original signal repeated P times, and the P -length spreading code $\mathbf{c}^{(k)} \in \mathbf{C}^{N_t \times P}$. Therefore, the spread signal has the following equation:

$$\mathbf{q}^{(k)} = \mathbf{x}_j^{(k)} \odot \mathbf{c}^{(k)} \quad (5)$$

Afterwards, in each k^{th} transmitter side, the resulted signal is transformed to the time domain by applying the inverse fast Fourier transform (IFFT). Hence, a cyclic prefix (CP) is added to eliminate the ISI between the OFDM symbols, to be conveyed over the channel [4]. The multipath signal received at the ρ^{th} antenna element from the ν^{th} transmit antenna could be written as:

$$\mathbf{y}_\rho(t) = \sum_{i=0}^K \mathbf{h}_{\rho\nu}^k(t) \otimes \mathbf{x}_\nu^k(t) + \mathbf{w}(t) \quad (6)$$

where $\mathbf{x}_\nu^k(t)$ is the signal emanating from the ν^{th} activated antenna of the k^{th} user, $\mathbf{w}(t)$ is the additive white Gaussian noise vector with $CN(0, \sigma^2)$ elements, $\otimes$ denotes the time convolution operator and $\mathbf{h}_{\rho\nu}^k(t) = [h_{\rho\nu}(t)^1 \ h_{\rho\nu}(t)^2 \ \dots \ h_{\rho\nu}(t)^L]$ stands for the $L \times 1$ channel vector between each pair of transmit-receive antennas encompassing the L significant multipath channel components.

$\mathbf{H}(t)$ is a block matrix containing a set of $N_r \times N_t$ vectors each of length L , $\mathbf{H}(t)$ sets to:

$$\mathbf{H}(t) = \begin{pmatrix} \mathbf{h}_{1,1}(t) & \mathbf{h}_{1,2}(t) & \dots & \mathbf{h}_{1,N_t}(t) \\ \mathbf{h}_{2,1}(t) & \mathbf{h}_{2,2}(t) & \dots & \mathbf{h}_{2,N_t}(t) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{h}_{N_r,1}(t) & \mathbf{h}_{N_r,2}(t) & \dots & \mathbf{h}_{N_r,N_t}(t) \end{pmatrix} \quad (7)$$

Each vector of $\mathbf{H}(t)$ corresponds to the L^{th} multipath channel gains.

The few research works addressing the performance of SM-based systems adopt Rayleigh distribution for the channel fading. However, Rayleigh distribution fails to fit the channel behavior over long distances and high frequencies, Nakagami then suggested a parametric gamma distribution-based density function, to describe the experimental data he obtained. Later, it was also shown by different researchers that real-life data was best fitted by the model provided by Nakagami. Basically, Rayleigh distribution is sufficient to model amplitude in urban areas, whereas, Rician distribution suits better sub-urban areas where LOS components exist. By contrast, Nakagami- m distribution is a generalized case which includes different distribution

cases. From [5], the envelope of the Nakagami- m fading channel is shown to be distributed according to the law:

$$f_{|\omega_l|}(x) = \frac{2}{\Gamma(m_l)} \left(\frac{m_l}{\Omega_l}\right)^{m_l} x^{2m_l-1} \exp\left(-\frac{m_l}{\Omega_l} x^2\right) \quad l = \{1, \dots, L\} \quad (8)$$

where L is the number of paths, $\Gamma(\cdot)$ is the gamma function defined as $\Gamma(m) = \int_0^\infty x^{m-1} e^{-x} dx$. The parameter $m \geq \frac{1}{2}$ verifies the relation and indicates the fading severity, while $\Omega_l = \mathbb{E}\{|\omega_l|^2\}$ designates the mean-square value of the l^{th} path signal and ω_l represents the multipath gain. A variety of fading conditions are generated by varying the fading or Nakagami scale parameter m_l . For critical cases which are represented in Fig. 2, we have:

- ★ $m_l = \frac{1}{2}$ represents one sided Gaussian channel with PDF $f_{|\omega_l|}(x) = \sqrt{\frac{2}{\pi\Omega_l}} \exp\left(-\frac{x^2}{2\Omega_l}\right)$.
- ★ $m_l = 1$ describes equivalent Rayleigh fading channel with PDF $f_{|\omega_l|}(x) = \frac{2x}{\Omega_l} \exp\left(-\frac{x^2}{\Omega_l}\right)$, there is no single line-of-sight path for this distribution, it can cause severe distortion or fading.
- ★ $m = 1.5$, in this case, a LOS path exists, the fluctuations of the signal strength are reduced compared to Rayleigh fading; hence, Nakagami tends to Rician distribution.
- ★ For $m_l = \infty$, the Nakagami- m fading channel converges to a non fading AWGN with a PDF $f_{|\omega_l|}(x) = \delta(x - \Omega_l)$.

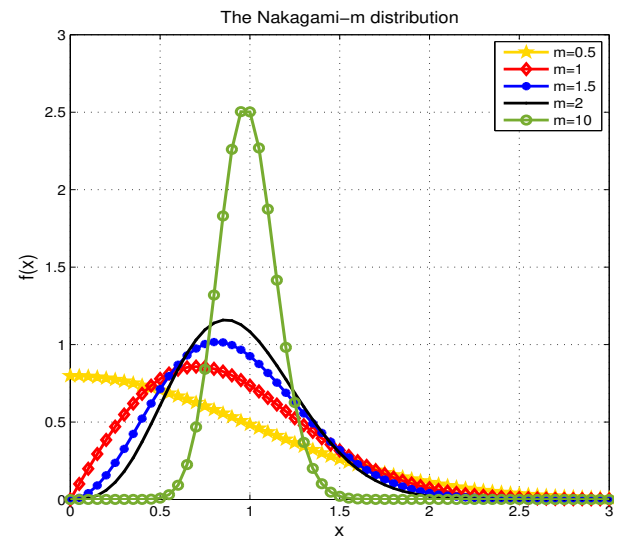


Fig. 2: Nakagami- m distribution.

1— Channel impairments: Imperfect channel state information (I-CSI)

Most of research works assume that the channel estimation errors are mitigated, thereby adopting error free (perfect) or erroneous (imperfect) CSI. In practice, this should be

considered by formulating the estimated channel matrix as follows [6]:

$$\hat{\mathbf{H}} = \rho \mathbf{H} + (1 - \rho) \lambda^{N_r \times N_t} \quad (9)$$

where λ is a normal distributed random variable with zero mean and unit variance. The coefficient $0 < \rho < 1$ is a factor that determines the similitude of the estimated CSI to the actual one. Perfect CSI estimation is achieved when $\rho = 1$.

where the overall channel model in eq. (9) is obtained by the variation of the parameter ρ .

III. SIMULATION RESULTS

Fig. 3 compares the achieved BER performance for different values of m , and different CSI conditions, when adopting SM-STBC-OFDM-CDMA scheme over Nakagami- m fading channel. Rayleigh channel scenario, obtained when $m = 1$, is compared against Rician and Gaussian cases, simulated with $m = 1.5$ and $m = 0.5$ respectively. Furthermore, ideally estimated channels are herein taken as a reference and are obtained by setting the value of ρ to 1. BPSK

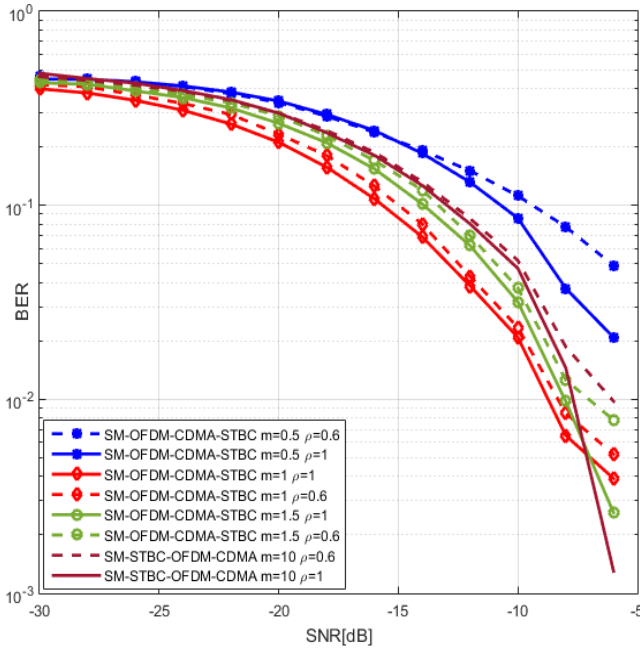


Fig. 3: Comparison of the achieved BER performance of SM-STBC-OFDM-CDMA architecture over Nakagami- m fading channel.

scheme is used. Other simulation parameters are fixed as follows: the number of users $K = 2$, the length of spreading code $P = 128$, the number of paths $L = 3$, and the number of sub-carriers $N_c = 128$. It can be observed from Fig. 3 that, in ideal channel conditions, Rayleigh channel is more favorable to communication than the general Nakagami channel where $m = 10$. Gaussian channel exhibits the lowest performance. The attained data reliability in all channel conditions remains quite interesting since low BER values are provided at low SNR range. However, as shown in this figure, increasing slightly the imperfection levels,

has a drastic degradation effect on the data reliability, more particularly in Gaussian channel.

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Wireless sensor network deployment optimization for a smart farming application: comparison of two Multi-Objective Evolutionary Algorithms

Soumaya FERHAT TALEB, and Nour El-Houda BENALIA

Abstract—As it is one of the main means of insuring food security, agriculture has employed different technologies such as the Internet of Things and wireless sensor networks (WSN), to improve the quality and quantity of agricultural products, while preserving natural resources. But unfortunately, in agricultural plots where we have large surfaces of interest, the optimization of node deployment in a WSN remains among the major problems to be solved. In this work, we proposed a WSN node deployment optimization model for an agricultural application according to classical constraints of coverage, over-coverage, connectivity and nodes number, in addition to the nodes separating distance constraint which affects the quality of physical parameters models. We have applied two variants of Multi-Objective Genetic Algorithms, the Non Sorting Genetic Algorithm II (NS-GA II) and the Strength Pareto Evolutionary Algorithm II (SPEA II). As a result, for a 100 m² plot, both algorithms ensured a communication rate of 100% while SPEA 2 presented a lower sensor number and over-coverage rates with a smaller separating distance, and the execution time of NSGA II was shorter with 11 s. Besides, both of them were greedy in terms of computation time with the increase in the size of the plots.

Keywords—Precise Agriculture, Wireless Sensor Network, Node Deployment, Strength Pareto Evolutionary Algorithm II, Non Sorting Genetic Algorithm II.

I. INTRODUCTION

The needs of food production are increasing with the growing population (expected to reach 9.4 and 10.2 billion by 2050 [1]). Therefore, new techniques were adopted in the agricultural field in order to solve the restricted land, water assets, and the climate change issues which represent a huge danger for accomplishing the sustainable farming objectives. PA aims to optimize the yield production by monitoring the irrigation operation, the application of fertilizers, pesticides and the crop growth with the help of direct or indirect measurement of several variables that gives immediate information about the crops [2]. For that purpose, different technologies were used such as the satellite remote sensing [3] and the WSN [4].

The WSN technology uses a group of components to collect, monitor and analyze the sensed data [5]. So as in the field of agriculture, WSN can optimize the crops quality and preserve the natural resources with the use of the deployed sensors which monitor the vital value of physical parameters such as humidity, temperature, and PH level, to define the exact requirements of plants. Furthermore, there are several types of WSNs in agriculture, which are classified according to their mode of deployment into two classes, the first class is the Terrestrial Wireless Sensor Networks (TWSN) where the sensors are deployed above the ground and the second one is Underground Sensor Networks (WUSN) where sensors are deployed inside the soil. The first mode presents more powerful features compared to the second

one such as low cost, energy consumption, and the high communication range which can go up to 100 meters [6]. However, the choice between the two modes is influenced by the WSN application type in agriculture. Each WSN is made up of a set of sensor nodes and base stations which work together to detect, transmit, process and interpret information. In addition, each sensor node is composed of a processing unit, a storage unit, a power source, a sensor and a communication module as shown in Figure 1. The components of sensor nodes are defined according to their field of application. For example the most adopted communication technology in agriculture is the Zigbee technology thanks to its properties such as the communication range which varies between 10 and 100 m, and its low energy consumption [7, 8]. Moreover, the WSN confronts several prob-

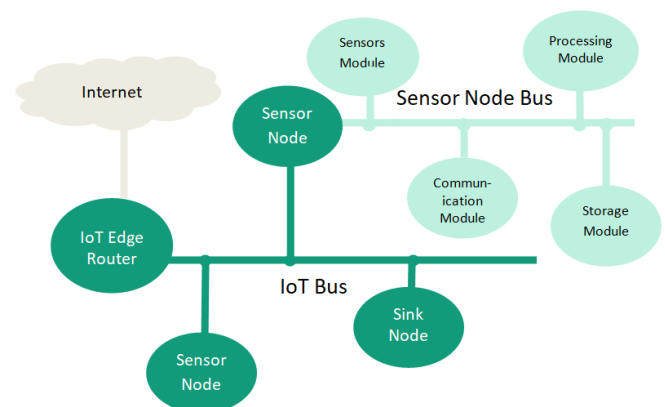


Fig. 1: Architectural model of a WSN.

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lems that can affect its performance, such as the Quality of Service (QoS), the WSN lifetime, security and the WSN cost. These lasts are related to each other, for example, we have to compromise with the WSN lifetime if we want to guarantee a full area coverage [9]. Therefore, in order to solve this problem

we have to deal with a multi-objective issue and for that specific algorithms and techniques have to be applied, the choice of the appropriate technique depends on the functioning of the algorithm, the desired time, the aimed precision and the issues types; Figure 2 illustrates the most important issues of WSN. The cost

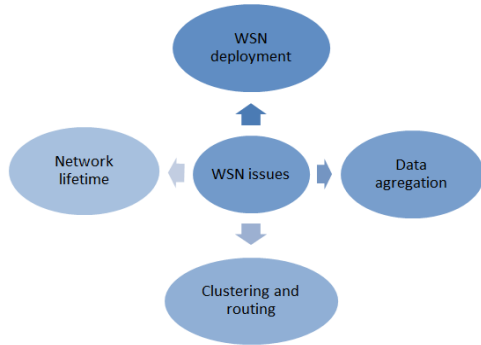


Fig. 2: The most important types of WSN issues.

of installing a WSN is one of the most important criteria for node deployment, of which the number of the used sensor and sink nodes has an impact on the cost of the WSN. Therefore, in this work we were interested in the problem of node deployment in a WSN according to several criteria that we will present in the following sections. In order to optimize the deployment, we chose one of several techniques that we will discuss in the next section. WSN can be defined as a specially appointed organization of an enormous number of nodes, which are miniature sensors equipped for sensing physical phenomena (e.g. sound, light, temperature, motion, seismic action, etc), gathering and sending information in a wireless self-sufficient way, to one or more data collection tanks called sinks [5]. Recently, this innovation plays a key role in several applications such as in PA and became an integral part in the IoTs [10]. One of the most important WSN design's complicated aspects is the node deployment in the region of interest (RoI), because it influences practically the entirety of its presentation measurements, such as connectivity, coverage, quality of service, and network lifetime. So, in order to place nodes in a WSN, there are several strategies which can be categorized in static or dynamic methods.

However, the static methods contain two classes, random and deterministic strategies, the first one aims to distribute the nodes aleatory [11], while the deterministic one aims to decide the location of WSN nodes under some given constraints in order to achieve some WSN goals [12]. Moreover, the dynamic methods are devised in two classes too; the centralized deployment uses a cluster-head to transmit the deployment algorithm to other sensors and the distributed deployment allows each sensor to use its nearby information, like distances to adjoining sensors and obstacles to run the conveying algorithm and self-convey in the upgraded positions.

In literature, the algorithms used for modeling, optimizing and solving the deployment issue, were classified in four main categories based on their mathematical methodology, Genetic Algorithms (GAs), Computational Geometry (CG), Artificial Potential Fields (APF) and Particle Swarm Optimization (PSO). The multi-objective GAs aim to improve the design of a WSN, with typically more than one organization objective [13], while the CG-based Algorithms such as the Voronoi Diagram and Delaunay Triangulation which were demonstrated valuable in

evaluating area coverage and discovering coverage holes [14]. In addition, the APF relies on motions, so they are most of time used for Mobile WSNs and conveyed nodes in an ideal manner so that to stay away from any obstacle in artificial field [15], and finally for the PSO, it focuses on artificial intelligence behavior and use a population (swarm) of search points (particles) that move stochastically in the boundaries of the search space optimization problem, in order to calculate moving position of mobile sensors [16].

Furthermore, the four categories can be used in static deterministic deployment or in dynamic centralized deployment, but only CG and APF algorithms can be used in dynamic distributed deployment. So in the case of multiple design objectives being defined, GA and PSO approaches are more qualified for conveying WSNs than CG and APF because multiple objectives can be easily determined using the objective functions of GAs and PSO algorithms [12].

As a result, in our case, we were interested in finding the best Multi-objective GAs between the Non Sorting Genetic Algorithm II (NSGA II) and Strength Pareto Evolutionary Algorithm II (SPEA II), to find the most efficient WSN statistic deterministic node deployment which best meets the constraints of coverage, over-coverage, connectivity, nodes number and nodes separating distance for an agricultural application.

To the best of our knowledge, it is true that in the literature these constraints have already been applied for WSN node deployment optimization as in the [17] study, but the novelty of our study is that they will all be used at the same time. Considering that in agriculture, each of them can have an impact on the performance of the application. For example, if the node separating distance constraint is not taken into consideration, there will be coverage gaps in the deployment plans, so the quality of the physical parameters surrogate models will not be perfect.

Both of the NSGA II and SPEA II has enabled as to find node deployment plans that responds to the 5 previous constraints at the same time. For a 100 m² plot, the SPEA II presented better results, but the NSGA II converges faster. The findings of our suggested model were good for agricultural plots of small areas, but the larger the areas, the lower the quality of the solutions of our model and required more computational skills. In addition, our document is arrange as following, the second section presents the related works, the third one explain the details of our proposed model, the fourth discusses the deployment results. Finally, we are going to end our article with some perspectives and future works.

II. RELATED WORKS

Several works in the literature have been interested in the problem of sensor nodes deployment in WSNs for different fields of use such as smart buildings, military applications, and agriculture, etc. In our study, we were interested in works that have applied GAs in their deployment models.

The GAs are the most famous family of EAs, thanks to their flexibility of application and adaptation to many kinds of complexe problems in practice [18]. In the optimization problems, the GAs evolve a set of solutions according to an already defined number of generations as presented in Figure 3, where each solution is a possible candidate for an optimum of the optimization issue. The representation of solutions in GAs family is in the form of strings of values; they are called vectors in the continuous case and bit strings in the bits case. Constantly, GAs apply

genetic operators as selection, crossover and mutation in order to guarantee diversity in the research space. In the case of WSN

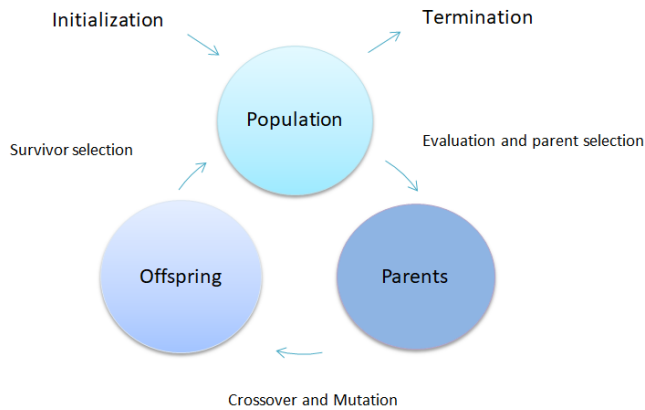


Fig. 3: General Scheme of GA.

deployment issues, where the constraints of coverage, connectivity and the network lifetime play a vital role in the performance and the functioning of WSNs [19], the GAs were widely used, because of their adaptiveness and their support of multiobjective optimization [20]. In literature, we found that Single Objective GAs (SOGAs), Bi-Objective GAs (BOGAs) and Multi-objective GAs (MOGAs) were applied to optimize the positioning of WSN nodes to response to different objectives as shown in Table 1. However, Figure 4 illustrates a comparison between the 16 works presented in Table 1 according to the number of objectives taken into account for the WSNs deployment. We found that 43.75% were SOGAs [21] [22] [17] [23] [24] [25] [26], 25% were BOGAs [27] [28] [29] [30] and 31.25% were MOGAs [31] [13] [32] [33] [34]. The difference between these three types is in the number of constraints to be taken into consideration when deploying nodes. As there are multiple parameters that need to be simultaneously optimized and may potentially conflict with each other. For example, in each WSN deployment, sensors must assure the highest coverage of the interested area with the smallest cost (sensor number). In addition, connectivity and network coverage can be maximized at the same time, but this maximization risks influencing the network lifetime. Therefore, it is remarkable that the more we decrease the number of constraints the more the GAs become more efficient in terms of calculation time, but the quality of the solutions found in the case of SOGAs and BOGAs cannot satisfy all the requirement constraints that affect the sensors locations as the MOGAs solutions.

The presented works aim all to find the optimal sensor nodes deployment according to defined constraints. However, there is a difference between them in the desired objectives. Figure 5 presents the percentage of WSN deployment constraints extracted from the previous works. According to this study, we found that the maximization of coverage and connectivity were the most desired objectives with the percentage of 75% and 50% respectively, followed by the maximization of energy efficiency objective with 25%, than the minimization of sensor nodes number with 18,75% and finally the minimization of over-coverage area with 12,5%. The cited works have proved the advantage and the possibility of applying GAs for sensor nodes deployment optimization in WSN according to single or multiple fitness functions in different fields of use, such as for

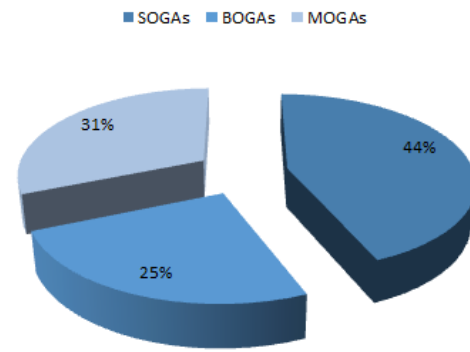


Fig. 4: Comparison of GAs types for WSN deployment.

agriculture. The choice of the GAs type to use, depends on the considered constraints number when deploying WSN. Besides, the computational time and solution qualities are also related to the constraints number. In our study, since it wasn't tested before, we aimed to compare the functioning of two variants of multi-objective GAs according to 5 constraints at the same time for an agricultural application.

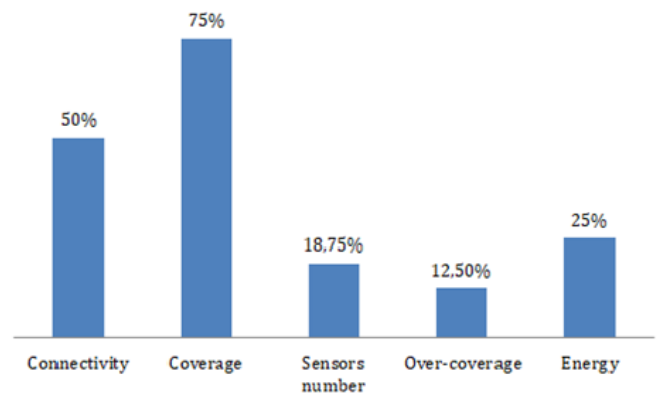


Fig. 5: Percentage of use of the different constraints in the 16 studied works.

III. PROPOSED MODEL

In nature, there are real optimization problems which are based on several criteria. These last are often contradictory and must be optimized simultaneously as in the case of our study. Therefore the optimization in this case consists in finding a vector of decisions that satisfy the 5 constraints and optimize the objective vector whose elements represent the objective functions, this is called multi-objective optimization.

The NSGA II (Deb et al. 2002 [35]) is made in such a way as to integrate the fronts obtained in the following populations, the crowding distance and the selection by tournament contribute to the obtaining of the diverse populations without losing good solutions, as illustrate the pseudo-code in Algorithm 1.

In addition, in SPEA II there is the mating choice stage, where individuals from the association of population and archive are chosen through twofold competitions. Every individual in the archive has a higher opportunity to be chosen than any population part, as presented in Algorithm 2.

Multi-objective optimization using multi-objective GAs deal with several objective functions at the same time, for this it

Table. I
GAS APPLICATIONS FOR WSN DEPLOYMENT.

Works (year)	Used Genetic Algorithm	Objectives	GA type
Hoffmann, Medina, and Wolisz. (2011) [21]	Hybrid GA with centralized sorting of base stations	Latency Minimization	SOGA
Tripathi et al. (2013) [27]	Genetic Algorithm and Genetic Programming	Coverage and network lifetime maximization	BOGA
Yoon and Kim. (2013) [22]	Genetic Algorithm	Coverage maximization with varying sensing radius of different sensors.	SOGA
Rebai et al. (2015) [28]	Genetic Algorithm	Coverage and connectivity maximization	BOGA
Gupta, Kuila, and Jana. (2016) [17]	Genetic Algorithm-based approach	Maximization of connectivity	SOGA
Gupta, Kuila, and Jana. (2017) [31]	MO-Genetic Algorithm	Maximization of coverage and connectivity and minimum number of sensor nodes	MOGA
Dai and Wang. (2017) [29]	Improved Genetic Algorithm	Maximization of connected confident information and coverage	BOGA
Benatia et al. (2017) [13]	MO Genetic Algorithm	Maximization of coverage and connectivity and minimization of node numbers and over-coverage	MOGA
Karatas. (2018) [23]	Genetic Algorithm-based scheme	Hybrid coverage of heterogeneous WSNs	SOGA
Liang and Lin. (2018) [24]	Genetic Algorithm	Maximization of coverage strategy in MWSN	SOGA
Panhwar et al. (2018) [25]	Genetic Algorithm	Distance based energy optimization	SOGA
Perez. (2018) [30]	Non-Sorting Genetic Algorithm 2 (NSGA II) with local search heuristics	Total number of devices used in the placement and total energy dissipated by the placement	BOGA
Hanh et al. (2019) [26]	Multi-Island Genetic Algorithm (MIGA) and Virtual Force Algorithm (VFA)	Maximization of area coverage	SOGA
Harizan and Kuila. (2019) [32]	Improved Genetic Algorithm	Maximization of connectivity and coverage and energy minimization	MOGA
ZainEldin et al. (2020) [33]	An improved dynamic deployment technique based on genetic algorithm (IDDT-GA)	Coverage Maximization with the lowest number of nodes and minimizing overlapping area	MOGA
Pal et al. (2021) [34]	Non Sorting Genetic Algorithm II	Strength of received signal, coverage and over-coverage	MOGA

Algorithm 1 Pseudo-code of NSGA II.

```

 $t \leftarrow 0$ 
 $g \leftarrow N$ 
 $s \leftarrow K$ 
Initialize population of chromosomes  $P(g=0)$  randomly
size  $K$ 
Evaluate the  $P(g=0)$  objectives according to 5 constraints
Assign Rank level based on Pareto sorting
Generate child population
Select children with binary tournament
Crossover  $P(g=0)$ 
Mutate  $P(g=0)$ 
while  $g \leq N$  do
   $gn \leftarrow g + 1$ 
  for each parent and child in population do
    Assign Rank level based on Pareto sorting
    Generate sets of non-dominated solutions
    Determine crowding distance
    Loop inside by adding solutions to next generation start-
    ing from the first front to  $K$  size
  end for
  Select on the lower front with higher crowding distance
  Generate child population
  Select  $P(gn)$  from  $P(gn-1)$  with binary tournament
  Crossover  $P(gn)$ 
  Mutate  $P(gn)$ 
  Evaluate  $P(gn)$ 
end while

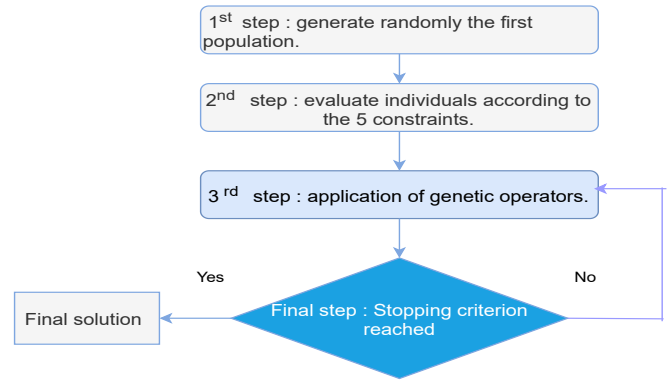
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Algorithm 2 Pseudo-code of SPEA II.

```

 $t \leftarrow 0$ 
 $g \leftarrow N$ 
 $s \leftarrow K$ 
Initialize population of chromosomes  $P(g=0)$  randomly
size  $K$ 
Create empty external archive  $V$ 
while  $g \leq N$  do
   $gn \leftarrow g + 1$ 
  Evaluate the  $P(g=gn)$  and the archive  $V$  individuals
  according to 5 constraints
  Copy all non-dominated solutions to archive  $V$ 
  if capacity of  $V$  is exceeded then
    remove individuals from  $V$  with truncation operator
  else
    Fill  $V$  with dominated solutions in  $P(gn)$ 
  end if
  Select  $P(gn)$  from  $P(gn-1)$  with binary tournament
  Crossover  $P(gn)$ 
  Mutate  $P(gn)$ 
end while

```

**Fig. 6:** Our model general evolutionary diagram.

uses the notion of optimal compromise (A solution that satisfies several functions at the same time). The solutions are separated based on the notion of dominance in the sense of Pareto. One solution may be better than another on some goals and less good on others. So there is usually no single solution that simultaneously provides the optimal solution for all of the goals. Therefore, in order to establish a comparison between the SPEA II and NSGA II functioning for an agricultural application, we had followed the same general evolutionary diagram as presented in Figure 6. Each step will be explained in next subsections.

A. First step

The modeling of the deployment space is done by dividing it into grids with cells of 1 square meter in size. By applying the binary representation to formulate the chromosomes of the different populations, of which the 1 represents the presence of a node in the cell, and the 0 signifies its absence. We made sure that the first population of individuals is generated randomly, in order to guarantee diversity in the solutions and to avoid at the same time falling on local optima.

B. Second step

Our Node deployment problem is considered as a multi-criteria problem, the WSN node network have to minimize the number of sensors used, areas of over-coverage and the nodes separating distance, and maximize coverage and connectivity at the same time [36]. After the generation of the initial population, each individual thereafter will be evaluated by the different 5 objective functions, and we will have as results evaluation vectors of 5 dimensions. Each criterion was therefore represented by a mathematical function as shown in Table 2.

C. Third step

In this step, genetic operators will be applied, the first is the selection of individuals who will participate in the offspring of new individuals using the rotation method. Individuals who have better fitness values will be selected, and in case two parents have a similar evaluation, another metric will be applied, which is the Crowding distance [37].

Once the parents are selected, we move on to the crossing operator with a certain probability, the latter defines the proportion of parents in the population that will be used by a crossing operator. in our case we have chosen the crossing at a single point.

After the crossing is applied, we move on to the mutation which

Table. II
MATHEMATICAL REPRESENTATIONS OF EVALUATION CRITERIA.

Evaluation Criteria	Mathematical function	Symbols interpretation
Sensor nodes number rate	$N_{sr} = \frac{N_s}{L \times l} \times 100 \quad (1)$	N_s represents the number of sensor nodes L , the length of the plot l , the width of the plot
Sensor nodes separating distance	$Dist = \sum_{N_s} \sqrt{(x_i - x_k)^2 + (y_i - y_k)^2} \quad (2)$	(x_k, y_k) , the coordinates of the K sensor, (x_i, y_i) , the coordinates of other sensors
Coverage rate	$C_{vr} = \frac{\sum_x \sum_y Det(x, y)}{L \times l} \times 100 \quad (3)$	$Det(x, y)$, detection coverage matrix
Over-coverage rate	$O_v - C_{vr} = \frac{\sum_x \sum_y O - Det(x, y)}{L \times l} \times 100 \quad (4)$	$O - Det(x, y)$, detection over-coverage matrix
Communication rate	$C_{on} = \frac{\sum_x \sum_y C_n(x, y)}{L \times l} \times 100 \quad (5)$	$C_n(x, y)$, connectivity matrix

allows us to introduce a disturbance into the solution with a mutation rate.

D. Final step

Given that the constraints of our deployment are very contradictory, for example we want to maximize coverage while minimizing the number of individuals. We don't already have a deployment model that we want to achieve. Therefore, the stopping criterion of our algorithms will be to set a generation number that allows the algorithms to properly explore the search space while minimizing the calculation time. Knowing that the number of generations is linked to the size of the agricultural plot.

IV. COMPARISON BETWEEN SPEA II AND NSGA II

A. Definition of MOGA hyper-parameters

In order to establish a comparison between the two variants of MOGA which are NSGA II and SPEA II, we made experimental tests to be able to define the good hyper-parameters of MOGA like the number of generations and the size of the populations. However, these two parameters depend in the first place on the size of the agricultural plot, the larger it is the higher these two parameters are to be able to explore all the research space. Thereby, we chose to work on a plot of dimensions 10 by 10 m². therefore our chromosomes will be 100 bits in length.

In addition, to define the hyper-parameters, we have varied the size of the population from 10 to 80 with a step of 10 and generations from 1 to 80. As a result, we found that the execution time is proportional to the population size as shown in Figure 7 and that our model starts reaching a global optimum with 60 individuals in the population at the 70 generation, and reaches it

with 70 and 80 individuals as shown also in Figure 7. Therefore, we decided to fix the population size at 70 individual in order to guarantee the convergence of our model with the minimum execution time, and the generation number at 100 in order to give more opportunity to our model to explore the research space.

B. Results and discussion

After having defined the different hyper-parameters of the MOGA, we evaluated the functioning of NSGA II and SPEA II for a 100 m² plot, and we evaluated their performance through comparing the fitness value and time efficiency of the two MOGA optimization algorithms. The two algorithms were implemented using Python, and were executed on a PC with an Intel Core i5-4300U, 2.50 GHz CPU and 4,00 GB RAM in order to have a fairly comparison between them. We ended up finding that both of them can give us good results in terms of fitness score, but it turns out that the non-dominated solutions obtained by SPEA II are better than those of NSGA-II in terms of fitness score. Figure 8 shows us the evolution of the 5 objective functions compared to the generation for the two algorithms. In addition, compared to the quality of the solution, the optimal solution presented by the SPEA II offered us better rates of connectivity and over coverage, while minimizing the number of deployed sensor nodes and the distance between the different sensor nodes, while the NSGA II was better in only one objective which is coverage rate, as shown in Figure 9. And according to the execution time, as presented in Table 3, the NSGA II converges to optimal solutions a little faster than the SPEA II. As a result we can conclude that although SPEA II gives better results, NSGA II still provides acceptable results, and still manages to be effective and faster. Figure 10, shows the plan of deployment of the Non-dominated solution of SPEA II.

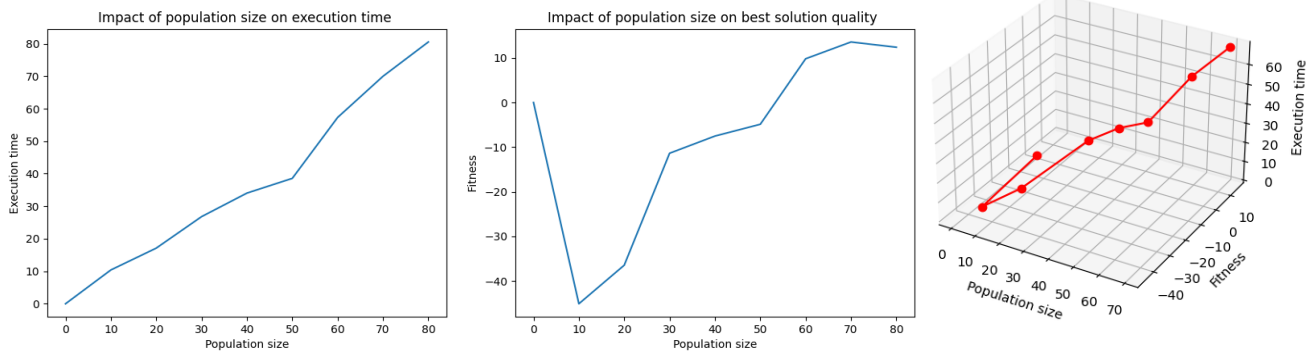


Fig. 7: Impact of population size on best solution quality and execution time.

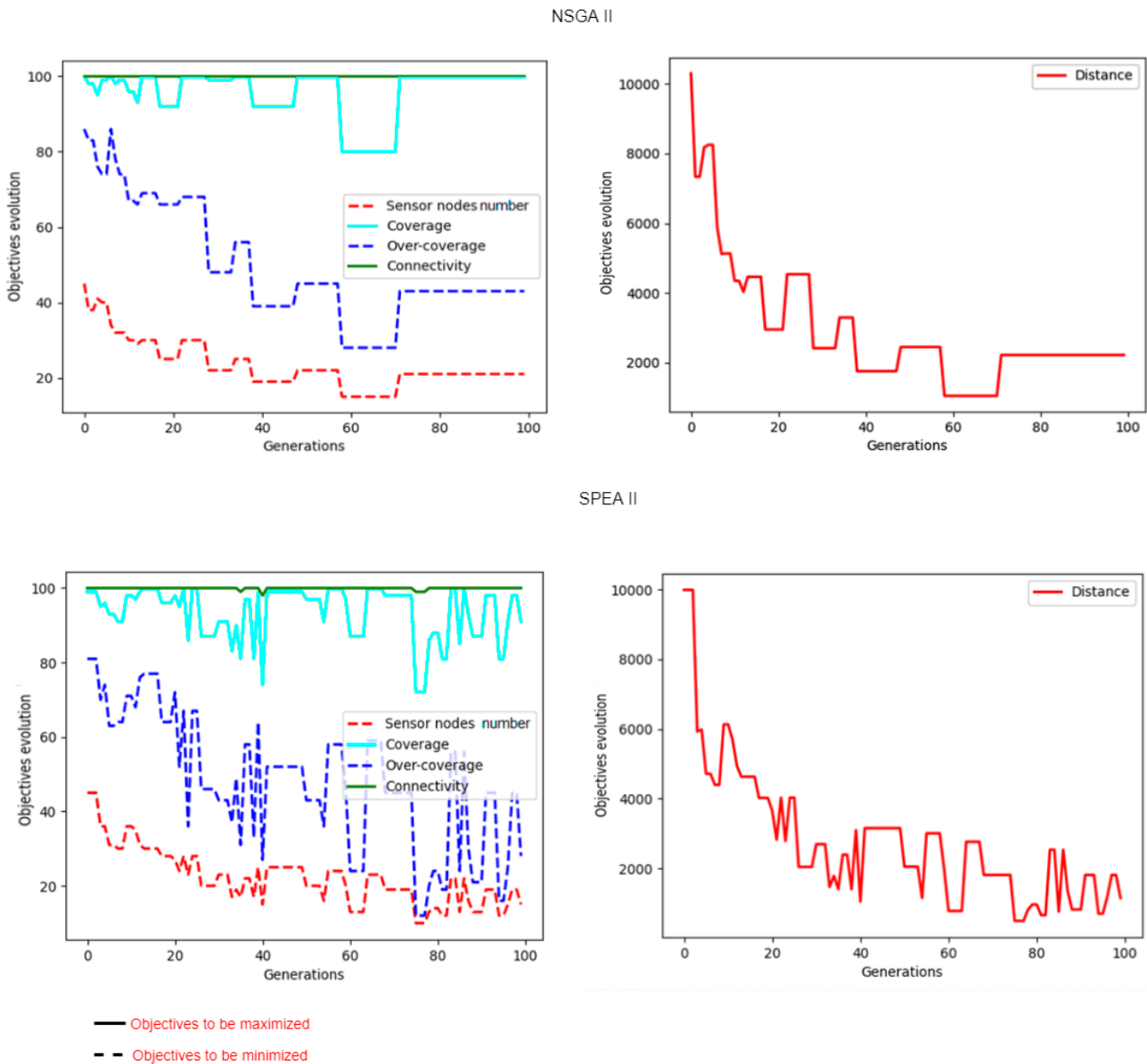


Fig. 8: Comparison between NSGA II and SPEA II according to their objectives functions evolution.

C. Results Generalization

Our proposed model has proven its performance in the deployment of nodes for an agricultural zone according to the 5 constraints described above. But however, we have found that it shows a drastic increase in the computation time, when we increase the size of the agricultural plot as shown in Table 4 (

For 400 m² the execution time was for about 40 min). The results presented in table 5 are obtained by the execution of the model for different plots of different sizes using the MOGA, SPEA II algorithm. As it presented better results in our tests, the phenomenon of the increase in time can be explained by the increase in the size of the individuals who build the popu-

Table. III

COMPARISON BETWEEN SPEA II AND NSGA II DEPENDING ON THE SOLUTIONS QUALITY AND THE EXECUTION TIME (IN SECONDS).

MOGAs	Execution time (s)	Sensors number (%)	Coverage (%)	Over-coverage (%)	Connectivity (%)	Distance (m)	The weighted sum of the fitness objectives
SPEA II	150.54	15.0	93.0	24.0	100.0	1098.98	-188.99
NSGA II	139.12	21.0	100.0	43.0	100.0	2214.94	-415.78

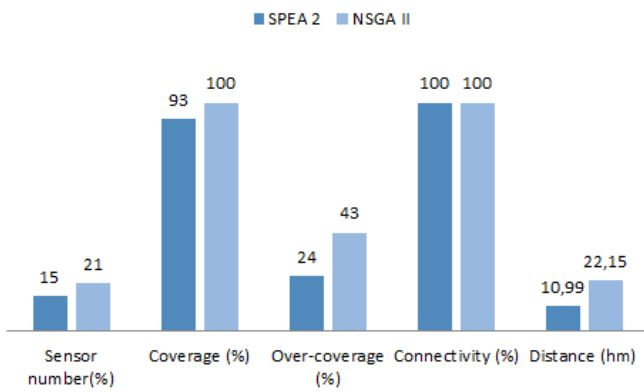


Fig. 9: Comparison between NSGA II and SPEA II non-dominated solutions according to their objective function.

lation because the execution time is proportional to the size of the plot. Thereby, the growing size of individuals will affect the objective function evaluation computation time (Figure 11), which is known as the most time-consuming phase in the GA process [38]. And considering the quality of the solutions, it is essential that if the size of the plot is large, the size of the population and the number of generations must increase more and more in order to better explore the research space. As shown in Table 4, the evaluation of objective functions deteriorates with increasing plot size. Because in our tests, we kept the same hyper-parameters (population size, number of generations) in all the tests. So, we can compare them in a fair way.

V. CONCLUSION

The application of Multi-objective GAs for WSN node deployment optimization in agriculture using NSGAI and SPEA II showed good results regarding the rates of the different constraints of coverage, over-coverage, connectivity, sensor nodes number, and the nodes separating distance.

However, we found that the SPEA II was more efficient but also more demanding in terms of calculation time compared with the NSGA II. The execution time of the two algorithms increases considerably and their solutions quality decreases too, with the increase of the size of agricultural plots. Therefore, as future works, we propose to use a fitness approximation tool. This proposition comes from the fact the evaluation step is the most expensive in terms of calculation time and requires very impor-

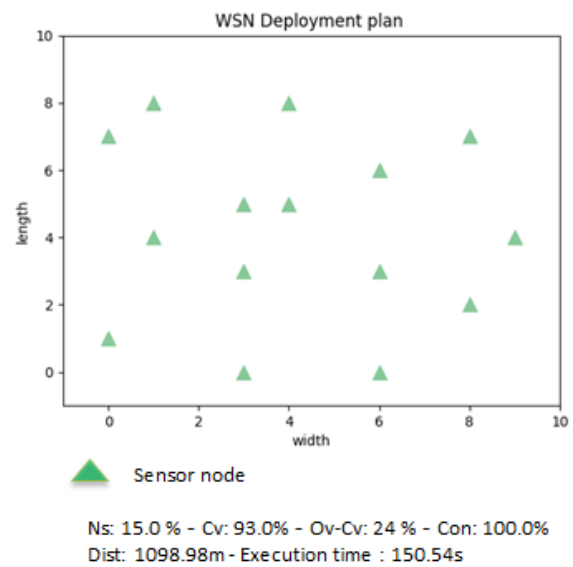


Fig. 10: Sensor node deployment plan for an agricultural plot of 100 m² size.

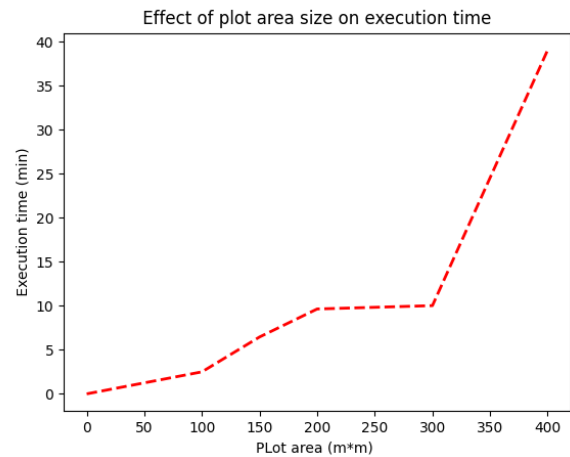


Fig. 11: Effect of plot area size on execution time.

tant computational skills. Besides, we can take advantage of the parallelization nature of GAs, to implement our algorithms

Table. IV

THE RELATION BETWEEN THE INCREASE IN THE SIZE OF THE PLOTS, THE EXECUTION TIME AND THE QUALITY OF THE SOLUTIONS FOR THE SAME HYPER-PARAMETERS OF THE MOGA .

Plot dimensions (m ²)	Execution time (s)	Evaluation of objective functions (N_{sr} , C_{vr} , $O_v - C_{vr}$, C_{on} , Dist)
10 × 10	150.54 (2.50 min)	(15.0, 93.0, 24.0, 100.0, 1098.98)
10 × 15	387.72 (6.46 min)	(19.33, 99.33, 43.33, 100.0, 5520.67)
10 × 20	578.63 (9.64 min)	(22.5, 93.5, 54.0, 100.0, 15584.02)
15 × 20	601.33 (10.02 min)	(22.0, 94.0, 50.5, 99.5, 14793.52)
20 × 20	2342.59 (39,04 min)	(30.5, 99.75, 77.25, 100.0, 151859.30)

on specific hardware computation platforms such as the Multi-Processor System-on-Chip (MPSoC), Field Programmable Gate Array (FPGAs) accelerators, Multicore or Manycore systems and Graphics Processor Unit (GPU).

VI. COMPLIANCE WITH ETHICAL STANDARDS

The authors have no relevant financial or non-financial interests to disclose.

The authors have no conflicts of interest to declare that are relevant to the content of this article.

All authors certify that they have no affiliations with or involvement in any organization or entity with any financial interest or non-financial interest in the subject matter or materials discussed in this manuscript.

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Performance Evaluation of Multilayer Perceptron Neural Network and Adaptive Neuro-Fuzzy Inference Systems for Reservoir Operation Optimization: a Case Study of Cheffia Reservoir, Algeria

Noureddine MEZENNER, Abdelmalek BERMAD, Tarik BENKACI, and Noureddine DECHEMI

Abstract– Artificial Intelligence based prediction has wide applications, including hydrology, water resources management and particularly reservoir operation. Thus, two black box models based on Artificial Neural Networks and Fuzzy Logic methods are implemented and tested in forecasting reservoir operation; the first is a Multilayer Perceptron Neural Network and the second is an Adaptive Neuro-Fuzzy Inference System combining the two methods. The developed models consist of predicting evaporation, inflows and reservoir storage from their historical records and that, with aim of providing the best fit between predicted and observed values and of improving operating rules on storage and releases. The performance of achieved results demonstrated the pertinence of Artificial Neural Networks and fuzzy Logic methods in predicting cyclical state variables, such as evaporation and storage, while the prediction of inflows to reservoir generally gave better results compared to other research works available in open literature on one hand. On other hand, it is deduced from testing different prediction models that these methods are unable to predict random variables.

Keywords– Modelling, Prediction, Reservoir, Inflow, Artificial Neural Networks, Fuzzy logic.

NOMENCLATURE

ANFIS	Adaptive Neuro-Fuzzy Inference System
ANN	Artificial Neural Networks
CANFIS	Co-Active Neurofuzzy Inference System
CC	Correlation Coefficient
DENFIS System	Dynamic Evolving Neural-Fuzzy Inference System
LR	Learning Rate
MAE	Mean Average Error
MCM	Million Cubic Meter
MLPNN	Multilayer Perceptron Neural Network
NC	Nash Coefficient
NN	Neurons Number
RMSE	Root Mean Square Error
RN	Rules Number

I. INTRODUCTION

Water is a vital natural resource. Though it is renewable, it is quantitatively limited and its distribution is stochastic in a semiarid regions. It can be stored in dam reservoirs during wet periods of precipitations for satisfying water demand during low water periods, that why reservoir releases regularization remains a difficult task in the conditions of uneven spatiotemporal distribution of rainfalls accentuated by the

climate change. Reservoirs operation is subject to operating rules associated with plant objectives that impose a minimal or maximal bound on the total flow able to be released from reservoir. These operating rules are considered as function of inflows, reservoir storage and releases.

Generally, operating inflows, as a state random variable, is a difficult task. Particularly when they are overexploited or underexploited for making decision about releases, whose estimation and allocation between users constitutes the objective of reservoir system exploitation and that, through taken options at the beginning of each decision period for satisfying as far as possible target objectives imposed to reservoir system. Depending on inflows, the reservoir releases are conditional on the storage and they are determined simply to satisfy water demand and to meet their target storage by insuring minimum flow requirements. Their systematic evaluation is subject to uncertainties. With aim of reducing the uncertainty and improving reservoir operation, different models exist for describing and analyzing the behavior of reservoir releases operation through the assessment of various scenarios. Several tools are used in matter of reservoir operation modelling; from dynamic programming to linear and nonlinear programming, to stochastic dynamic programming and to artificial intelligence with artificial neural networks (ANN) and fuzzy logics that emerged as viable tools for reservoir operation modelling and forecasting (Labadie, 2004) [1]. Likewise, multiple models based on fuzzy logics have been carried out in hydrology and water resources management. Recently, Panigrahi and Mujumdar (2000) [2] attempted to provide an implementable single purpose reservoir operation policy, using fuzzy logic methods. They found that the latter might remain limited to a single reservoir operation, because of the curse of dimensionality that is observed in a fuzzy rule based model. As the number of fuzzy sets increases, the dimensionality of the problem grows multiplicatively. What has been pointed out by Russell and Campbell (1996) [3] and it implies that the fuzzy

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logic approach is not an alternative to the more conventional optimization techniques.

Adaptive NeuroFuzzy Inference System (ANFIS) method, merging ANN and fuzzy logics, has been used for reservoir operation forecasting. For instance, Chang and Chang (2001) [4] combine Genetic Algorithms (GA) and ANFIS methods for determining the optimal real time reservoir release operation. Learning from the input-output patterns obtained from GA, they estimate the reservoir release in predefined conditions using ANFIS model. First, their work consists of using the subtractive fuzzy clustering to reduce the number of input rules; as the number of variables increases, the number of parameters of membership functions increases highly. As to the second problem, the objective function does not express the reservoir operation characteristics. Several optimal solutions have the same minimum value in the objective function of reservoir operation. From another side, El Shafie and all (2006) [5] developed an ANFIS model with a high accuracy in forecasting average inflow events of the Nile River at Aswan High Dam, especially for extreme events, given a high uncertainty in future inflows, making of developing optimal release policies of a multi-purpose reservoir a complex process. On the contrary, Chang and all (2014) [6] explored the effectiveness of multiple sources of rainfall characterized by a complex temporal heterogeneity, and they found if the ANFIS model is fed with the assimilated precipitation, it provided reliable and precise rainfall forecasts and can be a great help on floods warnings. On the other hand, a neuronal model based on the radial function using inflows and releases as inputs for predicting water losses, might give better results, depending generally on the data size and the neural model parameters (Samad and all, 2015) [7]. In the same field of reservoir operation modeling, Kisi and al (2017) [8] demonstrated the accuracy of the dynamic evolving neural-fuzzy inference system (DENFIS) in modeling pan evaporation compared with the classic ANFIS model. The latter differs from DENFIS new model by using an evolving clustering method, a triangular membership functions as fuzzy sets and an alternative weighting scheme for local learning of consequent parameters. Salih and all (2019) [9] applied ANFIS and Co-Active Neurofuzzy Inference System (CANFIS) models successfully in predicting monthly reservoir evaporation; a good accuracy is reached using only two inputs of air temperature and relative humidity. Alquraish and al (2021) [10] in predicting inflows to reservoir, succeeded in improving the forecasting performance of ANFIS model using an add-in optimization algorithm based Genetic Algorithms methods. In the present case of Cheffia reservoir, a performance evaluation of artificial neural networks and fuzzy logic methods is made in predicting evaporation, inflows and reservoir storage in order to define appropriate operating rules on releases.

II. MATERIAL AND METHODS

1. Study Area

Cheffia dam reservoir (Fig.1) is set up on Bounamoussa River, situated in the Northern East of Algeria. Watershed reservoir covers an area of 575 km² and received an average monthly rainfall of 61 mm with a standard deviation of 70 mm. Reservoir is intended to satisfy agricultural needs in water through releases modulated from spring to autumn and reinforced in a summer for the irrigation, and likewise to supply the region in drinking water. Reservoir storage is 180,8 MCM. Observed data covers the period of forty years and are collected from the reservoir operating budget (1979-2019). Five variables are assumed as input state variables, namely reservoir storage, inflows to reservoir, evaporation and seepage, and one as an output decision variable, namely releases. The maximum and the minimum of the reservoir storage are 202 MCM and 30

MCM corresponding respectively to the forced storage volume and reservoir dead volume or ecological volume. By the monthly inter annual average, the water consumption and evaporation represent respectively and nearly 59% and 12% of the inflows.

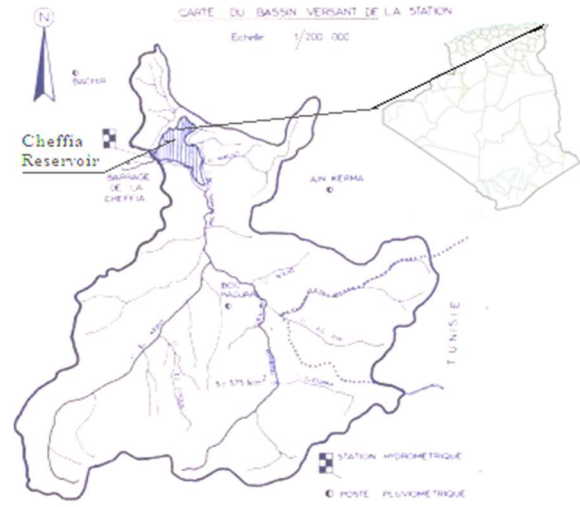


Fig.1: Map of Cheffia Reservoir and its watershed

The operating mass balance of Cheffia reservoir is derived from a continuity equation as:

$$S(t+1) = S(t) + I(t) - E(t) - WC(t) - SEEP(t) \text{ (MCM)} \quad (1)$$

Where $S(t+1)$ is storage at time step $t+1$; $S(t)$ is storage at time step t ; $I(t)$ is inflow at time step t ; $E(t)$ is evaporation at time step t ; $WC(t)$ is water consumption at time step t and $SEEP(t)$ is water seepage.

The permissible water storage is given as: $S_{min} \leq S(t) \leq S_{max}$. Through the continuity equation of the operating mass balance, the natural processes such as inflows and evaporation are the preponderant inputs to the system; they are uncertain events estimated systematically. Furthermore, the releases are conditional on the storage that depends on the inflows characterized by the randomness.

2. Architecture of MLPNN model

The objective of prediction consist in providing manager a decision making tool for a rational water management. Thus, the adopted MLPNN model is a conventional feedforward neural network with three layers (Rumelhart and all, 1986) [11].

The supervised learning is implemented iteratively; at each iteration, the synapses weights and the networks bias are modified using the gradient descent method, a criteria in minimizing the quadratic error is defined as the gap between the observed and the predicted activated functions, expressed as follows:

$$E = \frac{1}{2} \sum_{i=1}^n (y_i - \hat{y}_i)^2 \quad (2)$$

Equation (2) is the reduced form of the mean squared error E or usual function minimized in least squares regression and where n is the patterns size, y_i represents the observed activated function of output neuron i and $\hat{y}_i$ is the predicted activation function of that neuron. Once a network converges to a solution, it is then able of classifying each unknown input pattern with other patterns that are close to it in terms of the same distinguishing features. From the mathematical point of view, MLPNN may be considered a multivariate nonlinear

nonparametric statistical method (White, 1989, Ripley, 1993) [12] [13]. A typical MLPNN with five hidden layers containing M neurons is based on the following equation:

$$\hat{y}_i = f\left(\sum_{j=1}^n w_{ij}x_j + e_j\right) \quad (3)$$

Where $\hat{y}_i$ is the output value of the neuron, expressed as a function f of the sum of the weighted inputs of the hidden neurone j and the relating bias. N is the total number of input neurones, w_{ij} is the weight from input neurone i to the hidden neurone j , x_i is a value of the i th pattern input and e_j is the bias (or threshold) for neuron j .

3. Architecture of ANFIS model

An adaptive network, as its name implies, is a network structure consisting of nodes and directional links through which the nodes are connected. Moreover, parts or all nodes are adaptive, which means each output of these nodes depends on the parameters belonging to this node and the learning rule specifies how these parameters should be changed to minimize a prescribed systematic error. ANFIS is a multilayer feed-forward network where each node performs a particular function on incoming signals. Both square and circle node symbols are used to represent different properties of adaptive learning. To perform desired input–output characteristics, adaptive learning parameters are updated based on gradient learning rules (Jang, 1993) [5]. For simplicity, we assume the fuzzy inference system under consideration has two inputs, x and y , and one output z .

Suppose that the rule base involves two fuzzy if–then rules of Takagi and Sugeno's type:

$$\text{Rule1} = \text{If } (x \text{ is } A_1) \text{ and } (y \text{ is } B_1) \text{ Then } (f_1 = p_1x + q_1y + r_1) \quad (4)$$

$$\text{Rule1} = \text{If } (x \text{ is } A_2) \text{ and } (y \text{ is } B_2) \text{ Then } (f_2 = p_2x + q_2y + r_2) \quad (5)$$

Where x and y are inputs, A_i and B_i are the fuzzy sets, f_i are the outputs within the fuzzy region specified by the fuzzy rule, p_i , q_i and r_i are the design parameters that are determined during the training process. The ANFIS architecture to implement these two rules is shown in Fig.2, in which a circle indicates a fixed node, whereas a square indicates an adaptive node.

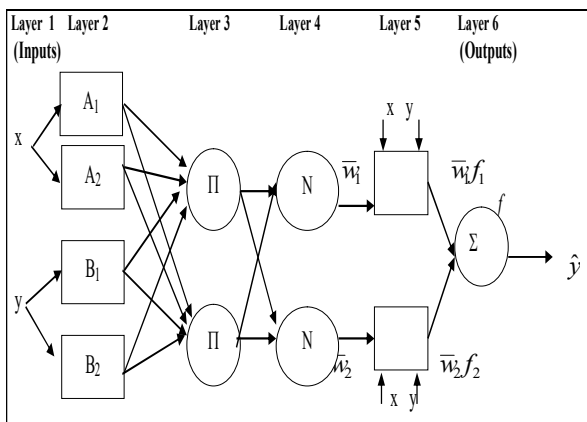


Fig.2: ANFIS Model

Layer 1: this layer transmits data inputs to the fuzzification layer.

Layer 2: Each node of this layer is a square node with a function expressed by:

$$O_i^2 = \mu_{A_i}(x) \quad (6)$$

Where x is input to node i , and A_i is the linguistic state associated with its function, in others words, O_i^2 is the pertaining degree of x to A_i in Jang model; membership functions are Gaussian.

Layer 3: Each node i in this layer is a circular node labelled Π that induces as outputs the product of its inputs; this product represents the activation degree of a rule expressed by:

$$O_i^3 = w_i = \mu_{A_i}(x)\mu_{B_i}(y) \quad (7)$$

In others words, each node of this layer corresponds to a Sugeno fuzzy rule; it receives neurons outputs of fuzzification and computes its activation.

Layer 4: In this layer, the nodes are adaptive nodes. The output of each node in this layer is simply the product of the normalized activation degree of a rule i given by:

$$O_i^4 = \bar{w}_i f_i = \left(\frac{w_i}{(w_1 + w_2)} \right) \quad (8)$$

Layer 5: each node in this layer is a square node with a function given by:

$$O_i^5 = \bar{w}_i f_i = \bar{w}_i(p_i x + q_i y + r_i) \quad (9)$$

Where $\bar{w}_i$ is the output of Layer 3 and $(\{p_i, q_i, r_i\})$, called consequent parameters, are the output parameters set of this node (Jang, 1993) [14]. Each neuron i of this layer is relied with a corresponding normalization neuron and to initial network inputs.

Layer 6: the single node of this layer is a circular node that sums the node's outputs in the previous layer to be the outputs of the whole network given by:

$$O_i^6 = \sum_{i=1} \bar{w}_i f_i \quad (10)$$

The output of the ANFIS is calculated by employing the consequent parameters found in the forward pass. The output error is used to adapt the premise parameters using a standard backpropagation algorithm. It has been proven that this hybrid algorithm is highly efficient in training the ANFIS (Jang, 1993)[14].

4. Performance Metrics For Models Calibration and Validation

a. Correlation coefficient

$$CC = \frac{\sigma_{xy}}{\sigma_x \sigma_y} \quad (11)$$

Where σ_x : is the standard deviation of the observed values ;
 σ_y : is the standard deviation of the predicted values;
 σ_{xy} : is the standard deviation of the observed and predicted values. $-1 \leq CC \leq 1$

b. Root Mean Square Error

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_i - y_i)^2} \quad (12)$$

When RMSE converges to zero represents the perfect fit

c. *Mean Average Error*

$$MAE = \frac{1}{N} \sum_{i=1}^N |x_i - y_i| \quad (13)$$

Others metrics are important for testing or validating the model, they're as follows:

d. *Nash Coefficient(NC)*

It is expressed as follows:

$$NC = \left[1 - \frac{\sum_i^n (x_i - \hat{y}_i)^2}{\sum_i^n (x_i - \bar{x})^2} \right] * 100 \quad (14)$$

x_i : Observed values;

$\hat{y}_i$: Predicted values;

$\bar{x}$: Average of observed values.

This metric is first used for deeming a model operating by episodes (Nash and al, O'Connel et. al, 1970) [15] [16]. It expresses the variance percentage of the measured series, explained by the model.

e. *The Biased Average*

The biased average is the difference between the averages of the observed and the predicted values (Legates and all, 1999) [17] given as follows:

$$\{B\} = \bar{y}_i - \bar{x}_i \quad (15)$$

$\bar{y}_i$: Average of predicted values;

$\bar{x}_i$: Average of observed values.

When $\{B\}$ converges to zero, the averages coincide.

f. *Variation index*

The validation of the used model may be completed by comparing the variation coefficients of the observed and the predicted values.

$$I = \frac{cv_y}{cv_x} \quad (16)$$

cv_y : Variation coefficient of the predicted values;

cv_x : Variation coefficient of the observed values.

If the variation index I is near to unity, the prediction is perfect.

III. RESULTS DISCUSSION

The reservoir operation efficiency depends on the decision making on releases through reservoir operating rules, developed from an accurate knowledge of preponderant state variables, such as evaporation, inflows to reservoir and storage, which they should be closely linked. For assessing the prediction model of these parameters, the period of the model testing or validation is three years (2015-2018). The adopted model is assessed according to neurons number, lag time, learning rate and validation. The prediction of state variables is expressed by the following equation:

$$X_t = a_0 + a_1 X_{t-1} + a_2 X_{t-2} + a_3 X_{t-3} + a_4 X_{t-4} + a_5 X_{t-5} + a_6 X_{t-6} \quad (17)$$

Where X_t : predicted values at time t with a lag time of t-2, t-4 and t-6.

t: Time step (month), $a_0, a_1, \dots, a_6$: Coefficients.

The adopted neural model « MLPNN » has four options of the neurons number (NN) corresponding to five, ten, twenty and one hundred, respectively, the lag time is t-2, t-4 and t-6 and a learning rate (LR) is varying from 10% to 90%. As to ANFIS model; it includes three options corresponding to a rules

number (RN = 2), a learning rate (LR= [10%: 90%]) and a lag time (t-2 t-6).

1. Evaporation prediction

Such as a cyclical preponderant variable acting on storage; its significance increases from a semiarid climate to arid. In the case of Cheffia reservoir situated in the semiarid region, by a monthly inter annual average, it is 0,579 MMC representing 12% of inflows to reservoir.

The achieved predicted values are better given the remarkable performance metrics whether in a calibration or validation phase (Table.I); the best model MLPNN which gives the best predicted values is the third model (Model-3) with a neurons number, a learning rate and a lag time equal to 5, 90% and t-4, respectively.

Thus, the model validation has given the best correlation between observed and predicted values of evaporation, whose CC, RMSE, MSE, NC, biased average {B} and variation index (I) are 0,990, 0,003, 0,001, 89%, 0,27 and 1, respectively. Fig.3, 4 and 5 show the best fit between observed and predicted evaporation values.

Table I
Performance metrics of the evaporation prediction using MLPNN model. (C: Calibration, V: Validation)

		Lag time	NN	LR %	CC	RMSE	MSE	NC	{B}	I
Model -1	C	t-2	5	50	0,815	0,016	0,011			
		t-2	10	50	0,805	0,021	0,026			
		t-2	10	50	0,740	0,020	0,019			
	V	t-2	5	50	0,926	0,012	0,021	83%	0,031	1,007
		t-2	10	50	0,901	0,001	0,001	80%	0,027	0,988
		t-2	10	50	0,943	0,003	0,002	88%	0,021	1,040
Model -2	C	t-4	5	50	0,889	0,013	0,004			
		t-4	10	50	0,894	0,015	0,016			
		t-4	20	50	0,881	0,015	0,009			
	V	t-4	5	50	0,957	0,009	0,008	86%	0,013	0,922
		t-4	10	50	0,940	0,003	0,002	84%	0,013	1,018
		t-4	20	50	0,953	0,004	0,003	85%	0,005	0,933
Model -3	C	t-4	5	90	0,944	0,013	0,026			
		t-4	10	90	0,944	0,014	0,014			
		t-4	20	90	0,927	0,015	0,009			
		t-4	100	90	0,844	0,021	0,012			
	V	t-4	5	90	0,990	0,003	0,001	89%	0,027	1,003
		t-4	10	90	0,962	0,005	0,001	87%	0,013	0,948
		t-4	20	90	0,940	0,042	0,004	83%	0,005	0,962
		t-4	100	90	0,955	0,027	0,003	72%	0,010	0,940
Model -4	C	t-6	5	90	0,914	0,013	0,026			
		t-6	10	90	0,895	0,014	0,014			
		t-6	20	90	0,889	0,015	0,009			
	V	t-6	5	90	0,932	0,003	0,001	89%	0,027	1,003
		t-6	10	90	0,967	0,005	0,001	88%	0,013	0,948
		t-6	20	90	0,932	0,042	0,004	86%	0,005	0,962

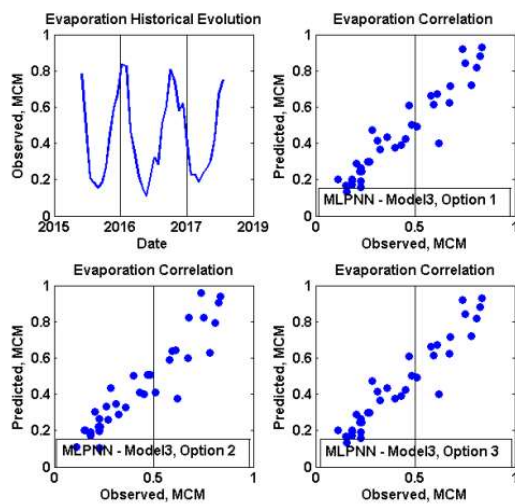


Fig.3 : Correlation of the observed and predicted evaporation

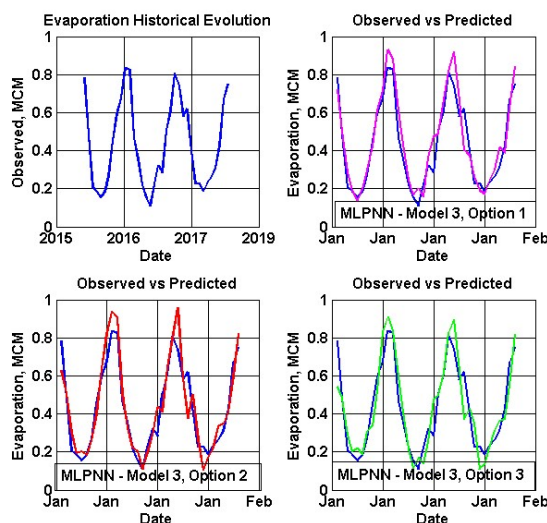


Fig.4: Confrontation of the observed and predicted evaporation

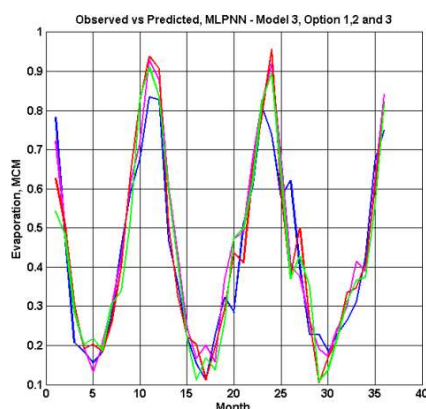


Fig.5: Observed vs predicted evaporation

As to ANFIS model, the best-adopted model involves a rules number (RN), a learning rate (LR) and a lag time of 2, 90%, and t-4, respectively. For assessing the predicted values of evaporation relative to observed ones, obtained performance metrics of CC, RMSE, MSE, NC, {B} and I corresponding to 0,972; 0,004 ; 0,006 ; 93% ; 0,009 and 0,850, respectively. The achieved performance metrics are greatly promising, particularly a Nash coefficient (NC) of 93% higher than 89% resulted from MLPNN model (Table.II). Moreover, by comparing the achieved results, it is clear that both MLPNN and ANFIS models give better outputs. However, ANFIS

model is more performing and accurate, as it is shown in the Fig.6, 7 and 8, where the predicted values of evaporation obtained using ANFIS model with two fuzzy rules are more performing than the predicted ones resulted from MLPNN model.

Table II

Performance metrics of the evaporation prediction using ANFIS model (C: Calibration, V: Validation)

	Lag time	NN/ RN	LR %	CC	RMS E	MSE	NC	{B}	I
Model-I	C	2	2	50	0,826	0,016			
		2	2	90	0,843	0,019			
		4	2	90	0,816	0,021			
	V	2	2	50	0,947	0,010	0,011	89 %	0,027
		2	2	90	0,947	0,004	0,002	88 %	0,033
		4	2	90	0,972	0,004	0,006	93 %	0,009

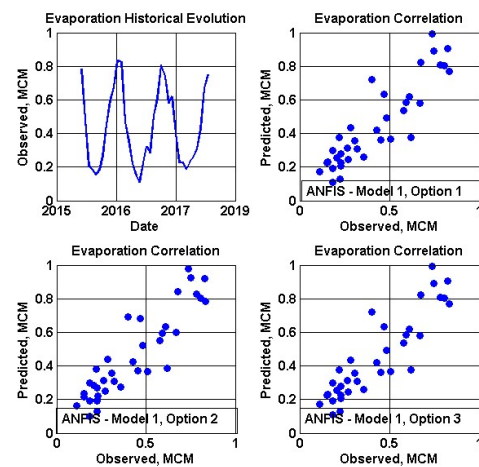


Fig.6: Correlation of the observed and predicted evaporation.

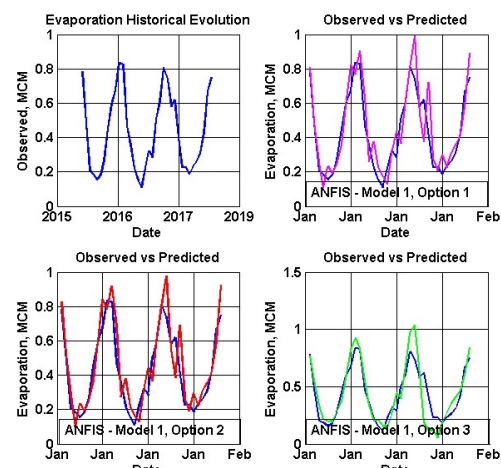


Fig.7: Confrontation of the observed and predicted evaporation.

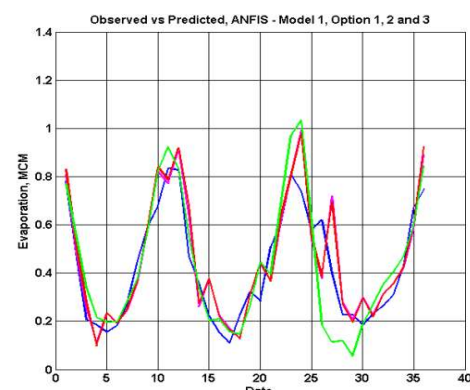


Fig.8: Observed vs predicted evaporation.

2. Inflows prediction

Given the random character of inflows to reservoir, the developed model consists as far as possible, in exploring the performance of ANN methods in predicting random variables. In this case, the adopted multilayer neuronal model « MLPNN », whose NN and LR are 20 and 90%, respectively, has given fair results (Table. III), as it is shown clearly in Fig.9, 10 and 11, a slight difference between the observed and the predicted values with a RMSE lesser than unity in a calibration phase and lesser than two in a validation phase. Nevertheless, a weak correlation is observed. Likewise, ANFIS model gave acceptable predicted values and that, according to performance metrics (Table. IV) and as it is shown in Fig.12, 13 and 14.

Table III

Performance metrics of the inflows prediction using MLPNN model
(C: Calibration, V: Validation)

	Lag time	NN	LR	CC	RMSE	MSE	NC	{B}	I
C	t-2	20	90	-0,745	0,231	0,373			
	t-4	20	90	-0,920	0,413	0,730			
	t-6	20	90	-0,316	0,405	0,735			
V	t-2	20	90	0,212	1,673	2,969	50 %	3,342	1,767
	t-4	20	90	0,145	1,733	2,737	65 %	3,467	1,792
	t-6	20	90	0,273	1,703	3,858	34 %	4,593	1,965

Table IV

Performance metrics of inflows prediction using ANFIS model
(C: Calibration, V: Validation)

	Lag time	NN	LR	CC	RMSE	MSE	NC	{B}	I
C	2	2	50	0,244	0,923	0,177			
	2	2	90	-0,394	0,237	0,329			
	4	2	90	-0,326	0,357	0,527			
V	2	2	50	0,257	1,244	1,016	-53%	1,193	1,321
	2	2	90	0,240	1,609	1,969	-49%	2,298	1,519
	4	2	90	0,130	1,878	2,197	-48%	2,723	1,367

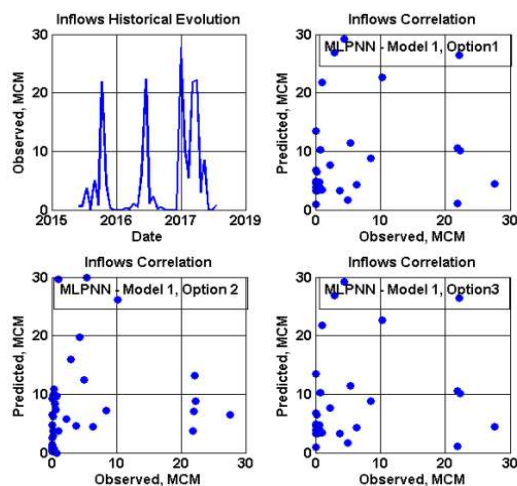


Fig.9: Correlation of the observed and predicted inflows.

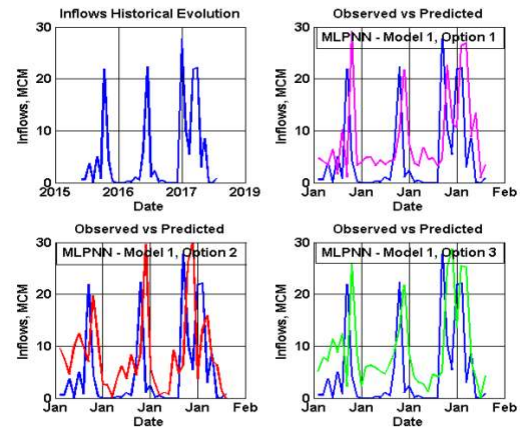


Fig.10: Confrontation of the observed and predicted inflows.

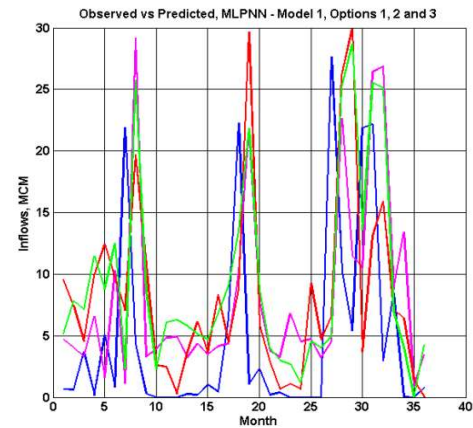


Fig.11: Observed vs predicted Inflows.

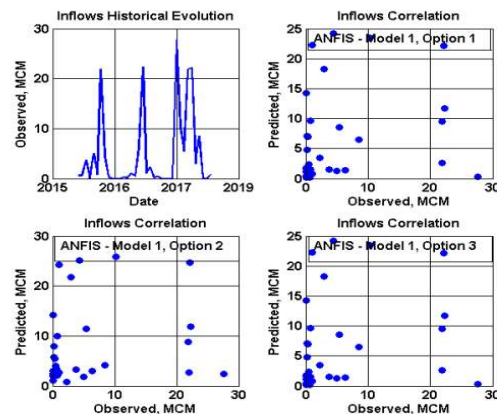


Fig.12: Correlation of the observed and predicted inflows.

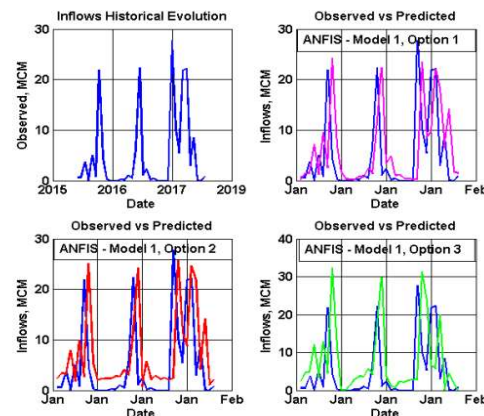


Fig.13: Confrontation of the observed and predicted inflows.

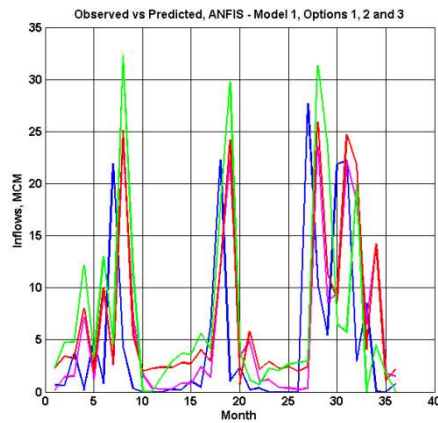


Fig.14: Observed vs predicted inflows.

3. Reservoir storage Prediction

Given the fair results obtained from inflows prediction, on the contrary, the prediction of a reservoir storage attempted with aim of verifying the performance of MLPNN and ANFIS models, is conclusive. Concerning MLPNN model, the second model with 5, 10 and 20 hidden neurons and learning rate of 90% is promising in predicting the reservoir storage according to the achieved performance metrics (Table.V). Thus, the observed and the predicted reservoir storage values are highly correlated with a CC and NC about 0,963 and 93%, respectively. Fig.15, 16 and 17 confirm the robustness and the performance of MLPNN model in predicting reservoir storage.

Table V
Performance metrics of the storage prediction using the model MLPNN (C: Calibration, V: Validation)

		Lag time	NN	LR %	CC	RMSE	MSE	NC	{B}	I
Model -1	C	t-2	5	50	0,884	1,907	3,503			
		t-2	10	50	0,861	2,045	3,878			
		t-2	20	50	0,793	2,328	3,598			
	V	t-2	5	50	0,945	0,898	1,107	89 %	4,609	1,024
		t-2	10	50	0,962	0,726	0,582	93 %	4,460	1,027
		t-2	20	50	0,951	0,834	0,769	91 %	4,367	1,042
Model -2	C	t-4	5	90	0,870	2,046	0,184			
		t-4	10	90	0,895	1,773	0,078			
		t-4	20	90	0,871	1,944	0,088			
	V	t-4	5	90	0,972	0,306	3,904	94 %	4,088	0,989
		t-4	10	90	0,963	0,178	3,293	93 %	3,372	1,029
		t-4	20	90	0,961	0,253	4,156	92 %	4,244	0,999
Model -3	C	t-6	5	90	0,891	1,744	2,044			
		t-6	10	90	0,897	1,672	2,550			
		t-6	20	90	0,862	2,002	3,480			
	V	t-6	5	90	0,953	0,138	0,076	92 %	1,968	0,980
		t-6	10	90	0,950	0,311	0,286	91 %	2,835	1,030
		t-6	20	90	0,936	0,598	0,598	88 %	4,078	0,990

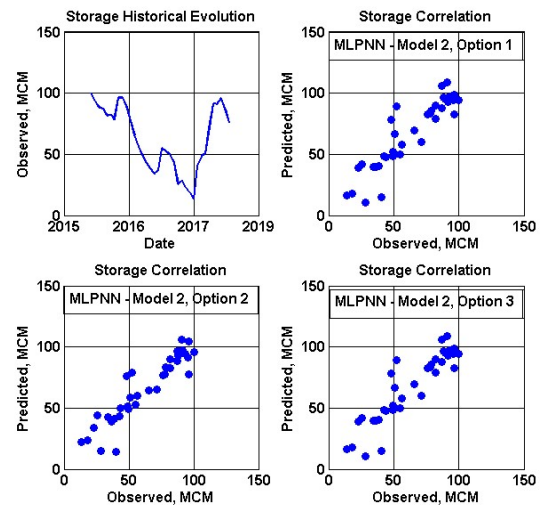


Fig.15: Correlation of the observed and predicted storage.

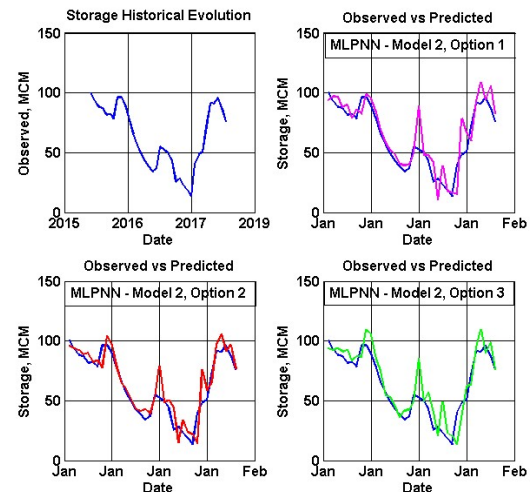


Fig.16: Confrontation of the observed and predicted storage.

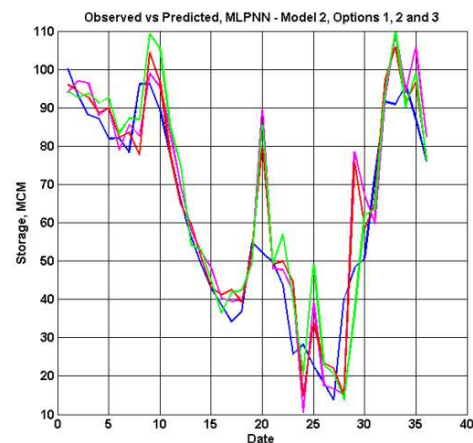


Fig.17: Observed vs predicted storage.

As to ANFIS model, the second option of the first model with two fuzzy rules, a learning rate of 90% and a lag time of t-2 has reached performing metrics by comparing predicted and observed storage values (Table.VI).

The robustness and the accuracy of achieved predicted storage values is confirmed through their confrontation with the observed values as it's shown in Fig.18, 19 and 20.

Table.VI

Performance metrics of the storage prediction using the model ANFIS (C: Calibration, V: Validation)

	Lag time	NN/ RN	LR %	CC	RMSE	MSE	NC	{B}	I
Model-I	C	2	2	50	0,865	1,862	2,457		
	V	2	2	50	0,953	0,797	0,118	79 %	2,575
	C	2	2	90	0,873	2,032	3,701		
	V	2	2	90	0,956	0,091	0,153	81 %	3,548
	C	4	2	50	0,746	2,718	2,887		
	V	4	2	50	0,962	0,190	0,146	70 %	3,033

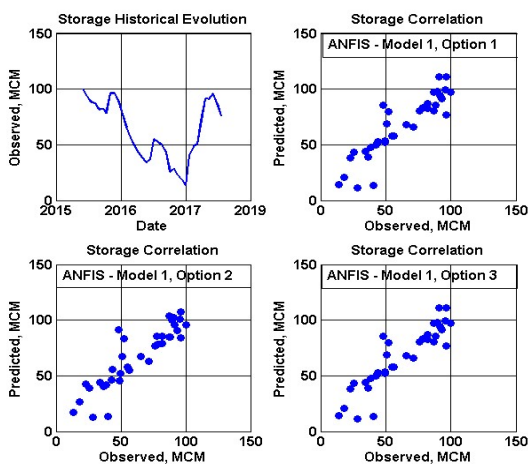


Fig.18: Correlation of the observed and predicted storage.

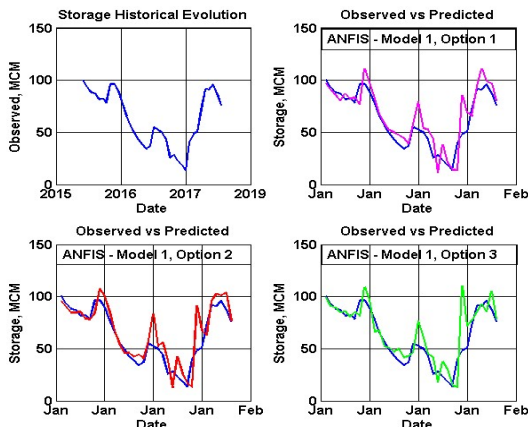


Fig.19: Confrontation of the observed and predicted storage.

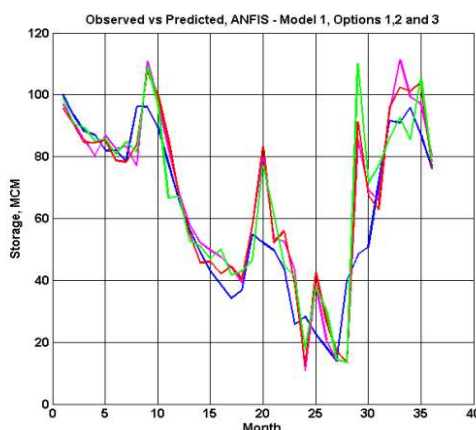


Fig.20: Observed vs predicted reservoir storage.

IV. CONCLUSION

Our approach through this case study consists of predicting state variables such as evaporation, inflows and reservoir storage, with aim of reducing a systematical uncertainty. This is necessary for optimizing the current reservoir operating policy, within the scope of a sustainable and secure water resources management, and therefore, the achieved models based artificial neural networks and fuzzy logic constitute, a decision making tool for a reservoir operator in order to manage releases efficiently. In this respect, given the performance of ANN methods in extrapolating linear or nonlinear data, the developed MLPNN feed forward model based on the supervised learning is tested in predicting reservoir state variables. Generally, the performance of the MLPNN and ANFIS models is demonstrated through the promising reached results related to evaporation and reservoir storage prediction, given the performance metrics of correlation and Nash coefficients widely persuasive. As to inflows, their prediction gave fair outputs, which confirms the inability of conventional MLPNN and ANFIS methods in predicting the random variables. Moreover, I have to note that the developed models enabled in predicting inflows with a root mean square error lesser than unity. Nevertheless, the correlation between observed and predicted values is lesser than unity; this reached result is singular compared with the achieved results related to the random variable prediction based artificial neural networks and fuzzy logic, available in the international literature on web. However, applying artificial neural networks and fuzzy logic helped in improving slightly reservoir-operating policy, in spite of predicting random variables remains a difficult task with these methods, whose classic solutions are not always completely satisfactory and are costly in computation time. In perspective, the future challenge in a matter of random variable prediction with artificial neural networks and fuzzy logic methods is to explore deeply their learning capacity, with aim of developing decision-making tools in a matter of reservoir operation and that, relative to the climate change, region aridity arid conditions of uneven rainfall distribution in space and in time.

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Machine Learning for Predicting the Stock Price Direction with Trading Indicators

Md. Siam ANSARY

Abstract—There are a number of possible advantages in utilising trading indicators and machine learning to predict the direction of stock prices. It is crucial to remember that stock price prediction is difficult by nature and that there is no way to ensure accuracy. The financial markets are extremely information-rich, dynamic, and complex. Large datasets may be processed and analysed by machine learning algorithms far more quickly than by people, which makes it possible to spot patterns or trends that might not be immediately obvious. By using historical price and volume data, the algorithms can be trained to identify patterns that could predict future moves. Because they offer insights into possible market moves, predictive models can help with risk management. Because ML models are always learning from fresh data, they can adjust to changing market conditions. This flexibility is essential in markets where a variety of factors impact the market. Several machine learning models have been used in this experimental effort to monitor the direction of stock prices, and the outcomes are extremely encouraging.

Keywords—stock, dhaka stock exchange, yahoo finance api, machine learning, classification.

NOMENCLATURE

ENP	Ecole Nationale Polytechnique.
ML	Machine Learning.
AI	Artificial Intelligence.
SVM	Support Vector Machine.
KNN	K Nearest Neighbor.
RF	Random Forest.
DT	Decision Tree.
MLP	Multilayer Perceptron.
AdaBoost	Adaptive Boosting.
API	Application Programming Interface.
MFI	Money Flow Index.
RSI	Relative Strength Index.

I. INTRODUCTION

A number of factors, such as complexity, volatility, and the abundance of variables that might affect market movements, make stock market research difficult. Financial markets are intricate systems that are impacted by a wide range of variables, such as emotions among investors, firm performance, economic indicators, and world events. A thorough and multidimensional approach is necessary to comprehend the interactions among these elements. Every day, the stock market produces enormous amounts of data. It can be very difficult to go through all of this data and find trustworthy and pertinent sources. To make wise selections, traders and investors must be able to discern between irrelevant signals and meaningful ones. Due to their dynamic nature, financial markets are susceptible to sudden changes in

news, events, or macroeconomic conditions. It's never easy to stay current with information and adjust to shifting market conditions. Market movements are significantly impacted by investor behaviour, which is driven by herd mentality, emotions, and market psychology. Research on stock markets becomes even more challenging when one attempts to comprehend and forecast human behaviour. It is difficult to forecast future price movements in financial markets due to the inherent risk and uncertainty involved. Economic downturns, market corrections, and unforeseen events can all have a big influence on the results of investments. Financial markets are becoming more interconnected and susceptible to global events and developments as a result of globalisation. Events in one region of the world that are political, economic, or geopolitical may reverberate throughout markets worldwide. The regulatory landscape is subject to change, which has an effect on how businesses function and how investments are managed. Maintaining up-to-date knowledge of regulatory modifications is essential for precise market analysis. A balance between quantitative analysis (financial statements, ratios, etc.) and qualitative analysis (industry trends, management quality, etc.) is necessary for stock market research. The research method becomes more complex when both elements are integrated. Examining price charts, patterns, and indications is part of technical analysis. Although it can offer insightful information, chart interpretation is a discipline that takes expertise and skill. Meaningful analysis requires data that are accurate and reliable. It is imperative for traders and investors to guarantee that the data they depend on is current, precise, and accurately depicts the actual market conditions. Traders and investors are susceptible to biases or overconfidence that impair their judgement. Effective market research requires retaining objectivity and being conscious of cognitive biases. It takes a combination of experience, knowledge, analytical abilities, and a methodical approach to decision-making to successfully navigate these hurdles. A lot of market players also use technology, such as data analytics and machine learning, or consult experts to improve their research skills. Compared to people, machine learning algorithms can process and analyse massive datasets much more quickly, which makes it easier to spot patterns or

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trends that might not be immediately obvious. Automated market data analysis and fast prediction generation are possible with machine learning models. This can be especially useful in quick-paced marketplaces where making decisions quickly is essential. Patterns in past price and volume data can be used to train machine learning algorithms to identify possible future movements. This involves seeing patterns on charts or technical indicators that are hard for people to interpret. With their ability to forecast future market moves, predictive models can help with risk management. In order to reduce the risks brought on by market volatility, traders and investors can modify their methods in light of the anticipated direction. A variety of indications and factors that can affect stock values can be quantitatively analysed thanks to machine learning. More unbiased and data-driven decision-making may result from this. To evaluate the performance of machine learning models, past data can be used for backtesting. This enables investors and traders to assess the performance of their methods in various market scenarios. Because machine learning models are always learning from fresh data, they may adjust to shifting market conditions. In markets where a variety of factors impact the market, this adaptability is essential. In this experiment, we used a variety of trading indicators to attempt to forecast the direction of the stock price in order to make investment decisions. Prediction techniques should be flexible enough to accommodate the rapid fluctuations that the stock exchange can encounter. For these kinds of trials, methods other than machine learning models are less appropriate because ML classifiers are more adept at learning from datasets and adapting to potential changes.

We review the previous research on our experiment, its methodology, and its results in the parts that follow.

II. LITERATURE REVIEW

Several studies have focused on the use of machine learning algorithms to forecast stock values. Various strategies have been investigated in order to detect patterns in historical data and predict future price changes. Effective feature engineering is critical for enhancing machine learning model performance. As potential elements for predicting stock prices, researchers studied several financial indicators, technical analysis metrics, and sentiment analysis from news or social media. Random forests and gradient boosting are two common ensemble learning approaches. These methods combine numerous models' predictions to improve overall accuracy and resilience. Chen and Hao [1] concentrated on stock market index prediction, which is important for investing and applications that aim to maximise earnings at minimal risk. In order to effectively predict stock market indexes, their study highlights the use of machine learning (ML). Specifically, they propose a hybrid framework that combines feature weighted support vector machines and feature weighted K-nearest neighbours. To evaluate the effectiveness of the suggested approach, the researchers ran tests on the indices of the Shanghai and Shenzhen stock exchanges. The findings show that for short-, medium-, and long-term predictions of the Shanghai Stock Exchange Composite Index and Shenzhen Stock Exchange Component Index, the created model has the ability to attain higher prediction skills. A Fuzzy Metagraph (FM) based method for stock market classification, prediction, and decision making for short-term investors in the Indian stock market was put out by Anbalagan and Maheswari [2]. The research employed the FM technique to train the system while

using Technical Indicators. Stocks listed on the Indian Bombay Stock Exchange (BSE) are used to assess the success of the suggested model. According to the findings, the FM-based model produces performance that is satisfactory and has very little risk error. He et al. [3] conducted research on pharmaceutical companies' stock price movement prediction during the COVID-19 pandemic. The researchers gathered extensive financial data unique to the company, including historical and present stock prices, key financial indicators, worldwide industry indices, and information about COVID-19. They had gathered information from a variety of sources, including Our World in Data [4] and the Yahoo! Finance API [5]. These disparate datasets were combined into a single, day-oriented dataset that could be used to train several predictive models. The researchers used worldwide industry indexes and created a set of characteristics especially for COVID-related data. The models' capacity for prediction was much improved by the addition of these features. The findings showed that adding features from industry indexes increased AUROC by an average of 1.7%. Additionally, the addition of COVID-related features led to a 6.5% average AUROC rise. The research investigated how the exclusion of early COVID era data affected the performance of the model. With an average increase in AUROC of 3.9% and an average increase in accuracy of 1.9%, the results showed significant gains. The decision tree model using the gradient boosting approach was able to attain a 70% AUROC and 66% accuracy as a result of this modification. Ballings et al. [6] attempted to forecast the direction of stock prices using information obtained from European companies that were publicly traded. Their primary goal was to determine which kind of algorithm worked better for single learners or ensemble learners. Logistic Regression, Neural Networks, K Nearest Neighbour, Support Vector Machine, Random Forest, AdaBoost, and Kernel Factory were the techniques employed in this study. Random Forest fared the best among the methods. For cross validation, they had employed five times the twofold. As features of this experiment, various financial indicators such as liquidity indicators, solvency indicators, and profitability indicators were used. Kara et al. [7] used ANN and SVM to forecast the direction of a stock price index. The Istanbul Stock Exchange data was used in the study. Artificial Neural Networks obtained 75.74% accuracy, while Support Vector Machines achieved 71.52% accuracy. In the experiment, various indicators were used. Basak et al. [8] used Random Forest and Gradient Boosted Decision Trees to forecast the direction of stock market prices. As features for the ML models, the work used the Relative Strength Index, Stochastic Oscillator, Williams Percentage Range, Moving Average Convergence Divergence, Price Rate of Change, and On Balance Volume. Parmar et al. [9] used the Yahoo Finance dataset to predict stock prices using Open, High, Low, Close, and Volume information. 20% of the dataset was used for testing, while the remaining 80% was used to train models. For stock price movement classification, Naik and Mohan [10] used data from the National Stock Exchange of India. As features, indicators such as the Simple Moving Average, the Exponential Moving Average, the Momentum Indicator, the Stochastic Oscillator, the Moving Average Convergence Divergence, the Relative Strength Index, the William R, the Accumulation Distribution Index, and the Commodity Channel Index were employed.

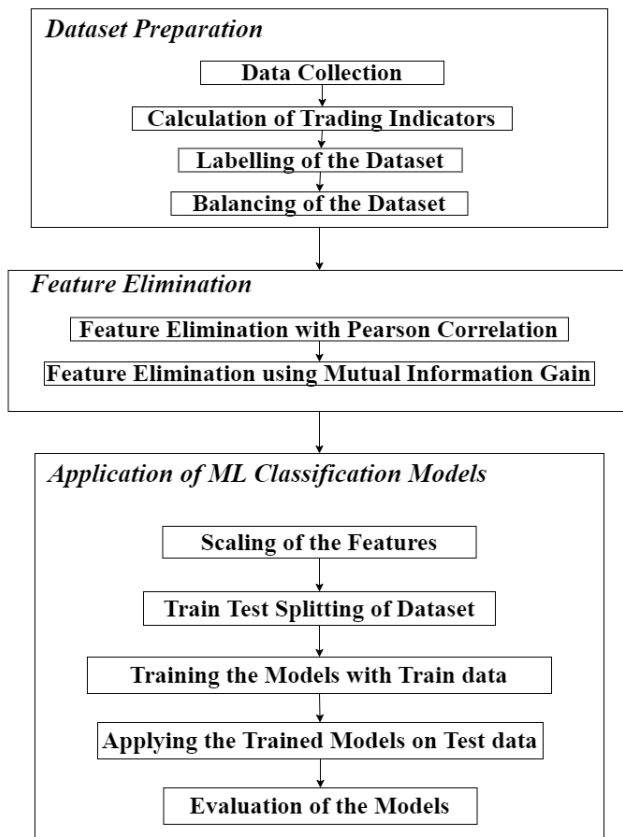


Fig. 1: Steps of Proposed Methodology

III. PROPOSED METHODOLOGY

A. Dataset Preparation

Firstly, stock data has been collected. This collected data has the Open, High, Low, Close, Volume values of each of the stocks. We have extracted important features from the dataset. Using Technical Analysis Library [11], based on the Open, High, Low, Close and Volume values of each stock, different trading indicators have been calculated so that they can be used as features. The calculated indicators are as below.

- Moving Average (MA)
- Balance of Power (BOP)
- Money Flow Index (MFI)
- Momentum (MOM)
- Rate of Change (ROC)
- Relative Strength Index (RSI)
- Weighted Close Price (WCP)
- Average Price (AP)
- Median Price (MP)
- Typical Price (TP)
- On Balance Volume (OBV)

With the MFI and RSI values of each stock, the stocks have been labeled. If the value of MFI is greater than 80 or the value of RSI is greater than 70, a stock is overbought. Overbought condition signifies the market is upwards. If the value of the MFI is less than 20 or the value of RSI is less than 30, a stock is oversold. Oversold signifies that at the current moment, the stock is going downwards. The stocks that are neither overbought nor oversold, are neutral. In such a manner, all the stocks have been labeled. After labelling the stocks as upwards, downwards or neutral, we have observed most stocks to be neutral, some stocks to be of upwards direction and very few records to be of downwards direction. Hence, for classification task, the dataset is not balanced. Hence, at first, we have used Random Undersampling on Neutral Stocks so that number of neutral records come down to the number of upwards stocks and then, applied Random Oversampling so that number of downwards stocks match the numbers of upwards stocks. We have used two datasets for our work. For the first dataset, we have at first collected data using the Yahoo! Finance's API [5]. Initially, we have collected 5630 samples. The samples are of the period from 01 January 2000 to 17 May 2022. After labelling the stocks, we have observed 5033 stocks to be neutral, 426 stocks to be of upwards direction and 171 records to be of downwards direction. The total number of records of the balanced dataset becomes 1278 where each class has same amount of records. For the second dataset, we used data of Dhaka Stock Exchange (DSE). Initially, there are 90041 records in the collection. Following the classification of stocks as moving upwards, downwards, or neutral, we discovered that 66004 stocks were in a neutral state, 14648 stocks were traveling upward, and 9389 records were heading downwards. The balanced dataset contains 43944 records overall, with an equal number of each kind.

B. Feature Elimination

After ensuring that the dataset is balanced, we attempted to determine whether all of the characteristics in our dataset are important, as unnecessary features might raise the complexity of the problem and cause overfitting. On the dataset, we used the Pearson Correlation method. Figure 2 shows the heatmaps of the values for the first dataset and Figure 3 shows the heatmaps of the values for the second dataset. The threshold values for the two datasets have been set to 0.8 and 0.9, respectively, to remove superfluous features. We used the Mutual Information Gain technique on the existing columns. The threshold value has been set to 0.5 in order to further exclude inconsequential features, suggesting that features with information gains less than 0.5 have been deleted.

C. Application of ML Models

Following feature elimination, we used Standard Scaling on the remaining features. Standard scaling was used to bring all features into a similar value range because various feature ranges can produce undesired bias. Then, we divided our dataset into two parts. 80% of the dataset was utilised for training, while the remaining 20% was used for testing. Five classification models were used. They are listed below.

- K Nearest Neighbor Classifier: The KNN classifier is a simple and effective supervised machine learning technique. KNN is a sluggish, non-parametric learning method. Non-parametric means that no assumptions are made about the

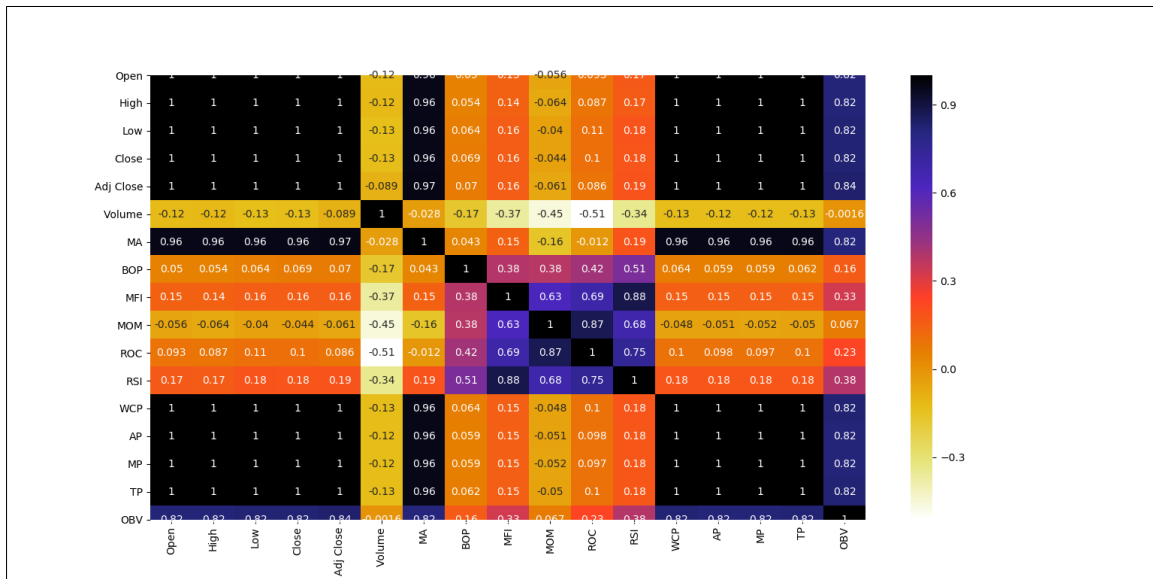


Fig. 2: Pearson Correlation Heatmap of Features for first Dataset of Yahoo! Finance's API

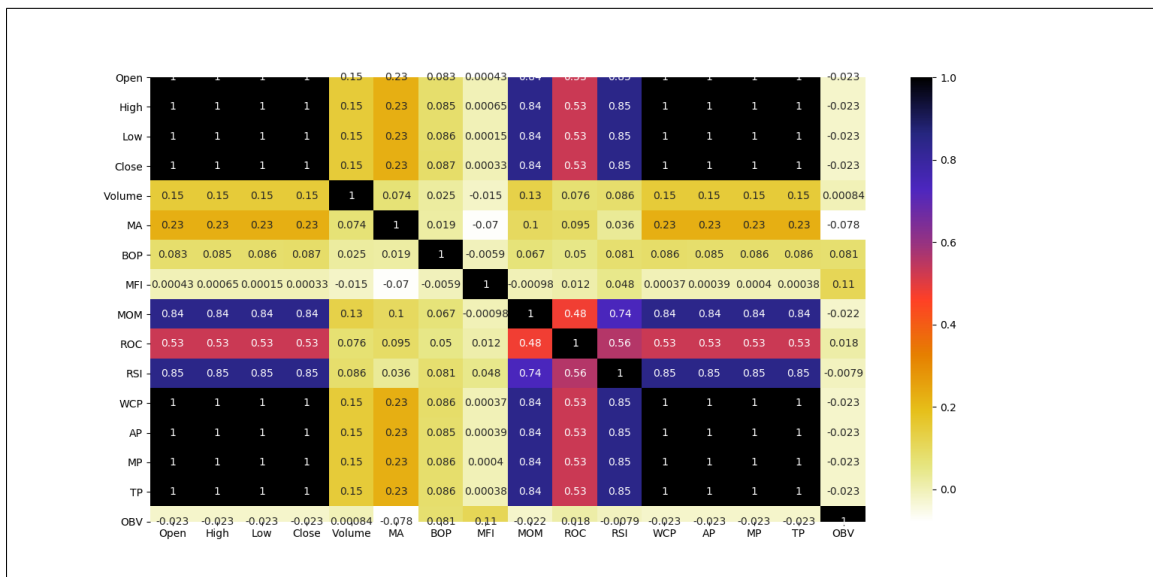


Fig. 3: Pearson Correlation Heatmap of Features for second dataset of Dhaka Stock Exchange

underlying data distribution, and lazy learning indicates that the method does not construct an explicit model during the training phase. It instead remembers the training data. Given a new, unknown data point, the method uses a distance measure to identify the K nearest data points in the training set. The parameter K (number of neighbours) must be chosen carefully because it has a substantial impact on the model's performance. A little K can cause overfitting, whereas a large K can cause underfitting. The ideal value of K is determined by the dataset's unique properties. The distance metric used to compare the similarity of data points is critical in KNN. While Euclidean distance is the most commonly used, alternative metrics such as Manhattan distance or Minkowski distance might be used depending on the nature of the data. Because the KNN method is sensitive to feature scale, feature scaling is frequently required. Normalisation or standardisation of the features guarantees that no single feature dominates the distance calculation. The main computational expense

of KNN occurs during the prediction phase, when the distances between the new point and all training points must be calculated. Large datasets can be computationally expensive. The curse of dimensionality can impair KNN, which means that as the number of features increases, the distance between points tends to become more uniform, potentially diminishing the algorithm's effectiveness. Techniques like as feature selection and dimensionality reduction can help to alleviate this problem. KNN is a versatile and intuitive method, but its effectiveness is dependent on careful tweaking of hyperparameters and consideration of data properties. It is frequently used as a baseline model for comparison with more complex algorithms, and its simplicity makes it excellent for short exploratory data analysis.

- **Decision Tree Classifier:** The Decision Tree Classifier is a common supervised machine learning method. It is a tree-like model in which each core node represents a characteristic or attribute, the branches represent decision

rules, and each leaf node corresponds to the anticipated outcome. A decision tree is a hierarchical structure in which nodes indicate decisions based on attributes. The root is the top node, and the leaves are the bottom nodes, each representing a class label or a numeric value. In the tree, decision nodes reflect questions regarding the input data. These queries lead to succeeding nodes, and the structure continues until a leaf node is reached, offering the ultimate judgement or forecast. To select the appropriate split for each node, the method employs metrics such as Gini impurity or information gain. The tree-building procedure entails recursively partitioning the dataset based on the features chosen. This process is repeated until a stopping requirement, such as a preset tree depth, a minimum amount of samples in a leaf, or no further improvement is possible, is met. It explores the decision tree from root to leaf, using the decision rules, to create a forecast for a new data point. The prediction is subsequently assigned to the class label or value of the leaf node.

- **Random Forest Classifier:** The Random Forest Classifier is a classification technique that creates numerous decision trees during training and reports the mode of the classes. To generate numerous subsets, the training dataset is randomly sampled with replacement. The forest's decision trees are then trained on one of these subsets. A random subset of features is considered for each split in a decision tree rather than all features. This adds to the randomness and diversity of the trees. The class indicated by the majority of trees is chosen as the final prediction in classification problems. When compared to individual decision trees, Random Forests are less prone to overfitting. The variety generated by training on diverse subsets of data and features aids in generalisation to previously unseen material. Because of its durability, ease of application, and strong generalisation performance, Random Forests are commonly utilised in practice.
- **AdaBoost Classifier:** AdaBoost is an ensemble learning method that combines the predictions of numerous weak learners to generate a strong and accurate predictive model. It lends additional weight to occurrences that were incorrectly classified by earlier models, allowing subsequent models to focus on the faults. Each data point in the training set is given a weight, which is changed at each iteration based on the performance of previously trained models. Misclassified points are weighted more heavily, emphasising them in subsequent iterations. Weak learners are trained in a sequential manner, with each new learner correcting the mistakes of the preceding ones. The final forecast is a weighted average of the individual weak learners' predictions. AdaBoost is especially successful in dealing with complex cases that are difficult for a single weak learner to correctly classify. AdaBoost prioritises troublesome cases by applying higher weights to misclassified instances. The algorithm is adaptive in the sense that it adapts its strategy based on the mistakes made by previous models. AdaBoost's versatility enables it to continuously enhance its performance. Each weak learner's contribution is weighted based on its accuracy during the final prediction. More precise models have a greater impact on the final decision. AdaBoost is a sophisticated algorithm that has gained popularity due to its ability to

improve the performance of poor learners and generate robust models. It is frequently employed as a building block in ensemble methods and has the potential to dramatically increase forecast accuracy.

- **Gradient Boosting Classifier:** Gradient Boosting is an ensemble learning strategy that combines the predictions of numerous weak learners, typically decision trees, to create a powerful predictive model. The Gradient Boosting Classifier, in particular, is utilised for classification problems. Gradient Boosting gradually constructs an ensemble of weak learners. Each new learner corrects the mistakes committed by the existing learners as a group. Using gradient descent optimisation, the approach minimises a loss function. It computes the loss gradient with respect to the ensemble prediction and modifies the model parameters to minimise the loss. Gradient Boosting is concerned with minimising the residuals, or mistakes, of the existing ensemble's predictions. To predict the residuals, new trees are trained, eventually reducing the overall error. A shrinkage parameter, often known as the learning rate, governs each tree's contribution to the ensemble. A slower learning rate necessitates more trees for the same degree of complexity, but it can improve model robustness. Gradient Boosting involves overfitting prevention approaches such as limiting tree depth, introducing a penalty term for tree complexity, and subsampling data during training. Gradient Boosting is a powerful technique that has shown effectiveness in a variety of disciplines. When high predicted accuracy is required, it is a common alternative, and careful adjustment of hyperparameters is often required for best performance.

D. Performance Evaluation

The performances of the ML models are evaluated with Accuracy, Precision, Recall and F-1 Score evaluation metrics. Figure 4 illustrates the performances of the models for the first dataset which has been created with the data of Yahoo! Finance API. The following Figure 5 shows how the models perform for the second dataset that has been made with the data of Dhaka Stock Exchange. From observations of the performances of the ML models, it can be noted that their attainments have been satisfactory.

IV. CONCLUSION AND FUTURE WORKS

Stock price direction is difficult to predict due to a number of factors, and financial markets are influenced by a complex interaction of various elements. Stock prices are vulnerable to erratic and chaotic movements that are influenced by a plethora of variables. Unexpected news, geopolitical events, economic statistics, and investor attitude, among other things, have an impact on the market. Because of these variables, properly forecasting the direction of stock values is difficult. According to the Efficient Market Hypothesis, markets assimilate all relevant information into stock prices rapidly and efficiently. If this hypothesis is correct, it indicates that predicting price fluctuations based on previous data or publicly available data is difficult. Human behaviour, including emotions such as fear

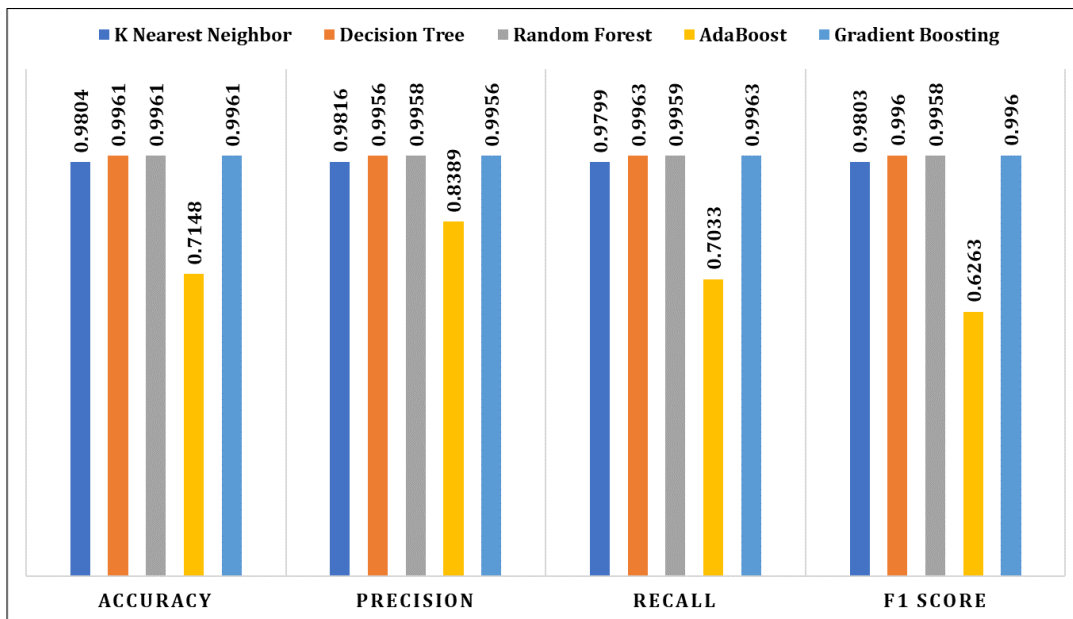


Fig. 4: Performance of ML Models for the first dataset

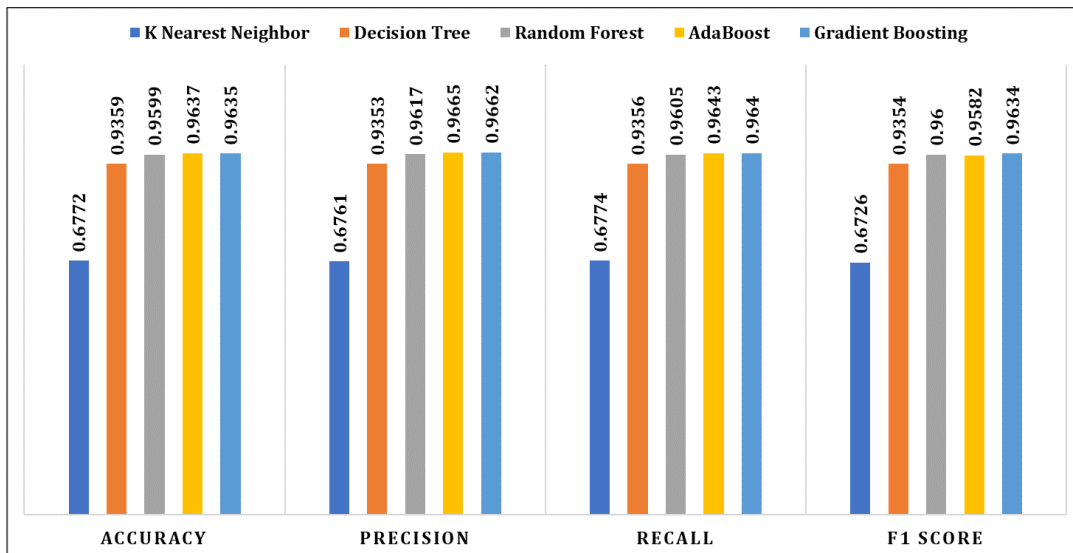


Fig. 5: Performance of ML Models for the second dataset

and greed, has a significant impact on stock values. According to behavioural finance, investors do not always make rational decisions, and market movements can be influenced by psychological factors that are difficult to quantify and anticipate. Not all market participants have simultaneous access to the same information. Insiders or large institutional investors may have access to privileged information, providing them an advantage over individual investors. Because of this information asymmetry, ordinary investors find it difficult to precisely predict stock price changes. In some circumstances, market manipulation occurs when specific businesses or individuals purposefully create misleading views in order to impact stock prices. This can add another element of uncertainty to the market. Stock prices can be greatly influenced by economic conditions, interest rates, inflation, and world events. It is difficult to correctly predict these macroeconomic factors, and their consequences on the market can be complex and varied. Financial markets create massive amounts of data, making it difficult to spot meaningful trends. Traditional models struggle to collect all essential

information due to the vast volume of data and the speed with which markets operate. Market dynamics are frequently nonlinear, which means that little changes can have disproportionately huge consequences. This makes using linear models for accurate predictions difficult. Price movements in the short term can be influenced by factors such as news and sentiment, whereas long-term trends can be influenced by fundamental factors such as earnings, economic growth, and industry trends. The combination of these elements makes projecting both short-term and long-term trends difficult. It is critical to recognise the inherent uncertainty and risk in projecting stock values. Despite the various challenges and difficulties, in our experiment it has been seen that Machine Learning models can predict the direction of movement of prices of stocks with great efficiency. Of course it is not always possible to predict correctly but the more accurate the prediction is, the less risks there are. In the future, we intend to continue training our models with the new data of Yahoo! Finance API and DSE. Also, we will employ more pre processing techniques, ML models on more Stock Datasets so that we can

scrutinize the capabilities of the ML approaches for predicting the direction of stock prices.

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Towards Optimal Frequency Plans: A Survey of Frequency Assignment Strategies, Models, and Methods

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Abstract—The frequency assignment problem in a telecommunication network refers to the task of assigning frequencies to transmitters to minimize or eliminate interference and to optimize at least one other parameter. The literature on this task is vast and examines many parameter optimization objectives. Numerous schemes, models, and methods have been proposed. In this article, we present an extensive survey of the published studies on the frequency assignment problem in the context of a wide range of applications. Different types of optimization objectives, models and solution methods are presented in a systematic way.

Keywords— Mobile computing; Telecommunication networks; Optimization; Heuristics and metaheuristics.

I. INTRODUCTION

The frequency assignment problem (FAP), also called the channel assignment, frequency, spectrum sharing, or spectrum management problem, is the task of allocating communication frequencies to users without compromising the quality of network communications. This problem arises in numerous real situations that have a direct impact on the quality of modern living. In an ever-increasing share of the world, the use of mobile phones and telecommunications in general (sound, image, and data) has become crucial for individuals and businesses to thrive. This may be considered one of the most significant socioeconomic advances in recent decades. Figure 1 depicts the evolution of telecommunications over time from 2020 and a forecast to 2025. The number of phone users and network service providers is growing steadily.

According to International Telecommunications Union subscription statistics, 6.42 billion mobile phones were in circulation in 2022 [1]. In 2017, Statista projected the number of mobile devices to reach 18.22 billion by 2025, which would be an increase of 4.2 billion from the count for 2020 [2]. While the numbers continue to grow, the spectrum of frequencies available to accommodate all these users remains the same.

Subject to certain constraints, frequencies are natural resources that can be shared and reused to manage the quality, coverage, and cost of telecommunication network services.

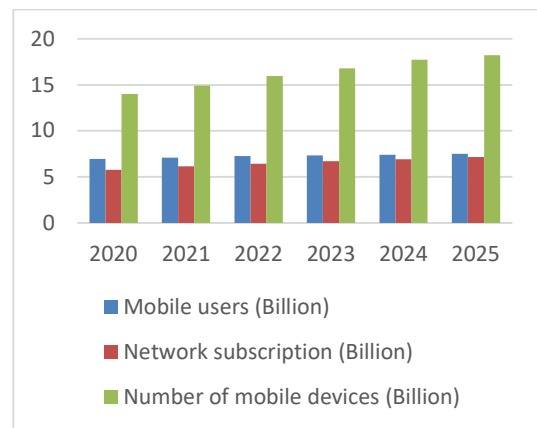


Fig. 1: Number of mobile devices, users, and network subscriptions evolution over time (image generated using data in [1–3]).

Subject to certain constraints, frequencies are natural resources that can be shared and reused to manage the quality, coverage, and cost of telecommunication network services.

The FAP was first addressed in its simplest form in 1977 [4] but became more and more complex with the growth in the number of users and technological advances in the telecommunications field [5]. The frequency assignment problem has since been proven to be an NP-hard problem [6]. Many models, algorithms, and theories have been introduced and studied in search of lasting solutions.

Rather than simply list all previous articles published in this subject area, we seek to propose a functional directory. No survey has yet provided a complete overview of the field. Most published surveys have focused on a specific category of FAPs or a specific application and formulation, and they do not give readers a comprehensive understanding of the problem overall, the research trends, or recently proposed solutions.

II. METHODOLOGY

To perform the literature review, works from the last twenty years were considered in addition to a few older references that are still important to the field. The works were mainly journal articles on the topic of modelling and solving the FAP that are

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listed in the SCOPUS database and were published in highly ranked IEEE, Elsevier, or Springer journals. Related topic conference papers were also considered from well-reputed conferences such as IEEE conferences and computer science related conferences.

The search words used to find related works were “frequency assignment problem” and “channel assignment problem” associated with one or more of the following words: modelling, optimization, solution, solving, methods, telecommunication, net-work, and wireless networks. Only papers dealing directly with the FAP and studies expanding on methodologies found in the literature were evaluated. Papers that presented generalities about the issue without mathematical breakthroughs or practical examples were not considered.

The papers were categorized according to the type of FAP, the modelling method, and the optimization method, as specified in Figure 2.

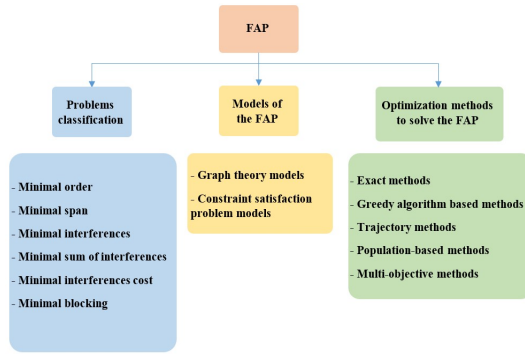


Fig. 2: Classifications of FAP problems, models, and optimization techniques found in the literature.

This article is structured according to Figure 3. Following a survey of FAP schemes and classes (Section 4), models (Section 5), and methods (Section 6), we propose a library of concepts to increase the effectiveness of FAP studies.

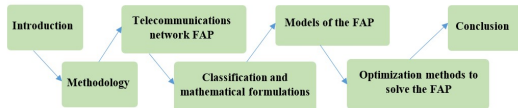


Fig. 3: The structure of the review.

III. TELECOMMUNICATION NETWORK FREQUENCY ASSIGNMENT PROBLEM S

A telecommunication network comprises a set of transmitters distributed over a set of utilization sites. The transmitters communicate with each other and with terminals (mobile phones and other devices) through radio links carried on frequencies or channels. Frequencies are scarce natural resources. The word scarce in this context means that, although they are reusable, the availability of frequencies is finite. All telecommunication networks must therefore share them in order to satisfy all the requests of the users and providers. However, the reuse of frequencies may cause deterioration of received signal quality. This deterioration may be mitigated by a suitable frequency assignment. A provider wishing to establish a network must implement a frequency plan capable of supporting the required coverage and service quality. In a functional network, the frequency assignment is updated continually as new requests are accommodated and current communications end.

The FAP arises at both the design and the deployment stages in

all telecommunication networks, including mobile telephony, TV and radio broadcasting, military and other wireless communication systems, the Internet of Things, and cognitive networks [5–11]. In its simplest and most generalized form, the assignment problem is equivalent to the graph colouring problem, which is a well-known NP-hard problem [5–9].

In the operations research literature, FAP in telecommunication networks has attracted much attention because of its practical relevance and close relationship with classical combinatorial optimization problems. Our aim here is to present the concepts and ideas that are common to all applications rather than focus on mobile telephone networks. We therefore use the word ‘network’ to specify any telecommunication network, including satellite and TV networks. Similarly, the word ‘transmitter’ may designate a satellite, a base station, or any other device with similar functionality.

A. Basic concepts

To guarantee the quality of network communications, the signals within and between all radio links or other connections must not deteriorate. Loss of signal is called interference and depends mainly on geographical distance, obstacles, hardware characteristics and configuration, climate, and other largely unavoidable factors [12]. Similar frequencies or channels can be used simultaneously by several transmitters if they transmit at frequencies different enough to avoid interference or keep it below an acceptable limit. The difference between assigned frequencies must be maintained for each pair of potentially interfering communications until these communications end. These separations are called interference constraints or electromagnetic constraints. They are expressed in the following form [11, 13]:

$$|f_i - f_j| \geq s_{ij}, \quad (1)$$

where f_i and f_j are the frequencies assigned to transmitters i and j , and s_{ij} is the required separation. Three main types of separation values are recognized:

- Co-channel separation, for which the same frequency or channel cannot be assigned to certain pairs of transmitters.
- Adjacent-channel separation, for which transmitters corresponding to adjacent sites cannot be assigned adjacent channels simultaneously.
- Co-site separation, for which the frequencies assigned to the same transmitter must be separated by a certain number of frequencies.

Other constraints have been described, including duplex constraints between the two paths (both communication directions) [14, 15], and six equality and inequality constraints have been defined [16].

B. Instances definition

A FAP is defined by a 4-tuple (E, F, R, I) whose elements are as follows:

- E is the set of n network transmitters $(e_1, e_2, \dots, e_n)$.
- F is the set of all available frequencies, defined as the union of all subsets F_i of available frequencies for each transmitter e_i ($1 \leq i \leq n$) such that $F = F_1 \cup F_2 \cup \dots \cup F_n$. In a small number of studies, these sets are considered as domain constraints [14].
- R is the function $R: E \rightarrow \mathbb{N}$ that defines a non-negative number of requests R_i for each transmitter e_i , and hence the

number of frequencies to assign to e_i .

Such as $\mathbb{N}$ is the set of whole numbers.

- I is the function $I: E \times E \rightarrow R$ defining for each pair of transmitters e_i and e_j the minimal separation I_{ij} to be respected by the assigned frequencies to avoid interference (as described in III.A.), meaning that

$$\forall i, j \in [1, n], \forall p \in [1, R_i], \forall q \in [1, R_j]:$$

$$|f_{pi} - f_{qj}| \geq I_{ij}, \quad (2)$$

such that f_{pi} is the p th frequency assigned to the transmitter e_i .

In this general formulation, the geographical distance separating the transmitters is implicit in the function I , whereas other formulations consider the distance as its own function $E \times E \rightarrow R$, with I as a function $I: R \rightarrow R$ such that $I(d)$ is the smallest separation to be respected by the frequencies assigned to any pair of transmitters that are d units apart (to avoid interference).

Some researchers define the interferences as their level in an $n \times n$ matrix $M = \{(\mu_{ij}, \sigma_{ij})\}_{n \times n}$, in which μ_{ij} and σ_{ij} represent the mean and the standard deviation, respectively, of a Gaussian probability distribution describing the signal-to-interference ratio when the sectors in which transmitters i and j are installed and operate on the same frequency [17, 18]. The mean must be kept as high as possible to ensure adequate communication quality [19].

Other classes of FAP consider cost as a function of either distance or pairs of transmitters. The cost function C_{sig} used in GSM networks is defined for each pair of transmitter sectors relative to the frequencies assigned to them [17, 18].

The solution of a problem thus formulated is the assignment of a_i frequencies to each transmitter $e_i \in E$ ($a_i \leq R_i$) from its nominal set F_i . Usually encoded as an assignment vector or a partition vector, this must satisfy at least some of the interference constraints defined by I and thus optimizes one or many of the objective functions of the assignment problem (considered in detail in the next section).

In assignment vector representation, transmitters are the entries, and each element is a vector containing the a_i frequencies assigned to transmitter e_i . In partition representation, the entries are the frequencies, and each element is a vector that includes the transmitters to which each frequency is assigned [8].

Coding the solution as a binary $n \times m$ matrix X , such that the cardinality of the set F $m = Card(F)$, has been suggested [12]:

$$X = \begin{pmatrix} x_{11} & \cdots & x_{1m} \\ \vdots & \ddots & \vdots \\ x_{n1} & \cdots & x_{nm} \end{pmatrix}. \quad (3)$$

Here, each element x_{ij} is defined as follows:

$$x_{ij} = \begin{cases} 1, & \text{if the frequency } f_j \text{ is assigned to the transmitter } e_i, \\ 0, & \text{otherwise.} \end{cases} \quad (4)$$

The following conditions must be satisfied:

$$\forall 1 \leq i \leq n, \sum_{j=1}^m x_{ij} = a_i \quad \text{and} \quad a_i \leq R_i. \quad (5)$$

In this coding, the relation between the assigned frequency and the corresponding fulfilled request does not appear. A new storage format for representing a solution has been proposed to reduce the space allocated to it [14]. Frequency assignment has also been addressed simultaneously with other problems such as realistic interference constraints [20], polarization [21, 22], and coverage planning via antenna localization [23].

IV. CLASSIFICATION AND MATHEMATICAL FORMULATIONS

The FAPs studied in the literature cover a wide range of scenarios with diverse characteristics. Additionally, vendors set their own policies, which are often proprietary information, making it impossible to share or compare results. This has led to the development of many types of problems and models, considering various aspects such as frequency availability, number of requests, communication between transmitters, interference management, and objectives to be optimized.

A. Frequency assignment problem schemes

Published frequency assignment algorithms are based mostly on one of three techniques.

(i) Fixed channel assignment

In fixed channel assignment, each transmitter is allocated a nominal set of frequencies from which frequencies to be assigned to specific requests are chosen [24]. For a given request, if no non-interfering frequency is available, as may occur in high-traffic scenarios [25], the request is blocked (rejected). The advantage of fixed channel assignment is that all transmitters act independently and no communication between them is required. However, blocking requests is an unsatisfactory solution, especially when adjacent transmitters have unused frequencies.

(ii) Borrowing-based assignment

Borrowing-based assignment is a scheme in which an unused nominal frequency assigned to a neighbour is borrowed when this does not interfere with previous assignments [26, 27].

(iii) Dynamic channel assignment

In dynamic channel assignment (DCA), all transmitters share the same set of frequencies and each assignment leads to an exchange of updates of current channel use to allow selection of a non-interfering frequency. Two dynamic channel assignment schemes are known:

- Centralized, in which a central authority is responsible for all assignments and updating the status of the entire network. The transmitters execute only the channel-use decisions made by the central base.
- Distributed, in which the network status and channel-use decision-making are distributed among the transmitters, and communications between them cannot be avoided.

No scheme is optimal in all scenarios. In all cases, the main criterion for choosing one over another is the denial ratio, that is, the proportion of rejected requests. A fixed channel scheme is adequate when the traffic is low. In especially low-traffic situations, borrowing performs better than fixed schemes. Dynamic schemes usually outperform fixed schemes in low-traffic situations but perform poorly with high traffic. This subject has been reviewed in detail [28].

B. Classification of frequency assignment problems

Since 1977, providers of telecommunication networks have defined several objectives and policies and hence FAP formulations. Engineers usually attempt to minimize one or more of the parameters described previously [29]. We begin

here by formulating a fixed scheme in which all frequency domains for all transmitters are the same:

$$\forall i, 1 \leq i \leq n : F = F_1 = F_2 = \dots = F_n. \quad (6)$$

(i) Number of assigned frequencies:

The number of assigned frequencies, or the minimal order, FAP was first introduced in the 20th century when channels were sold as single frequencies [4]. The objective was to find the assignment that meets all interference constraints and all requests using the smallest number of frequencies. This is formulated mathematically using a binary variable x_{ij} that defines whether transmitter e_i is using frequency f_j or not.

$$x_{ij} = \begin{cases} 1, & \text{if the frequency } f_j \text{ is assigned to the transmitter } e_i, \\ 0, & \text{otherwise.} \end{cases} \quad (7)$$

The objective is to minimize the number of frequencies such that all requests are fulfilled for each transmitter:

$$\min \sum_{j=1}^{Card(F)} \sum_{i=1}^n x_{ij}, \quad (8)$$

such that

$$\forall i, 1 \leq i \leq n, \sum_{j=1}^{Card(F)} x_{ij} = R_i. \quad (9)$$

Equation (8) can be transformed by introducing a variable y_j that indicates the use of frequency f_j :

$$\forall j, 1 \leq j \leq Card(F), y_j = \sum_{i=1}^n x_{ij}. \quad (10)$$

The new objective formulation is as follows:

$$\min \sum_{j=1}^{Card(F)} y_j. \quad (11)$$

(ii) Spectra of used frequencies

When frequencies began to be purchased as bandwidth, network providers needed to know the range of frequencies that they could use. This created the minimal span FAP, formulated as follows [30]:

$$\min (f_{\max} - f_{\min}), \quad (12)$$

such that the requests are fulfilled according to the conditions

$$\forall i, 1 \leq i \leq n, \sum_{j=1}^{Card(F)} x_{ij} = R_i. \quad (13)$$

And

$$f_{\min} = \min_{1 \leq i \leq n, 1 \leq j \leq Card(F)} (f_j * x_{ij}), \quad (14)$$

$$f_{\max} = \max_{1 \leq i \leq n, 1 \leq j \leq Card(F)} (f_j * x_{ij}). \quad (15)$$

An alternative formulation is applied if the smallest frequency used is designated as '1', thus allowing the reduction of (12) to the maximum-used FAP [31]:

$$\min f_{\max}. \quad (16)$$

Other formulations are known [32–35]. The minimal order and minimal span problems may not have interference-free solutions. If this is the case, one of the objectives described in the remainder of this section may be optimized.

(iii) The number of violations

One option is to find an assignment in which the number of interferences (constraint violations) is minimal. This is called the minimal interference or minimal violation FAP [30]. The following objective function is used when a transmitter pair e_1, e_2 is assigned frequencies f_1, f_2 :

$$\min(\sum_{f_1=1}^{|F|} \sum_{f_2=1}^{|F|} \sum_{e_1=1}^n \sum_{e_2=e_1+1}^n x_{f_1 e_1} * x_{f_2 e_2} * \text{Inter}(f_1, e_1, f_2, e_2)), \quad (17)$$

where

$$\text{Inter}(f_1, e_1, f_2, e_2) = \begin{cases} 1, & \text{if } |f_1 - f_2| \leq I_{e_1 e_2}, \\ 0, & \text{otherwise.} \end{cases} \quad (18)$$

Each request is assigned a frequency as in (13).

(iv) Cumulative interferences

Minimizing the number of interferences may be an unhelpful criterion if interferences are minor and tolerable. In this situation, the amount of interference measured as the signal-to-interference ratio may be more useful. The objective is to minimize the sum of the interferences. This is called the minimal cumulative (or sum of) interferences problem [12].

Using the interferences matrix M (described in Section III.B.), the objective is formulated similarly to (13) and (18) as follows:

$$\min(\sum_{f_1=1}^{Card(F)} \sum_{f_2=1}^{Card(F)} \sum_{e_1=1}^n \sum_{e_2=e_1+1}^n x_{f_1 e_1} * x_{f_2 e_2} * \text{Inter}(f_1, e_1, f_2, e_2) * M_{e_1 e_2}). \quad (19)$$

However, if M is defined as in previous studies [16, 17] (see Section III.B.), the sum in (19) must be maximized (or its negative value minimized), since the interference measurement decreases when the mean value of the matrix $M(\mu_{ij})$ increases. Extensions of this approach have been proposed that consider a quality threshold, a value that represents an acceptable signal-to-noise ratio [36–38].

The minimal cumulative interference problem may be considered as a generalization of the minimal interference problem [39, 40]. For frequencies f_i and f_j assigned to transmitters e_i and e_j , one measurement of interference would be the difference between the actual separation between these frequencies and the minimal allowable distance between constraint violations. The objective function is as follows:

$$\min(\sum_{f_1=1}^{Card(F)} \sum_{f_2=1}^{Card(F)} \sum_{e_1=1}^n \sum_{e_2=e_1+1}^n x_{f_1 e_1} * x_{f_2 e_2} * \max(|f_1 - f_2|, I_{ij})). \quad (20)$$

Another way to consider cumulative interferences is to attribute a weight or penalty to each constraint [41, 42]. The objective is then to minimize the sum of the weights associated with the violated constraints. This may result in a larger number of less critical violations. Weight factors can be defined at more than one level, depending on whether a constraint is violated by one channel or multiple channels.

Minimizing the weighted sum $\alpha|V| + \beta|W|$ of the co-site ($|V|$) and the remaining constraint violations ($|W|$) has been considered [15]. Usually, $\alpha > \beta$ since serious interferences are caused more often by channel selection constraint (CSC) violations than by other constraint violations [12]. Another formulation with multiple levels has been described elsewhere [43].

(v) Cost of cumulative interferences

In the minimal interferences cost FAP, the objective to be minimized is the sum of the costs that will be paid for every violated interference constraint [4]. For each pair of transmitters e_1 and e_2 assigned frequencies f_1 and f_2 , the cost to be paid is defined as follows:

$$\min \left(\sum_{f_1=1}^{Card(F)} \sum_{f_2=1}^{Card(F)} \sum_{e_1=1}^n \sum_{e_2=e_1+1}^n x_{f_1 e_1} * x_{f_2 e_2} * Inter(f_1, e_1, f_2, e_2) * C_{sig}(e_1, e_2) \right), \quad (21)$$

such that all the requests are fulfilled as in (13), where C_{sig} is the cost function (see Section III.B.). In this problem category, only the costs that exceed a given threshold are considered. Note that in the minimal interference, minimal cumulative interference, and minimal interference cost approaches, all communication requests are fulfilled completely [17–19].

(vi) Number of blocked calls

In some markets, service quality is critical and interferences cannot be tolerated. The constraint becomes the number of communication requests fulfilled. This forms the minimal blocking or maximal service FAP [44]. If for a given request, no frequency can be assigned without creating interferences, this request will be rejected. The objective of minimizing the number of rejected or blocked requests is formulated as follows:

$$\min \left(\sum_{i=1}^n (R_i - \sum_{j=1}^{Card(F)} x_{ij}) \right). \quad (22)$$

In this case, the solution is interference-free.

(vii) Network perturbation

Once an initial frequency assignment plan is established, channel allocation and release must be updated continually, as new requests arrive or ongoing communications end [9]. The objective then becomes to reduce the number of changes made to the initial assignments when managing variations in demand. This creates the minimal perturbation FAP [45, 46].

The different objectives of a FAP can be optimized individually in single-objective optimization or jointly in multi-objective optimization. Single-objective optimization has been examined previously in minimal order [47], minimal span [7, 8, 32–35], minimal interference, [3, 11, 27] and other FAPs [18, 19, 36–38]. In this form, the result is the optimal or nearly optimal frequency plan. Table 1 summarizes the types of FAPs with an indication of the existence of a solution and whether it has been discussed yet by the scientific community.

Multi-objective optimization typically provides superior solutions. In most applications, ensuring high-quality service at the lowest cost requires optimizing more than one objective. Multi-objective optimization has started to receive more attention in the frequency assignment literature but remains limited to considering two objectives, for example, minimizing

Table. I
FEASIBILITY OF SOLUTIONS TO THE DIFFERENT CATEGORIES OF FREQUENCY ASSIGNMENT PROBLEMS (FAPs).

Assignment Problem	Existence of Solution (Yes/No)	Under Discussion (Yes/No)
Minimal order	No	Yes
Minimal span	No	Yes
Minimal interferences	Yes	No
Minimal sum of interferences	Yes	No
Minimal interferences cost	Yes	No
Minimal blocking	Yes	No

the number of constraint violations and the corresponding sum of interferences [48], or combining the minimal order and minimal span problems. The output in the latter case is a solution or set of solutions depending on the decision-maker's choice and the form of optimization (a priori, a posteriori, or interactive) [20].

The strength of multi-objective optimization has motivated researchers to develop a multi-objectivized version of the FAP [49]. In this form, the problem is treated as a single-objective minimization task in a multi-objective framework with a main objective and additional objectives that measure diversity in terms of the distance to the closest neighbour [50] or the average distance to all individuals, and which may be maximized [51–53].

V. MODELS OF THE FREQUENCY ASSIGNMENT PROBLEM

The task of modelling real frequency assignment problems is complex due to the variation between the policies and preferences of telecommunications providers. Graph theory and constraint satisfaction are the main approaches employed in the literature.

A. Graph theory models

In graph theory models for FAPs, a simple undirected graph $G = (V, E)$ is considered, defined by a set of nodes V (also called vertices) and a set of edges E that connect the nodes. The graph G can also incorporate other information describing the peculiarities of the model.

The most widely used model of this type is graph colouring, which consists of assigning an integer colour to each node so that no two adjacent nodes (connected directly by an edge) receive the same colour and so that the number of colours used is minimized, this number being the 'order' of the solution (colouring).

The problem can be formulated as finding a function $C : V \rightarrow \mathbb{N}$, $i \rightarrow C_i$, such that:

$$\forall (i, j) \in E, |C_i - C_j| > 0. \quad (23)$$

Figure 4 illustrates an example of a simple graph and a feasible colouring with an order of three.

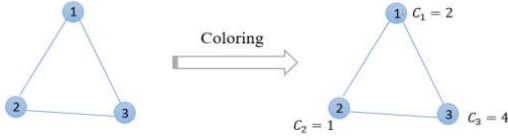


Fig. 4: A simple graph and a feasible colouring of its vertices..

Graph colouring for the FAP was first introduced in the 1970s [54, 55] and has since received much attention [56–59]. Each node represents a transmitter, and each edge connecting two nodes represents either an interference risk or requests between the transmitters, and hence a separation requirement to be satisfied between the pairs of colours (frequencies) assigned to the nodes. The minimal order problem described in III.B. can be modelled as a graph colouring problem when the following conditions hold:

- Each transmitter has exactly one request:

$$\forall i, 1 \leq i \leq n, R_i = 1. \quad (24)$$

- All transmitter nominal sets F_i are the same and equal to the global set F , which is the set of integers:

$$\forall i, 1 \leq i \leq n, F_1 = F_2 = \dots = F_n = F = \mathbb{N}. \quad (25)$$

- All interference separations are equal to 0. This can be expressed as follows:

$$\forall \text{ interfering } i \text{ and } j \in [1, n], |f_i - f_j| > 0. \quad (26)$$

Since simple graph colouring does not allow a complete representation of the FAP, several generalizations have been proposed [4], each seeking to meet additional FAP modelling requirements.

(i) Set or list colouring problem

One generalization assumes that, given a simple graph $G = (V, E)$, each node i of V is associated with a list L_i of allowed colours [51, 52]. The lists are not necessarily of the same length. The list colouring problem is then to assign to each vertex i of V a colour C_i from its set L_i so that two adjacent vertices receive different colours and the number of colours used is minimized. Mathematically, the set colouring problem is to find a function $C : V \rightarrow \mathbb{N}^+$, $i \rightarrow C_i$ such that relationship (23) is satisfied. The standard graph colouring problem is the special case in which all the sets L_i are the same. Set colouring is a perfect model for the minimal order problem when the conditions (20) and (22) apply and the sets F_i differ from one transmitter to another (condition (25) is not met).

Such as $\mathbb{N}^+$ is the set of natural numbers.

(ii) Multi-colouring problem

This generalization incorporates into a simple graph $G = (V, E)$ a set of demands $D = \{d_i, 1 \leq i \leq n\}$ that defines, for each node i , the required number d_i of distinct colours. The task is to assign the colours so that adjacent nodes do not receive similar colours. The order must be as small as possible. The problem can be formulated as finding a function $C : V \rightarrow \mathbb{N}^+$, $i \rightarrow C_i$ such that

$$\forall i, 1 \leq i \leq n, C_i = \{C_{i1}, C_{i2}, \dots, C_{id_i}\} \quad (27)$$

and

$$\forall (i, j) \in E, \forall 1 \leq k_i \leq d_i, \forall 1 \leq k_j \leq d_j:$$

$$|C_{ik_i} - C_{jk_j}| > 0. \quad (28)$$

This models the minimal order problem when conditions (25) and (26) apply, as described in [60, 61].

(iii) T-colouring problem

In T-colouring, the simple graph $G = (V, E)$ is weighted [4, 7]. A set D_{ij} of non-negative integers containing 0 is assigned to each edge E so that the absolute difference between the colours assigned to vertices i and j does not belong to D_{ij} . The problem is equivalent to finding a function $C : V \rightarrow \mathbb{N}$, $i \rightarrow C_i$ such that

$$\forall (i, j) \in E, |C_i - C_j| \notin D_{ij}. \quad (29)$$

If the sets D contain consecutive integers, condition (29) reduces to the following and is called a restricted T-colouring problem [62, 63] or bandwidth colouring problem [64]:

$$\forall (i, j) \in E, D_{ij} = \{0, 1, \dots, d_{ij}\}, |C_i - C_j| > d_{ij}. \quad (30)$$

The task is to find a feasible colouring of the vertices of G using the smallest colour bandwidth, called the ‘span’ [65–68]. This can model the minimal span problem when conditions (24) and (25) are met.

Many combinations of the above generalizations have been studied: set T-colouring, list T-colouring, multi-set-colouring [69–71] and various transformations [9, 56].

B. Constraint satisfaction problem models

The constraint satisfaction framework is now used extensively to describe and solve many optimization problems in which some constraints must be satisfied while the others must be prioritized, as is the case in planning, scheduling, and design. It has been applied to model [72], then to improve [73] the lower bound on the overall valuation of any partial frequency assignment. Table 2 summarizes the types of graph colouring that have been applied to FAPs along with a description of the separation constraints set, the number of nodes that are

Table. II SUMMARY OF THE FEATURES OF FAPS SUPPORTED BY GRAPH COLOURING.			
Problem	Separation constraints set	Requested number per node	Nominal sets
Simple graph coloring	$\{0\}$	1	$F_1 = F_2 = \dots = F_n$ $= F = \mathbb{N}$
Set coloring	$\{0\}$	1	Not necessarily equal
Multi-coloring	$\{0\}$	$\mathbb{N}$	$F_1 = F_2 = \dots = F_n$ $= F = \mathbb{N}$
T-coloring	$\mathbb{N}^*$	1	$F_1 = F_2 = \dots = F_n$ $= F = \mathbb{N}$

required, and the corresponding nominal set for each type of problem.

Such as $\mathbb{N}^*$ is the set of signed whole numbers excluding 0.

VI. OPTIMIZATION METHODS FOR THE FREQUENCY ASSIGNMENT PROBLEM

As mentioned above, frequency assignment is an NP-hard problem for which an optimal solution cannot generally be found in polynomial time. Furthermore, the current large number of applications in the telecommunications field and the limited frequency spectrum make interference-free assignment difficult and often impossible. Consequently, most recent studies propose approximately optimal solutions or a compromise between the different objectives. Upper and lower bounds of the objective function and some exact methods have nonetheless been proposed under special conditions. In this section, we present and categorize the proposed approaches to solving variants of the FAP.

A. Exact methods

Table. III

FEASIBILITY OF SOLUTIONS TO THE DIFFERENT CATEGORIES OF FREQUENCY ASSIGNMENT PROBLEMS (FAPS).

References	Methods	Objectives
[29]	Tree search algorithm	Interference handling
[46]	Integer linear programming with exact branch-and-cut technique based on strong cutting planes	Achieve an acceptable signal-to-noise ratio
[56]	Exact enumerative method	Achieve an acceptable signal-to-noise ratio
[74]	Quadratic 0–1 integer programming	Reduce interference
[75]	Two-layer conflict graph and branch-and-bound method	Partial constraint satisfaction
[76]	Russian doll search method	Bandwidth reduction
[77]	Polyhedral structure and $\{0, 1\}$ linear programming	Maximum satisfiability
[78]	Strong-edge graph coloring and butterfly, Benes, hyper tree, and honeycomb	Minimal required number of frequencies
[79]	Scheduling algorithm	Reduce the interference
[80]	Game theory	Reduce channel interference
[81]	Binary search	Minimizing the frequency utilization
[7], [64], [82], [85]	Graph theory, labelling, and colouring	Minimize the interference and the spectrum of used channels and maximize the throughput
[86]	Coherent Ising machine	Channel distribution with total-throughput maximization

Exact methods have been extensively used in the literature to solve various optimization problems and more specifically the FAP problem. Table 3 summarizes the exact optimization methods along with the optimization objectives that have been examined in the literature.

One of the most extensively studied methods of frequency assignment in the literature is the tree search algorithm [29]. This technique has two main steps: first constructing a tree and choosing a search process such as depth-first or best-first, then

processing the tree nodes using reduction techniques such as cutting-plane algorithms or combinatorial lower bounds. The tree search algorithm is applicable to all FAP variants, since they all share an important structure, namely frequency assignment and interference handling. However, their objective functions differ, and lower and upper bounds must be determined using appropriate means for each model.

Quadratic 0–1 integer programming has been used to solve interference cost minimization with some success compared to published benchmarks [74]. An exact tree search algorithm called BBFAB includes all elements mentioned above in a method that introduces a family of lower bounds and three reduction/dominance rules to strengthen and reduce the size of the problem.

A mathematical formulation of the FAP for mobile radio systems in a geographical region has been presented as an integer linear program using an exact branch-and-cut technique based on strong cutting planes [46]. Tested on 85 real-world instances provided by the Centro Studi E Laboratori Telecomunicazioni (CSELT), this technique was able to provide the exact solution of instances with up to 203 nodes in acceptable computation time.

The authors of [56] introduced an algorithm for frequency assignment in mobile phone, radio, and television systems. Frequency allocation must meet interference requirements to achieve an acceptable signal-to-noise ratio. To this end, an exact enumerative method was used by setting criteria that reduce the size of the instances. The well-chosen sub problem algorithm was used to solve larger instances by extending the obtained solutions. Applied to real-life situations, this approach has produced better results than previously reported methods.

The assignment problem has been modelled as a graph colouring problem in the cognitive radio network context, and solved using efficient centralized and distributed sequential algorithms based on dynamic vertex ranking with novel cognitive-radio-specific saturation metrics [7]. Interference susceptibility is represented as a two-layer conflict graph that determines the interference potential using the weights of the co-channel and adjacent channel edges. A dynamically updated local list of locked channels in the interference zone protects the primary users. In addition, frequency and bandwidth selection are considered instead of channel selection in the frequency assignment decision process.

The authors of [75] solved the partial constraint fulfilment problem using a branch-and-bound method, which was described using a genetic algorithm. In [76], the authors present the Russian doll Search method, which substitutes one search with n subsequent searches on nested subproblems. This resulted in good performance in both small and large cases.

In [77], the polyhedral structure of the partial constraint satisfying problem is investigated as a potential model of FAPs. The authors express the partial constraint fulfilment problem as a $\{0, 1\}$ linear programming problem and prove theorems for lifting aspects of a sub problem to facets of the entire problem.

The minimal required number of frequencies has been determined for interconnection networks, namely the butterfly, Benes, hyper tree and honeycomb networks, by treating the FAP as a strong-edge graph colouring problem [78].

To reduce interference in broadband wireless networks, an efficient channel assignment algorithm has been proposed to

avoid interference in multi-channel wireless mesh networks by optimizing the number of channels [79]. The algorithm uses scheduling to avoid primary interference, reduces the transmission delay for uplink/downlink networks, and prioritizes to prevent packet accumulation in multi-hop networks.

A game theoretic framework has been used to obtain a frequency assignment algorithm for networked unmanned aerial vehicles, whose optimality and computational complexity were validated through numerical simulation [80]. To maintain access to military communications and space navigation for combatant commanders, frequency assignment in two-way traffic across a link using two disjoint bands has been studied [81]. Two variants of this problem were considered as separate tasks: (i) minimizing the frequency utilization of the more congested air platform in the two frequency bands while meeting the traffic demand, without violating the frequency constraints, and (ii) the minimal span problem of minimizing frequency utilization in both bands. An optimal minimal span solution was found using a binary search, providing a feasible solution to the first task, which is a sub problem of the other, thus reducing the computational complexity of the solution.

A model that fits the FAP into graph theory is $L(3,2,1)$ -labelling. This model assigns integer frequencies to radio transceivers such that (i) transceivers located one unit apart get frequencies that differ by at least three, (ii) transceivers located two units apart get frequencies that differ by at least two, and (iii) transceivers located three units apart get frequencies that differ by at least one. A generalization of (s,t) -relaxed k - $L(2,1)$ -labelling is used to establish the optimal (s,t,r) -relaxed k - $L(3,2,1)$ -labelling for paths and certain cycles and for hexagonal and square lattices, thus constituting a solution for the FAP in these three cases [82, 83].

A method of finding the optimal solution to the channel assignment problem of mobile communication systems, with multiple base stations requiring the same number of channels, uses n -fold t -separated $L(j_1, j_2, \dots, j_m)$ -labelling when the corresponding interference graph consists of triangular and square lattices, and consecutive labelling when the corresponding interference graph is a complete graph or cycle [84].

The FAP has been also formulated as k - $L(p,1)$ -labelling of graphs [61, 64], which involves finding a function $f: V(G) \rightarrow \{0,1,2,\dots,k\}$ such that $|f(u) - f(v)| \geq p$ if $d(u,v) = 1$ and $|f(u) - f(v)| \geq 1$ if $d(u,v) = 2$, where $d(u,v)$ denotes the distance between vertices u and v . In the case of planar graphs without 4 or 6 cycles, the authors of [85] propose upper bounds that are obtained in a constructive manner that can be harnessed as a labelling method for the channel assignment problem and for other types of labelling and colouring problems.

Ultra-fast and rigorous channel distribution has been achieved via an entirely different approach using a coherent Ising machine for dense WLAN systems [86]. Based on a high-speed laser system, this material technique uses optoelectronic properties to solve Ising problems. The FAP is formulated first as a total throughput maximization problem for large-scale centralized WLAN systems, and then converted to an Ising machine problem. Experimental results obtained within milliseconds ('ultra-short' times) confirmed that total throughput was thus improved.

B. Greedy algorithm-based methods

A greedy algorithm is a constructive process whereby a solution is built piece by piece, selecting the best option at each stage. For FAPs, a greedy algorithm assigns frequencies iteratively by selecting a vertex and then assigning to it the best feasible frequency that optimizes the global objective function. The resulting solution is usually adequate but only sometimes optimal.

Distributed greedy and game theoretic approaches to the channel allocation and sharing problem have been considered in cognitive radio mesh networks for Internet of Things systems to maximize network energy efficiency while satisfying scalability and self-organization constraints [87].

A polynomial-time probabilistic greedy algorithm has been proposed for cellular networks using a complete edge-weighted FAP graph in which every call is represented by a vertex and an edge weight represents the minimal frequency separation required between calls. The objective is to assign frequencies (colours) to the vertices so that the requested span is minimized. The algorithm performs two main tasks: ordering the nodes in a probabilistic manner and executing an exhaustive vertex colouring strategy using an iterative compression phase to further reduce the obtained span. Its execution time and deviation from optimality compared favourably to the benchmarks [34]. Incorporating randomization and net filter discrimination effects into the communication model has been found to considerably improve the early greedy and fast heuristics [88].

C. Trajectory methods

Among the most common methods of solving NP-hard problems are algorithmic techniques called local search methods, which may be used alone or incorporated into other algorithms to explore solutions sequentially along a rational trajectory through the solution space. This type of approach needs to proceed from an initial feasible solution, which often means beginning by building a heuristic solution.

A real-time solution to the FAP has been sought using mathematical models that incorporate field measurements of real-time interference [18], as practised in existing GSM networks. A set of hybrid metaheuristics created for this purpose, and combining the advantages of global and local search methods, was found to perform better than previous solutions, based on tests with two real GSM networks. Two of the metaheuristics performed better than the others, namely a population-based scatter search algorithm and a trajectory-based (1+1) evolutionary algorithm [85].

A local search heuristic and a compound motion-based heuristic for fixed spectrum frequency assignment in telecommunication networks [89] has been found to outperform the dynamic list-based tabu search [90] and the heuristic manipulation technique [91] in 88% and 79% of tests, respectively.

A revised version of the tabu search algorithm including a dynamic tabu for the fixed spectrum frequency allocation problem is claimed to be efficient in tests against several benchmark sets and existing algorithms [92], improving on some of the best allocations available for COST259 benchmarks and proving very efficient when the best solution is required without constraints on the runtime. This approach is claimed to reduce the number of rejected calls in exchange for a small increase in interference.

D. Population-based methods

Metaheuristics are techniques for searching a space of feasible solutions in a structured manner to determine near-optimal solutions in a short time. They provide no guarantee of optimality, are often derived from natural processes, and are increasingly hybridized with other methods to mitigate their weaknesses. In particular, a strategy for avoiding getting stuck in a region of the search space may be necessary.

To solve the FAP, two iterative strategies of implementing an ant colony system for the graph colouring problem have been proposed, namely construction of feasible solutions and solution improvement through tabu search [57]. In extensive comparisons, this approach outperformed hybrid graph colouring algorithms on large instances of the Dimacs benchmarks.

The T-colouring problem is a generalization of the classical graph colouring problem. It has been applied to frequency assignment using an algorithm inspired by the mating process in real bee colonies [60] in order to solve the graph colouring, restricted T-colouring, and general T-colouring problems [7, 8]. A hybrid evolutionary approach combining the ant colony algorithm and tabu search is also described. A comparison between bee mating-based and ant colony-based optimizations demonstrates the effectiveness of the method using benchmark methods.

A well-adjusted ant-colony-inspired optimization algorithm has been shown to outperform an evolutionary algorithm at solving the FAP [17]. A continuous-domain ant colony algorithm [93] inspired by the natural behaviour of *Pachycondyla apicalis* ants [94] has also been found to be highly suitable in computations of standard problem instances, in direct comparison with tabu search and simulated annealing.

An extensive study of three hybridizations of the stochastic competitive Hopfield neural network and the genetic algorithm showed that when the neural network manages the constraints and the genetic algorithm searches for the highest-quality solutions with the lowest costs, the performance is similar or superior to other algorithms on five benchmark problems and 12 large, randomly generated problems [95].

Parallel cooperation between ‘marriage in honey bees’ optimization and two ant colony strategies for solving the graph colouring problem has been studied [96]. Various collaboration and parallelization modalities, executed using a simulated parallel machine on a cluster of PCs, showed that one ant colony-based construction strategy was faster than the other methods, thanks to the parallel implementation. Combining the two strategies gave better results than either algorithm alone.

The ability of artificial immune systems to solve optimization-related problems has been harnessed to design a cellular frequency allocation model [27] that was found to be efficient and able to reuse available non-conflicting frequencies. Another approach is combining particle swarm optimization with tabu search, which is claimed to reduce premature convergence to a local minimum to achieve global optimization [13].

A new low-cost model has been implemented to reduce optimization solution storage volume and thus evaluation time. In tests against most benchmarks, the proposed algorithms were found to be superior to particle swarm optimization, tabu search, ant colony optimization, simulated annealing, and

genetic algorithms [22, 97–99] in terms of convergence rate, number of iterations, solution storage volume, and execution time needed to converge to the optimal solution.

An algorithm that ensures frequency allocation in cells of a cellular network has been obtained by hybridizing the Hopfield neural network and the tabu search algorithm, while adopting the principles of short-term memory and the tabu search candidate list in the control of the neural selection process [100].

A diversity management strategy based on replacement has been applied to overcome the premature convergence that occurs when using some of the most recently developed evolutionary algorithms [101]. In addition to the possibility of transforming a single-objective problem into a multi-objective problem by considering diversity as an additional objective, the proposed extensions were tested on 44 instances of two FAP variants, which led to the updating of the benchmark method, based on superior solutions (slowing of population convergence) for 11 instances.

Three graph theoretic heuristics, namely greedy, greedy colouring-based, and clique colouring-based, have been tested for the joint assignment of antenna frequency and polarization in a cluster-based tactical communication environment [23]. These heuristics are of low complexity and are quasi-optimal in terms of chromatics and thus very suitable for practical use. When executed in artificial degenerating environments, the coloring-based approach clearly outperformed the greedy algorithm, with few exceptions. A clique coloring-based algorithm was able to find the optimal solution.

A particle swarm optimization augmented by an approach called guided search, designed to minimize an interference cost, was found to improve the particle location updating operation and thereby avoid local minima [40]. This enhanced the convergence rate, robustness, and stability of the results significantly. A gravitational search algorithm [102, 103] tested against benchmark combinatorial algorithms for military applications was also found to be satisfactory [104].

The static version of the single-objective routing and spectrum assignment problem in flexible-grid optical networks has been tackled using an efficient genetic algorithm-based metaheuristic described as providing neo-optimal solutions [105]. Compared to exact methods based on integer linear programming and other classical heuristics, the proposed method generated optimal solutions in a short execution time in a variety of scenarios [106].

An initial attempt to use an adapted version of the harmony search algorithm to solve the minimal interference problem was unsuccessful due to premature convergence and getting stuck repeatedly in local optima [73]. This was overcome by combining the algorithm with local search and opposition-based learning. Although the resulting performance was comparable to the literature results, the method needs improvement to be competitive.

A hybrid heuristic with no problem-specific parameters (unlike most metaheuristics) has been proposed, based on the non-dominated sort-II genetic algorithm and tabu search [107]. At each iteration, tabu search is applied to each element of the population to be substituted by the best solution in its neighbourhood, and the elements of the next generation are then selected using the non-dominated sorting algorithm considering

the interference and entropy of each solution. Experimental tests indicate that the proposed heuristic provides higher-quality solutions than four existing heuristics and has an equivalent convergence time.

In [108], the channel assignment problem was formulated as a time slot allocation problem and solved using a vertex coloring algorithm. This approach offered a significant improvement in network throughput, transmission delay, and packet loss rate compared to distributed coordination function and TMCA algorithms. In a study of the dynamic FAP, new requests were incorporated gradually, and frequencies were allocated to them while minimizing the number of reassignments [108]. The problem was formulated as a vertex colouring problem in a graph and treated using a heuristic approach based on both the canonical correlation analysis algorithm and a process for slot reservation. On dynamic datasets generated from static benchmark datasets, performance was superior to the state-of-the-art approach.

The problem of frequency distribution between wireless radio stations operating in a high- and a low-frequency band has been addressed using a model of the electromagnetic compatibility problem [109]. An effective solution was thus obtained faster than by the conventional backtracking method.

Game theory has been combined with centralized and distributed behaviour, while respecting net filter discrimination, to obtain a heuristic for allocating and utilizing frequency resources in communication systems [110]. This allowed maximizing the sum of the user signal-to-interference-to-noise ratios, minimizing the standard deviation thereof, and reducing the computational complexity.

The minimal interferences problem has been solved using three metaheuristics: variable neighbourhood search, stochastic local search, and a hybridization of these two, with the latter incorporated as a subroutine [111]. In empirical experiments, the hybrid method gave higher-quality solutions than either search method alone and offered an acceptable trade-off between scalability and diversification.

Particle swarm optimization of static channel assignment in maritime communication networks has been shown to be efficient at reducing interference in multi-radio simulations performed in NS-3, considerably improving the performance of both sparse and dense networks with fewer iterations than in previous studies [112]. The channel assignment was found to modify the routes, which can lead to unnecessary channel changes. A newer swarm-intelligence metaheuristic [113], based on an algorithm inspired by fireworks explosions in the night sky [114], has been shown to converge quickly to global optimal solutions to the FAP.

The dynamic FAP has been modelled as a combinatorial optimization problem, by separating it into two phases to develop a metaheuristic method of finding the solution for a tactical wireless communication network [115]. This involved optimizing the neighbourhood search process and using simulated annealing and greedy algorithms to facilitate the frequency regularization process and generate new solutions with fewer frequencies and reduced influence of interferences. The superiority of a simulated annealing approach to solving the minimal interference problem has been demonstrated in several instances [116] using two benchmarks [44, 117]. This study provided the first known instance/solution database for this FAP.

The dynamic FAP is studied in [118], in which new requests are gradually introduced and need frequency assignments. A heuristic approach is proposed to solve this problem using different solution methods. The gap technique is also used to improve efficiency.

The authors of [119] describe the FAP by considering the electromagnetic compatibility (EMC) problem. They concentrate on the many types of interference. When utilized in military situations, the authors discovered that their method was faster than the standard counterpart (the backtracking method).

A modified version of the recently described immune plasma algorithm, a metaheuristic inspired by a technique used to process SARS-CoV-2 coronavirus convalescent or immune plasma [120], has been proposed as a more efficient solution to the channel assignment problem [121] compared to brute force search. The latter work proposes a fairness-oriented semi-chaotic genetic algorithm-based channel assignment technique to solve the node starvation problem in wireless mesh networks. Unlike previously published methods, this technique, applied with a nonlinear fairness-oriented fitness function, ensured maximal link fairness and minimal link interference. A multi-criteria link-ranking channel assignment algorithm was used to create the primary chromosome engendering a population that searched for global minima. High node equity was maintained while network resource use efficiency was increased by up to 23%.

E. Multi-objective methods

Methods dedicated to solving the multi-objective frequency allocation problem are summarized in this subsection.

Approximation algorithms, namely tabu search, simulated annealing, and DDS, have been applied to simultaneously reduce the total interference cost and the frequency spectrum usage in the absence of interference [122], and have been compared in practical instances to using a nonlinear relaxation lower-bound technique to evaluate solution quality [123].

In [17], a multi-objective memetic algorithm based on the non-dominated genetic sorting algorithm II was applied to a single-objective FAP with an artificial objective attached to circumvent stagnation. This approach was validated in tests on real-world instances and shown to provide high-quality solutions in reasonable time and to improve on the best previously known frequency plans in those instances.

In [124], a variant of the multi-swarm particle swarm optimization (MS-PSO) method was proposed to optimize a bi-objective FAP [49]. A condition to be satisfied by the optimal solution was described and incorporated into the conventional MS-PSO algorithm to avoid convergence to a local minimum and to define the search direction. Simulations generated solutions that provide a best-fit compromise between the two objectives, and the embedded condition yields better solutions compared to the conventional MS-PSO algorithm [125, 126].

In another line of research, [127] focuses on the aspects and characteristics of real GSM networks to propose a hybrid parallel model that combines an island-based model with a hyper-heuristic to address a multi-objective version of the FAP. A thorough robustness analysis of the proposed model, as well as the results of its application to several real network instances, demonstrate its efficiency in obtaining high-quality solutions in a short time. Moreover, it offers a remarkable improvement over the best-known frequency plans in the literature.

An ant colony optimization method has been described for multi-objective channel assignment aimed at achieving high spectral efficiency in mobile ad hoc networks with a clustered topology, through a trade-off between maximizing spectral utilization and minimizing interference [128]. Stable and scalable solutions were thus found with a minimal number of assigned channels, independently of the number of network clusters.

The numerical parameters of a diversity-based multi-objective evolutionary algorithm have been fitted specifically to the FAP using a hybrid control scheme based on both fuzzy logic controllers and hyper-heuristics [28]. Based on evaluation in benchmark instances and real-world cases, this approach produced equivalent or superior frequency plans to those delivered by a significant number of standard diversity-based multi-objective configurations.

The problem of channel assignment in multi-cell wireless local area networks has been formulated as a mixed-integer linear program aiming to minimize the worst-case interference at the network access points [129]. Simulations showed that a proposed optimization model achieves the objective with better fairness than the pick-first greedy algorithm, the optimal model, and the max-min model.

The preminent notions of the Nash equilibrium from game theory and Pareto optimality have been combined to obtain a hybrid multi-objective genetic algorithm for solving the multi-objective FAP in mobile networks [130]. Several tests performed on benchmark instances (Philadelphia FAP and COST259) showed that this method provides high-quality solutions that are both Nash equilibria and Pareto non-dominated. Its performance was confirmed in comparisons with the most-referenced multi-objective optimization algorithms, such as the non-dominated sorting genetic algorithm II (NSGA-II) and the improved version of the strength Pareto evolutionary algorithm (SPEA2) on the same instances. A multi-objective immune memetic algorithm has been described for solving the FAP in cellular networks with the goal of minimizing network total interference, peak interference, and the number of frequencies used [131]. The proposed approach replaces the crossover and mutation evolutionary processes with a local FAP-specific search strengthened by a guided diversification strategy. The algorithm's exploration and exploitation capabilities are further enhanced by clonal selection inspired by artificial immune systems.

A hybridization of a multi-objective memetic algorithm and an artificial immune system hybrid algorithm has been investigated for solving the multi-objective FAP in cellular mobile networks [132]. The objectives considered simultaneously are minimizing total interference, peak interference, and the number of frequencies used. Exploration and exploitation capabilities were improved by integrating a local FAP-specific search into the evolutionary process, clonal selection, and receiver editing. Tests on COST259 instances showed the effectiveness of the proposed approach in generating high-quality solutions complying with the hyper volume metric. These results have found support in comparison with the most frequently cited algorithms in the literature. The impact of the key parameters and their relationships were analysed using statistical tools.

VII. CONCLUSION

We have surveyed the available literature on the FAP, a critical technical challenge for several industries, with specific focus

on network applications. The definitions of the problem and its variants as well as its myriad proposed solutions are summarized and presented along with their mathematical formulations. The FAP has been modelled using a wide variety of approaches. The literature is replete with descriptions of optimization-based methods used to find exact and approximate solutions to the FAP. This review is expected to be a reference and a guide for researchers and practitioners dealing with or interested in channel frequency allocation and management problems. It allows for the identification of prospective improvements in mathematical formulations and solutions, as well as potential advancements and contributions to the field.

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Estimation of Synchronous Machine Parameters by Stand Still Frequency Responses Testing

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Abstract– This paper presents a general method based on the standstill frequency response (SSFR) tests cited in IEEE Std 115-A. The used methodology consists of determining the frequency response of the measured values of the direct axis and quadrature operational inductances. Subsequently, we identify the different models of transfer functions, which are functions of time constants, using curve-fitting techniques. Then considering the time constants as inputs and the parameters of the equivalent circuits of the synchronous machine as outputs. We can identify all the parameters of different topologies of the models, using the numerical Newton Raphson method. The approach used is inexpensive, accurate, and reliable.

Keywords– Synchronous machine, standstill frequency responses tests, equivalent circuit, identification parameters.

NOMENCLATURE

s	Laplace's operator
V_d	d-axis stator voltage
V_q	q-axis stator voltage
V_f	d-axis field voltage
i_d	d-axis stator current
i_f	field current
v_{arm}	stator voltage during test
i_{arm}	stator current during test
$Z_d(s)$	d-axis operational impedance
$L_d(s)$	d-axis operational inductance
L_d	q-axis synchronous inductance
R_a, R_f	stator and field resistances
R_k, R_b, R_e	d-axis damper resistances
L_k, L_b, L_e	d-axis magnetizing inductances
L_a	armature leakage inductance
L_{md}	direct-axis stator to rotor mutual inductance
T_{d0}'	d-axis transient open circuit time constant
T_{d0}''	d-axis transient short-circuit time constant
T_{d0}'''	d-axis subtransient open circuit time constant
T_{d0}''''	d-axis subtransient short-circuit time (t) constant
T_{d0}'''''	d-axis sub-subtransient open circuit time constant
T_{d0}''''''	d-axis sub-subtransient short-circuit time constant
T_{d0}'''''''	d-axis sub-sub-subtransient open circuit t constant
T_{d0}''''''''	d-axis sub-sub-subtransient short-circuit t constant
$G(s)$	stator to field transfer function
$Z_{af}(s)$	stator to field transfer impedance

I. INTRODUCTION

Synchronous generators play a crucial role in power supply systems, and to study their stability and power control, understanding the parameters of synchronous machines is imperative [1]. Precise identification of these parameters is of utmost importance, and the literature presents various measurement techniques and identification methods for determining synchronous machine model parameters [2]. The graphical analysis of short-circuit tests [3,4] a classic method

outlined in IEEE standard 115 [5] enables obtaining parameters of the d-axis but does not identify q-axis parameters. Certain investigations [6-8] based on time analysis, such as Standstill Time, Response (SSTR) and Rotating Time-Domain Response (RTDR) described in IEEE standard 115 [5], have been explored. RTDR has been employed to determine machine parameters along both axes [9,10]. The rapid evolution of computers has given rise to several identification methods for synchronous generator models. On-line measurements during normal machine operation fall into two categories: "grey box" modelling, assuming a known model structure like orthogonal series [11,25] or Kalman filter [24], and "black box" modelling, where no model structure is assumed. In the latter, the goal is solely to establish the correspondence of inputs to outputs using methods like neural networks [14, 15] or Volterra series [16]. The prevailing approach for determining d-q model parameters involves Standstill Frequency Response (SSFR) tests introduced in [17]. During SSFR tests at standstill, the machine is stationary, and the rotor aligns with the d-axis or q-axis. Two stator phases are supplied in series by a sinusoidal voltage source with variable frequency. Machine parameters are then determined through a transfer function optimization process characterizing the d-q model. In spite of the widespread use of SSFR, detailed experimental setups are seldom provided in publications [18], including technical characteristics of measuring and recording devices, frequency range and number, and the magnitude of the source voltage. Such information is vital for obtaining satisfactory measurements for data analysis. Some authors have investigated factors that can impact SSFR results, such as the level of machine magnetization during testing [19, 20] and variations in stator resistance [21]. Beyond the experimental aspects, the SSFR method encounters another challenge related to parameter identification from the collected data. Similar to any system identification problem, selecting the model type, estimator, minimization algorithm, and initial values becomes essential. The IEEE standard 1110 [22] outlines various potential structures for the d-q model, with variations primarily contingent on the number of branches employed to depict the rotor circuit in each axis based on rotor construction type [23]. In this paper, we present a general procedure, with a judicious choice of a topology of the equivalent circuit of the synchronous machine. We start by using the curve fitting techniques to obtain the suitable model, then we use the Newton Raphson method to determine the parameters of the circuit. Our paper is structured in five sections. Section II is devoted to the development of models of the synchronous machine. In section III the stand still frequency

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responses tests (SSFR) are presented on d-q axis for data determination. Section IV develops the general process for extracting machine parameters. In section IV we present the obtained results and their validation. Finally, the conclusion is presented in section V.

II. DIRECT-AXIS MODEL STRUCTURE OF A SYNCHRONOUS MACHINE

The determination of synchronous machine parameters from the transfer function represents the operational inductance as a function of frequency. The operational inductance is derived from the machine impedance, which is measured across the stator terminals. This method involves three main steps:

- Conversion from impedance to operational inductance.
- Determination of the time constants from the operational inductance.
- Determination of machine parameters from time constants and inductances.

The conversion from machine impedance to operational inductance is based on the direct-axis equivalent circuit of a synchronous machine, Fig. 1.

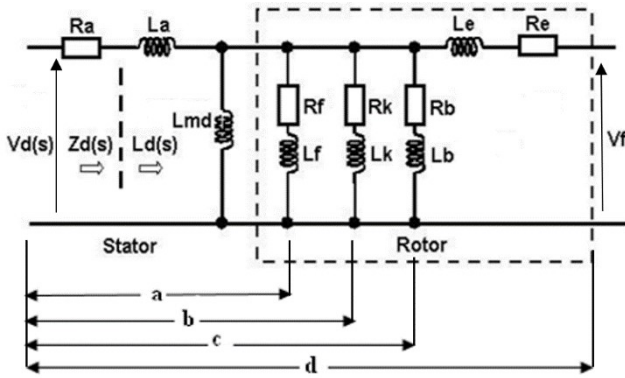


Fig. 1: Equivalent Circuit of a fourth order model for direct axis

A. Synchronous machine models

1. First order model (one rotor circuit)

The fundamental equivalent circuit is the one that incorporates one field circuit in the direct axis and one damper winding in the quadrature axis, depicted in Fig. 1.a. The operational inductance can be expressed in the following manner [12]:

$$Ld(s) = \frac{Rf(La + Lmd) + s(LaLmd + LaLf + LmdLf)}{Rf + s(Lmd + Lf)} \quad (1)$$

Equation (1) can be written in the following standard form:

$$Ld(s) = Ld \frac{(1 + sTd')}{(1 + sTd0')} \quad (2)$$

Where

$$Td' = \frac{Lf + La}{Rf} \quad (3.a)$$

$$Td0' = \frac{Lf + Lmd}{Rf} \quad (3.b)$$

$Lamd$ is the parallel combination of Lmd and La , and

$$Ld = La + Lmd \quad (3.c)$$

2. Second order model (two rotor circuits)

Figure 1.b represents the equivalent circuit of the operational inductance of the second order model. The operational inductance of this model can be written in the following form [12]:

$$Ld(s) = \frac{(La + Lmd)(Rf + sL)(Rk + sLk) + sLaL(Rf + sLf + Rk + sLk)}{(Rf + sL)(Rk + sLk) + sLmd(Rf + sLf + Rk + sLk)} \quad (4)$$

By simplifying equation (4), we get:

$$Ld(s) = Ld \frac{(1 + s(Td' + T1) + s^2Td'Td'')}{(1 + s(Td0' + T2) + s^2Td0'Td0'')} \quad (5)$$

where

$$T1 = (Lk + Lamd)/Rk; \quad (6)$$

$$T2 = (Lk + Lmd)/Rk.$$

The exact time constants of the zeros and poles of equation 6 are obviously determined by extracting the roots of the quadratic equations composing the numerator and the denominator. For the denominator, the roots are:

$$S_{1,2} = \frac{-(Td0' + T2)}{2Td0'Td0''} \pm \frac{1}{2} \sqrt{\frac{Td0' + T2}{Td0'Td0''} - \frac{4}{Td0'Td0''}} \quad (7)$$

and for the numerator

$$S_{1,2} = \frac{-(Td' + T1)}{2Td'Td''} \pm \frac{1}{2} \sqrt{\frac{Td' + T1}{Td'Td''} - \frac{4}{Td'Td''}} \quad (8)$$

Therefore, the operational inductance for a second-order model can be written as:

$$Ld(s) = Ld \frac{(1 + sTd')(1 + sTd'')}{(1 + sTd0')(1 + sTd0'')} \quad (9)$$

3. Third order model (three rotor circuits)

The effect of expanding the model to include other parallel branches on the rotor for other eddy current effects, caps and slot wedges etc.... Consequently add further pole-zero pairs into the frequency response, requiring the identification of additional pairs of time constants.

When the equivalent circuit of the rotor is composed of three branches in parallel, as shown in Figure 1c, the equation of the operational inductance is naturally a ratio of polynomials of third order, for which the denominator is given by

$$\begin{aligned} \text{Den} = [1 + s \left\{ \frac{Lf + Lmd}{Rf} + \frac{Lk + L}{Rk} + \frac{Lb + Lmd}{Rb} \right\} + \\ s^2 \left\{ \frac{Lf * Lb + Lb * Lmd + Lmd * Lf}{Rf * Rb} + \frac{Lb * Lk + L * Lmd + Lmd * Lb}{Rb * Rk} + \right. \\ \left. \frac{Lf * Lk + L * Lmd + Lmd * Lk}{Rk * Rf} \right\} + \\ \left. \frac{s^3 (Lf * Lb * Lk + L * Lk * Lmd + Lb * Lk * Lmd + Lb * Lf * Lmd)}{Rf * Rk * Rb} \right] \end{aligned} \quad (10)$$

The numerator has exactly the same shape as the denominator but with Lm replaced by Lam in each of the coefficients of s , s^2 and s^3 .

$$\begin{aligned} \text{Num} = & \left[1 + s \left\{ \frac{L_f + L_a}{R_f} + \frac{L_k + L_{am}}{R_k} + \frac{L_b + L_{am}}{R_b} \right\} + \right. \\ & s^2 \left\{ \frac{L_f * L_b + L_b * L_{am} + L_{am} * L_f}{R_f * R_b} + \frac{L_b * L_k + L_k * L_{am} + L_{am} * L_b}{R_b * R_k} + \right. \\ & \left. \left. \frac{L_f * L_k + L_f * L_{am} + L_{am} * L_k}{R_k * R_f} \right\} + \right. \\ & \left. \frac{s^3 (L_f * L_b * L_k + L_k * L_{am} + L_b * L_k * L_{am} + L_b * L_f * L_{am})}{R_f * R_k * R_b} \right] (L_a + L_{md}) \end{aligned} \quad (11)$$

And

$$L_d(s) = \frac{\text{Num}}{\text{Den}} \quad (12)$$

So the operational inductance for a third-order model can be expressed as:

$$L_d(s) = L_d \frac{(1 + sT_d')(1 + sT_d'')(1 + sT_d''')}{(1 + sT_{d0}')(1 + sT_{d0}'')(1 + sT_{d0}''')} \quad (13)$$

4. Fourth order model (fourth rotor circuits)

The equivalent circuit of the fourth-order model is shown in Figure 1.d. It follows from the previous analyzes that the transfer function of each model is constructed by adding a pair of zero poles to the lower model. The operational inductance is given by:

$$L_d(s) = L_d \frac{(1 + sT_d')(1 + sT_d'')(1 + sT_d''')(1 + sT_d''''')}{(1 + sT_{d0}')(1 + sT_{d0}'')(1 + sT_{d0}''')(1 + sT_{d0}''''')} \quad (14)$$

III. STANDSTILL FREQUENCY RESPONSE TESTS

Standstill frequency response testing is highly recommended for obtaining synchronous machines parameters compared to short-circuit testing methods. There are three main reasons that show the superiority of this method they are

- The capability to determine the parameters of the both axes, the direct axis and the quadrature axis.
- The ability to identify the parameters of higher order models.
- The results obtained using sudden short circuit tests are only relevant for the second order model (two rotor circuits). In addition these tests remain unable to determine the synchronous machines parameters in the q-axis.

The following subsections describe the general procedure of the SSFR method, as given in reference [5].

1. Measurable quantities

The d and q axis operational impedances are respectively

$$Z_d(s) = R_a + sL_d(s) \quad (15)$$

$$Z_q(s) = R_a + sL_q(s) \quad (16)$$

To obtain the frequency response characteristics of the operational quantities, the following measurements are made:

$$Z_d(s) = \left. \frac{\Delta V_d(s)}{s \Delta i_d(s)} \right|_{\Delta V_f = 0} \quad (17)$$

$$Z_q(s) = \left. \frac{\Delta V_q(s)}{s \Delta i_q(s)} \right|_{\Delta V_f = 0} \quad (18)$$

$$G(s) = \left. \frac{\Delta V_d(s)}{s \Delta V_f(s)} \right|_{\Delta V_f = 0} \quad (19)$$

From the equations (16) and (17) above, the operational inductances in d-q axis can be written as follows

$$L_d(s) = \frac{Z_d(s) - R_a}{s} \quad (20)$$

$$L_q(s) = \frac{Z_q(s) - R_a}{s} \quad (21)$$

The stator resistance R_a is defined by

$$R_a = \lim_{s \rightarrow 0} |Z_d(s)| \quad (22)$$

Or by

$$R_a = \lim_{s \rightarrow 0} |Z_q(s)| \quad (23)$$

We can also measure $sG(s)$. The principle of this test is based on measuring small variations in field and armature current when the field circuit is shorted as shown in equation (24).

$$sG(s) = \left. \frac{\Delta i_f(s)}{s \Delta i_d(s)} \right|_{\Delta V_f = 0} \quad (24)$$

In addition, the armature to the field transfer impedance Z_{afo} is measured, as follows

$$Z_{afo}(s) = - \left. \frac{\Delta V_f(s)}{s \Delta i_f(s)} \right|_{\Delta i_f = 0} \quad (25)$$

2. Measurement setup

The typical configuration of SSFR tests is composed of a sinusoidal signal generator, a power amplifier, the MUT (machine under test) and an analyzer capable of measuring at the same time the amplitude and phase differences of two input signals. The four schemes below show the test configurations of the measurable quantities $Z_d(s)$, $Z_q(s)$, $sG(s)$ and $Z_{afo}(s)$. We note that the frequency range varies from 0.001 Hz to 1000 Hz.

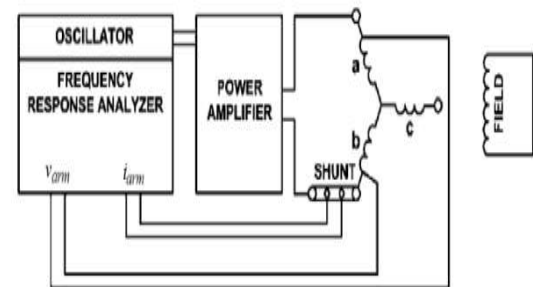


Fig. 2: Test schematic for the d-axis operational impedance, $Z_d(s)$

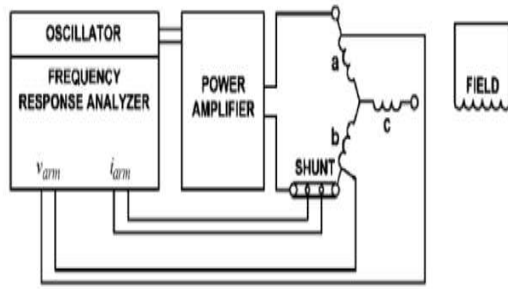


Fig. 3: Test schematic for the q-axis operational impedance, $Zq(s)$

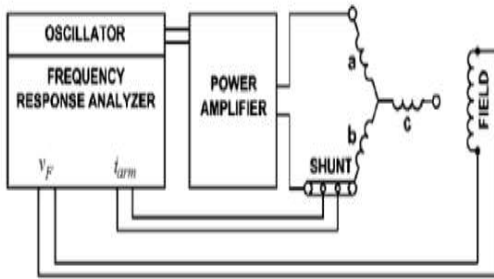


Fig. 4: Test schematic for the stator to field transfer function, $sG(s)$

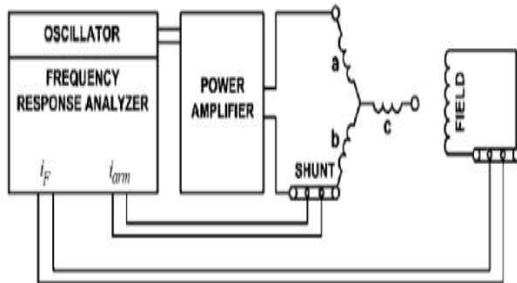


Fig. 5: Test schematic for the stator to field transfer impedance, $Zafo(s)$

IV. METHODOLOGY

The design methodology proposed in this study begins with the definition of the structure of the d-axis equivalent circuits. We are interested in the SSFR1, SSFR2, and SSFR3 models, it should be noted that this methodology can be correctly applied to any circuit topology for the q axis.

1. D-axis parameters from tests

In order to implement our method, a machine with a power of 277.8 MVA and a voltage of 16.5 KV was chosen. SSFR results are published in EPRI [13] and d-axis impedance has been introduced for the process of identifying the parameters of synchronous machine models. Fig. 6 represents the experimental measurements of the impedance $Zd(s)$.

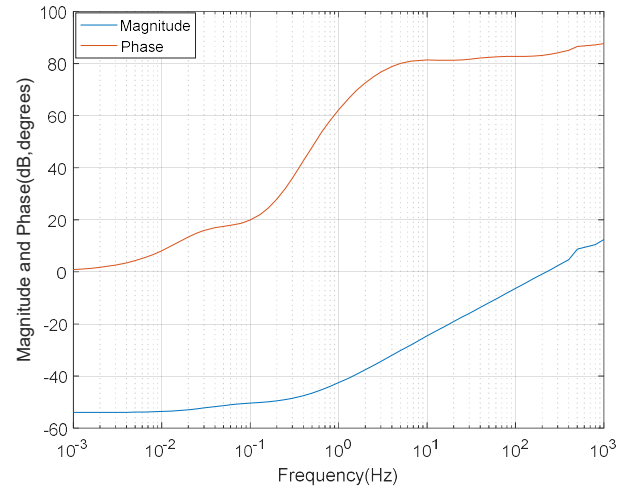


Fig. 6: Frequency response of the impedance (measured data)

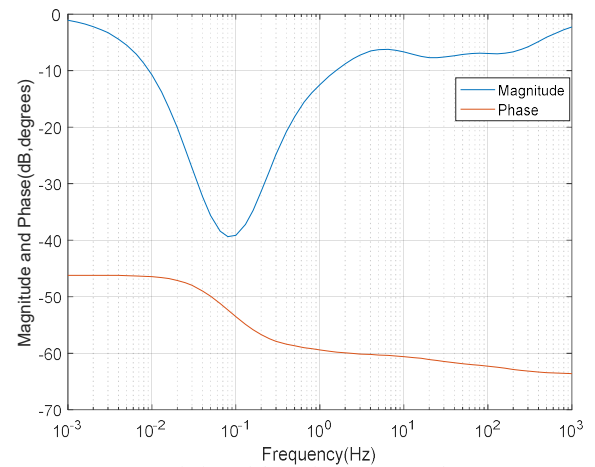


Fig. 7: Variation of the resistance versus frequency

By extrapolating the real part of the operational impedance at zero frequency, we obtain the stator resistance value $Ra = 0.002000 \Omega$, Fig. 7. Equation (21) makes it possible to trace the evolution of operational inductance magnitude in dB and its phase in degree. The extrapolation of the operational inductance $Ld(s)$ at zero frequency, allows calculating the synchronous inductance value $Ld = -46,1991 \text{ dB} = 0.004898 \text{ H}$, Fig. 8.

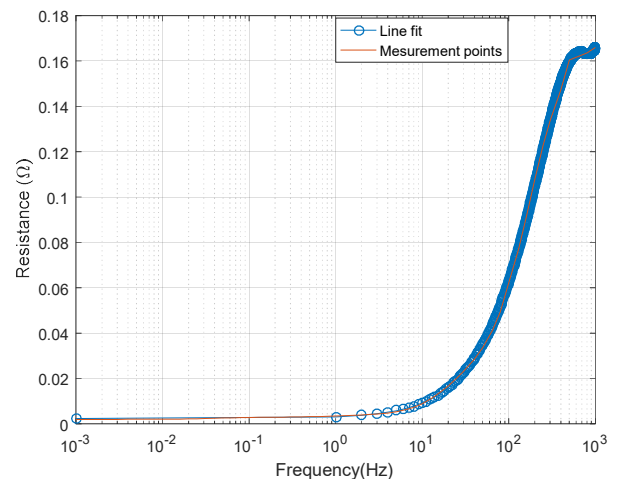


Fig. 8: Frequency response of the operational inductance in terms of magnitude (in dB) and phase in (degrees)

The mutual inductance of the stator in the direct axis to the rotor can be obtained by taking the difference between the d-axis synchronous inductance and the leakage inductance as follows:

$$L_{md} = L_d - L_a \quad (26)$$

The choice of the value of the leakage inductance in the technical literature [14] is taken $L_a = 8\% L_d$.

2. D-axis equations set

In this part, we try to find the relations between the time constants of the operational inductance and the parameters of the equivalent circuit of synchronous machine models. In this context, we are interested in the third order model (SSFR3). We proceed to the development of equation (13), then by identifying it with equations (10) and (11), we obtain the non-linear equations system, whose inputs are the time constants and outputs are the parameters of the equivalent circuit.

The system is:

$$L_{am} = \frac{L_a * L_{md}}{L_a + L_{md}} \quad (27)$$

$$T_{d'} + T_{d''} + T_{d'''} = \frac{L_f + L_{md}}{R_f} + \frac{L_k + L_{md}}{R_k} + \frac{L_b + L_{md}}{R_b} \quad (28)$$

$$\begin{aligned} & T_{d'} * T_{d''} + T_{d''} * T_{d'''} + T_{d'''} * T_{d'} \\ &= \frac{L_k * L_f + L_b * L_{md} + L_f * L_{md}}{R_f * R_b} \\ &+ \frac{L_b * L_k + L_j * L_{md} + L_k * L_{md}}{R_b * R_k} \\ &+ \frac{L_f * L_k + L_f * L_{md} + L_k * L_{md}}{R_k * R_f} \end{aligned} \quad (29)$$

$$\begin{aligned} & T_{d'} * T_{d''} * T_{d'''} = \\ & \frac{L_f * L_b * L_k + L_b * L_k * L_{md} + L_f * L_k * L_{md} + L_f * L_b * L_{md}}{R_f * R_b * R_k} \end{aligned} \quad (30)$$

$$T_{d0'} + T_{d0''} + T_{d0'''} = \frac{L_f + L_{am}}{R_f} + \frac{L_k + L_{am}}{R_k} + \frac{L_b + L_{am}}{R_b} \quad (31)$$

$$\begin{aligned} & T_{d0'} * T_{d0''} + T_{d0''} * T_{d0'''} + T_{d0'''} * T_{d0'} = \\ & \frac{L_k * L_f + L_b * L_{am} + L_f * L_{am}}{R_f * R_b} \\ &+ \frac{L_b * L_k + L_b * L_{am} + L_k * L_{am}}{R_b * R_k} \\ &+ \frac{L_f * L_k + L_f * L_{am} + L_k * L_{am}}{R_k * R_f} \end{aligned} \quad (32)$$

$$\begin{aligned} & T_{d0'} * T_{d0''} * T_{d0'''} = \\ & \frac{L_f * L_b * L_k + L_b * L_k * L_{am} + L_f * L_k * L_{am} + L_f * L_b * L_{am}}{R_f * R_b * R_k} \end{aligned} \quad (33)$$

3. Identifying direct Axis model time constants

Curve fitting techniques are used here, to identify the time constants of the models from the measured data of operational inductance magnitude and phase. The 'freqs' and 'invfreqs' functions in MATLAB convert measured data from the frequency response of the operational inductance to a transfer function for different models.

The following results were obtained with MATLAB:

First order model (SSFR1)

$$L_d(s) = 0.004500 \frac{(1 + 0.491032s)}{(1 + 2.301883s)} \quad (35)$$

Second order model (SSFR2)

$$L_d(s) = 0.004898 \frac{(1 + 0.820584s)(1 + 0.005902s)}{(1 + 3.858375s)(1 + 0.008495s)} \quad (36)$$

Third order model (SSFR3)

$$L_d(s) = 0.004899 \frac{(1 + 0.896976s)(1 + 0.084855s)(1 + 0.002473s)}{(1 + 3.944719s)(1 + 0.101208s)(1 + 0.003354s)} \quad (37)$$

4. Determination of equivalent circuit parameters

The set of nonlinear equations outlined in subsection III.2 underwent simulation through the Newton Raphson method in MATLAB, utilizing the time constants from the previously identified third-order model (SSFR3). This methodology was also employed to deduce the parameters of the SSRF1 and SSRF2 models.

While this method is simple and easy to execute, a significant drawback is its susceptibility to converging towards a local minimum based on the chosen model and initial values. The progression of SSFR3 model parameters over iterations, illustrates that total convergence is achieved around the 8th iteration, Fig. 9 to Fig. 14.

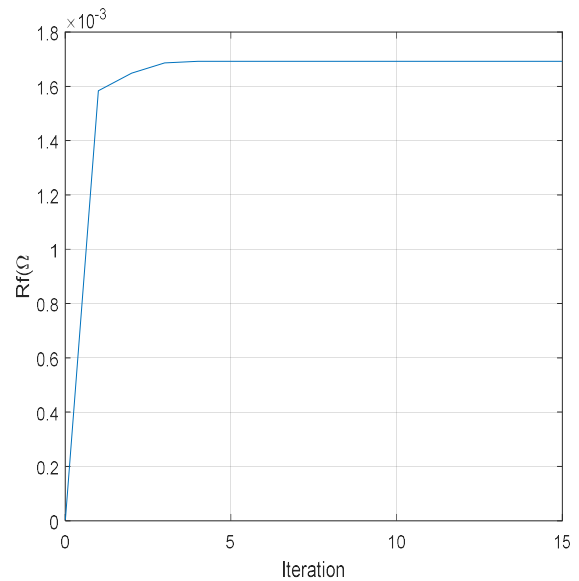


Fig. 9: Variation of resistance R_f versus Iteration

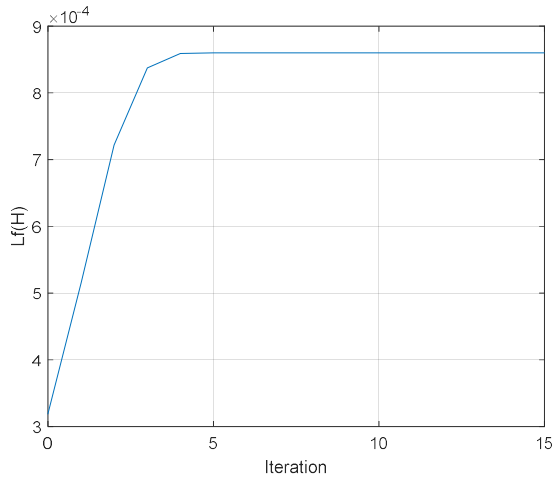


Fig. 10: Variation of inductance L_f versus Iteration

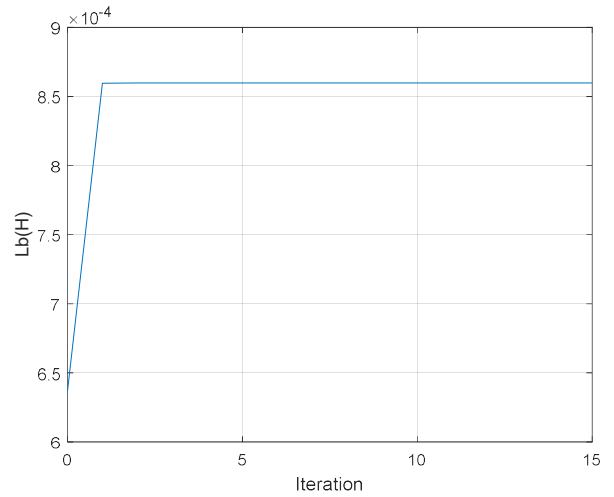


Fig. 13: Variation of resistance R_b versus Iteration

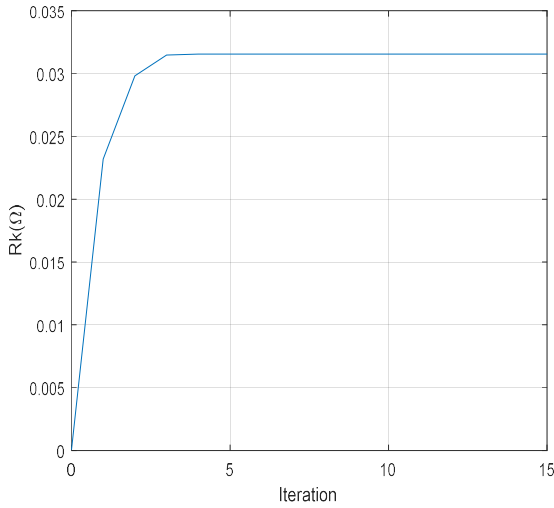


Fig. 11: Variation of resistance R_k versus Iteration

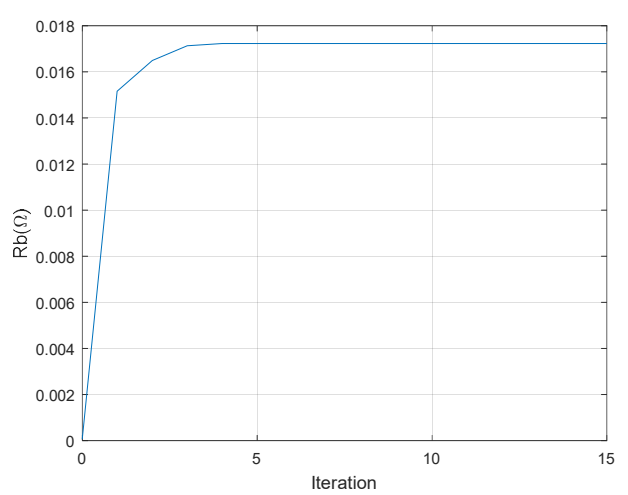


Fig. 14: Variation of inductance L_b versus Iteration

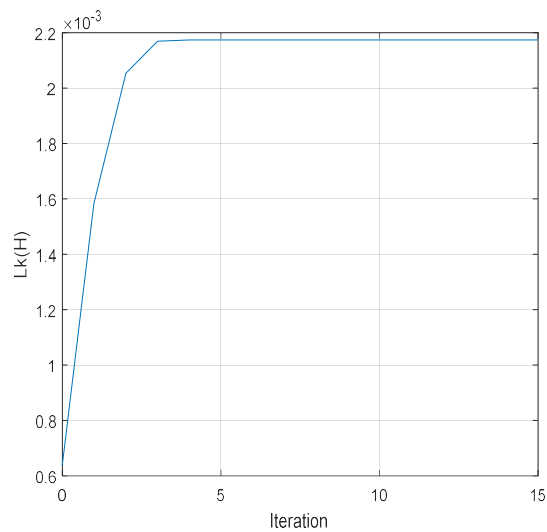


Fig. 12: Variation of inductance L_k versus Iteration

The parameters of the equivalent circuits of models SSFR1, SSFR2, and SSFR3 are summarized in Table 1:

Table 1. Estimation of equivalent circuits parameters

parametrs	SSFR1 model	SSFR2 model	SSFR3 model
$R_f (\Omega)$	0.0026169	0.0011697	0.001692
$L_f (H)$	0.000842	0.000759	0.000859
$R_k (\Omega)$	-	0.194508	0.017233
$L_k (H)$	-	0.000672	0.000859
$R_j (\Omega)$	-	-	0.031553
$L_j (H)$	-	-	0.002174

V. RESULTS AND VALIDATION

In order to validate the results obtained, the parameters estimated using the Newton Raphson method for SSFR1, SSFR2, and SSFR3 models, as outlined in Table 1, are incorporated into the transfer functions represented by equations (1), (4), and (12) respectively. The resulting findings

are illustrated in Figures 15 and 16. A direct comparison between the frequency responses of these models and the measured data distinctly showcases the effectiveness of the proposed approach. Upon closer examination of the curves in Figures 15 and 16, it becomes apparent that the SSFR2 model outperforms the other models. A notable resemblance is evident between the frequency response of this particular model and the measured data.

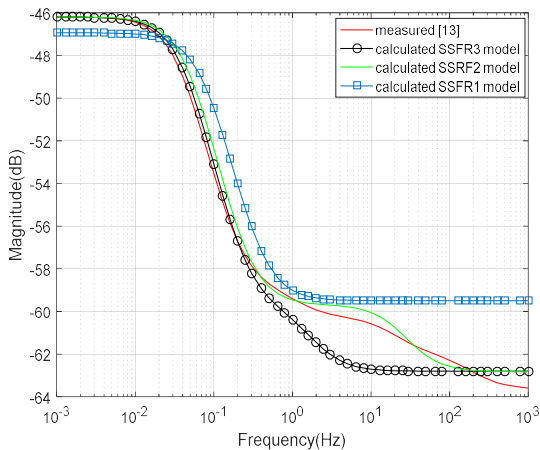


Fig. 15: Operational inductance magnitude versus frequency

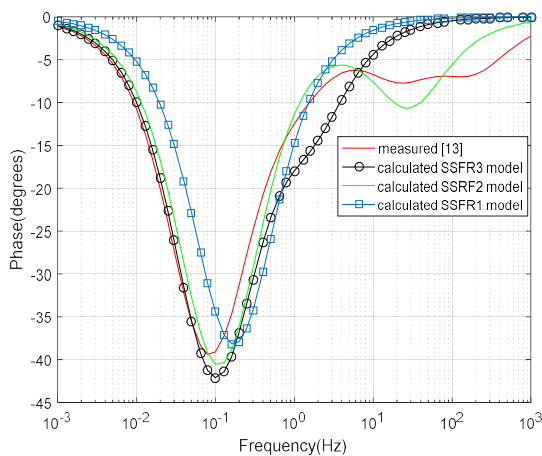


Fig. 16: Operational inductance phase versus frequency

5. CONCLUSION

This study introduced a methodology for computing parameters in the direct axis equivalent circuits of synchronous machines, demonstrating its applicability to various models. The values derived from this approach align well with experimentally reported data. In evaluating the frequency response, the phase discrepancy between the SSFR2 model and measured data for operational inductance ranges from -3.78 degrees to 2.75 degrees, with a magnitude error fluctuating between -0.34 dB and 0.77 dB. It was observed that implementing this method is straightforward, rapid, comprehensible, and reproducible.

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A Survey on Spatial Modulation OFDM: Optimal Detection and Performance Analysis

Fadila BERRAHMA, Khalida GHANEM, and Hicham BOUSBIA-SALAH

Abstract—Spatial modulation is a novel modulation scheme that aims to acquire a trade-off between the two adverse purposes in wireless communications, namely the improvements of data rate and data reliability, by adjuncting the operational transmit antenna index in the emitted bits. To fight against inter-symbol interference (ISI) present in frequency selective channels, orthogonal frequency division multiplexing (OFDM) has become an extensively accepted and used mechanism. We investigate in this work the performance of SM-OFDM modulation over frequency selective MIMO channel using quadrature amplitude modulation (QAM) scheme. At the receiver side, a group of detectors is proposed to recover the active transmit antenna index (Tx-antenna) and the emitted data sequence. In this paper, a new SM-OFDM-based scheme is proposed, which adds another element of information in the transmitted SM-data block consisting of a chosen constellation among the available rotated set. To attain this, an additional index is used to select the specific constellation. At the receiver, a set of detectors is proposed to jointly estimate the transmitted symbol, as well as the used constellation and the active transmit antenna indices. The comparison of the performance of this new scheme against the conventional SM-OFDM shows an increased spectral efficiency at the price of a slight degradation in bit error rate. The proposed scheme permits a trade-off between the quality of the communication and the achieved spectral efficiency. Some concluding remarks are drawn over simulation results.

Keywords—Spatial modulation, orthogonal frequency division multiplexing, MIMO, inter-symbol interference, optimal detector.

I. INTRODUCTION

Wireless Communication has invaded nowadays all the fields of aspects of modern life. Accessing the global network has become the most important behavior of our life style. Wireless communication is an ever developing field that still holds many possibilities in this area. Up to the current communications standards, demand for an increased throughput of wireless communication systems, has been fulfilled by the aid of multiple antenna systems at the transceiver block, the so called MIMO-Systems, or by combination of these latter with techniques such as OFDM [1]. In [2, 3], A.R. Kaye and D.A. George proved that transmitting over single antenna systems, could be extended to multiple antenna case. Since then, the efforts are focused on looking for methods that enhance the spectral efficiency, and low hardware and software complexities. MIMO communication systems offer higher data rates and enhanced BER performance, compared to single-antenna transmissions, at the cost of:

1. An increased level of complexity at the receiver, in-

duced by the interchannel interference emanating from the simultaneous transmission of many data sequences from multiple antennas;

2. The transmit-antenna elements need to be synchronized to exploit the advantages of multiple access MIMO communications;
3. To have the ability to convey many data symbols simultaneously, multiple RF chains are required at the transmitter side.

Motivated by these aspects, SM has been coined recently. Therefore, SM has been investigated for flat fading channels using multiple antennas in [4], and for OFDM transmission [5].

Moreover, a set of SM-based detectors have been introduced. First, an iterative maximum ratio combining (i-MRC) detector is used in [4], in order to retrieve both the emitted data stream and the TX-antenna index. Due to the sub-optimality of i-MRC-based demodulator, Jeganathan et al. have derived in [6] an optimal detector (OD) for SM system that utilizes ML metrics. It was illustrated that OD offers a high performance with a reasonable complexity.

An alternative multiple antenna-frequency selective transmission approach, called spatial modulation-OFDM (SM-OFDM), that eliminates interchannel interference (ICI) was introduced in [5], [7]. Furthermore, better performance can be achieved when applying soft-decision ML detector discussed in [8]. A low-complexity soft output DBD algorithm and a distance-based ordered detection (DBD) for

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SM-MIMO schemes are proposed in [9]. In the aim to reduce the complexity of OD approach at a viable BER performance, sphere decoder (SD) detector was also investigated in SM schemes in [10]. Generalized SM (G-SM) was afterwards proposed in [11], [12], where more than one transmit antenna are active simultaneously, to exploit the advantages of MIMO configuration. Furthermore, we analyse in this letter the performance of SM-OFDM-based scheme using a group of detectors, and a new cosine-rule-based detector is also proposed.

The paper is organized as follows. Section II describes the mapping paradigm of SM-MIMO based scheme. Design guidelines for SM systems are given in section III. Section IV details the operating principle of SM-OFDM based architecture. SM-OFDM detectors are described in sections V, including some numerical results. A new SM-OFDM-based detector is proposed in section VI. Conclusions are presented in the last section.

II. SPATIAL MODULATION (SM)

Spatial modulation (SM) is a novel mapping paradigm, where the antenna index is utilized as an additional source to send the information bits. At the transmitter, the information bits are divided into blocks, each one of them has the length:

$$b_{SM} = \log_2(N_t) + \log_2(M) \quad (1)$$

with $\log_2(N_t)$ presents the number of bits utilized to identify the activated transmit antenna among the possible ones in the antenna-array which are kept silent meanwhile, and $\log_2(M)$ is the number of bits used to indicate the symbol in the signal constellation emitted by the selected antenna as shown in Fig. 2. This idea was introduced for the first time in [4].

An example of SM is shown in figure 2, with $N_t = 2$ that uses 4QAM modulation, thereby using 3bits/symbol for each transmitted SM symbol. Assuming that the input data bits is $[1 \ 0 \ 0]^T$, which corresponds in SM mapping to the transmission of the symbol $-1.0000 + 1.0000i$ from the second transmit antenna.

The resulted signal is transmitted through the wireless channel using the following equation:

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w} \quad (2)$$

where: $\mathbf{y}$ is the complex received vector; $\mathbf{H}$ is the complex channel matrix; $\mathbf{w}$ is the complex AWGN noise vector; and $\mathbf{x}$ is the complex modulated transmitted vector.

The channel matrix $\mathbf{H}(k)$ can be presented as a group of complex coefficients between the transmit and receive antennas as illustrated in the following equation:

$$\mathbf{H} = \begin{pmatrix} h_{11} & h_{12} & \dots & h_{1N_t} \\ h_{21} & h_{22} & \dots & h_{2N_t} \\ \vdots & \vdots & \ddots & \vdots \\ h_{N_r,1} & h_{N_r,2} & \dots & h_{N_r,N_t} \end{pmatrix} \quad (3)$$

III. DESIGN GUIDELINES FOR SM-MIMO SYSTEMS

SM is a new modulation concept that aims to offer some benefits in terms of:

- **Rate:** due to the introduction of the spatial-position and the adoption of the three-dimensional constellation diagram, SM-MIMO can offer a high throughput and an increased SE, compared to SISO system. Hence, this design should be tuned accurately since the data rate could further enhanced by a proper data streams encoding ensuring ICI mitigation, but at the cost of an additional receiver complexity [13].
- **Capacity:** depends on the number of transmit antenna elements (N_t) [14].
- **Error performance :** it was shown in most of research papers that SM can provide a better BER performance than SISO counterpart, especially, when the number of TX-antennas is higher than four and the number of receive antennas is more than one [15].
- **Channel fading:** the BER performance of SM-based scheme is affected by the distribution of the wireless channel fading [15]. Thus, more bits can be encoded and transmitted if there is less channel fading or equivalently a higher Nakagami fading parameter [15].
- **Channel State Information:** based on the assumption that adequate channel estimators are used, it was illustrated that SM is robust to imperfect channel state information (CSI) [16, 17].
- **Demodulation:** the selection of the best demodulator depends on the offered performance and the detection complexity level at the receiver.
- **Energy-Efficiency:** SM-MIMO scheme uses one RF chain at each time instant, thus providing a better energy efficiency and minimizing the complexity [18].

IV. SM-OFDM BASED SCHEME

In the current and future generations of wireless communication networks, and for many applications, ISI resulted when using a frequency selective wireless channel. For a frequency-selective fading channel, OFDM is a popular modulation mechanism, which is utilized to eliminate ISI and to take advantages of the frequency diversity. Subsequently, coupling SM and OFDM is favourable to deal with frequency selective channels.

The operating principle of SM-OFDM is described in Figure. 3. After insertion of error correction codes in the original interleaved data to combat channel errors, the bit stream is mapped with SM. In general, the number of bits/symbol/sub-carrier or sub-channel b_{SM} that SM-OFDM based scheme can transmit for each block is given earlier in “(1)”.

Then, the output vectors from the SM mapping $\mathbf{R}(t)$ are converted to the time domain by using the inverse fast

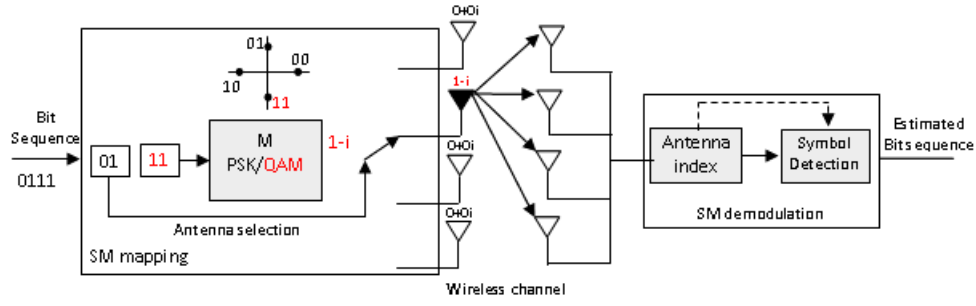


Fig. 1: Spatial modulation transceiver block diagram.

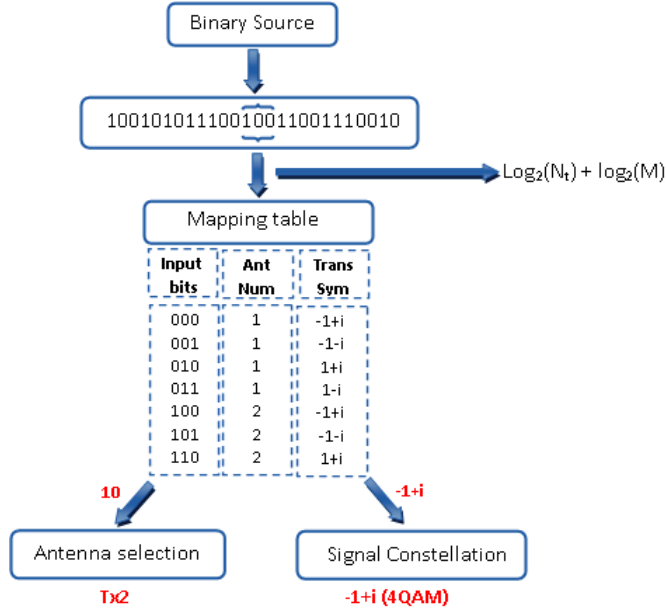


Fig. 2: SM: How it works "The Transmitter".

Fourier transform (*IFFT*). Afterwards, a cyclic prefix (CP) is inserted, to avoid the ISI between the OFDM symbols.

After that, the resultant vectors $\mathbf{R}(t)$ are conveyed simultaneously from the N_t active transmit antennas over the MIMO channel, denoted $\mathbf{H}(t)$, using the following equation:

$$\mathbf{Y}(t) = \mathbf{H}(t) \otimes \mathbf{R}(t) + \mathbf{W}(t) \quad (4)$$

$\mathbf{H}(t)$ sets to:

$$\mathbf{H}(t) = \begin{pmatrix} \mathbf{h}_{1,1}(t) & \mathbf{h}_{1,2}(t) & \dots & \mathbf{h}_{1,N_t}(t) \\ \mathbf{h}_{2,1}(t) & \mathbf{h}_{2,2}(t) & \dots & \mathbf{h}_{2,N_t}(t) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{h}_{N_r,1}(t) & \mathbf{h}_{N_r,2}(t) & \dots & \mathbf{h}_{N_r,N_t}(t) \end{pmatrix} \quad (5)$$

Each vector of $\mathbf{H}(t)$ refers to the multipath channel gains between each ν^{th} transmit and ϱ^{th} receive antennas as follows:

$$\mathbf{h}_{\varrho\nu}(t) = [h_{\varrho\nu}(t)^1 \quad h_{\varrho\nu}(t)^2 \dots h_{\varrho\nu}(t)^L] \quad (6)$$

where L is the number of paths. The satages are simply reversed at the receiver. The indices of transmit antenna and the emitted data stream can be recovered using one of the detectors that will be described in the next section.

V. SM-OFDM DETECTORS

The detection algorithms of SM-MIMO system could be divided into four fundamental categories as described in [19]: matched filter (MF) based detection in [7], maximum likelihood (ML) detection in [6, 8], sphere decoding (SD) [10, 20], and other detectors as signal vector based list detection (SVD) in [21]. SM-based detection techniques are presented in Fig. 4.

A. Minimum Mean Square Error (MMSE) detection

MMSE algorithm describes the approach which minimizes the mean square error (MSE) between two entities as described in the following equation.

$$\tilde{\mathbf{z}}^c(k) = (\mathbf{h}_j^c(k))^H [(\mathbf{h}_j^c(k))^H * \mathbf{h}_j^c(k) + \sigma_w^2 \mathbf{I}]^{-1} \mathbf{y}^c(k) \quad (7)$$

Then, the TX-antenna index is computed by:

$$\hat{j}^c = \arg \max_j |\tilde{\mathbf{z}}^c(k)| \quad (8)$$

Finally, the transmitted symbol is computed using the following quantization function:

$$\hat{q}^c = Q(\tilde{\mathbf{z}}^c(k)_{(j=\hat{j})}) \quad (9)$$

B. Optimal detection (OD)

This detector follows a different rule compared to the detectors mentioned above. Assuming that the conveyed signals are probably equally likely, as in [6], the indices $\hat{j}, \hat{q}$ are computed as follows:

$$[\hat{j}, \hat{q}] = \arg \max_{j,q} p_y(\mathbf{y} | \mathbf{x}_{j,q}, \mathbf{H}) \quad (10)$$

$$= \arg \min_{j,q} (\|\mathbf{g}_{j,q}\|_F^2 - 2\text{Re}\{\mathbf{y}^H \mathbf{g}_{j,q}\}) \quad (11)$$

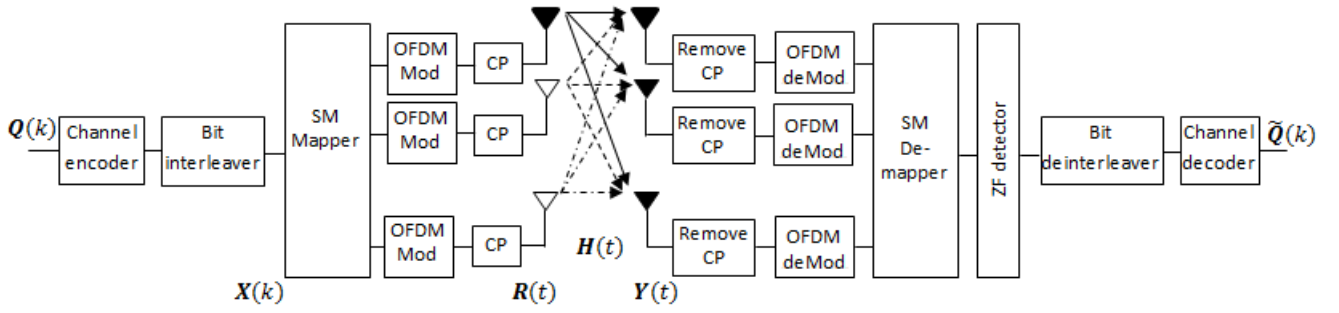


Fig. 3: SM-OFDM System model.

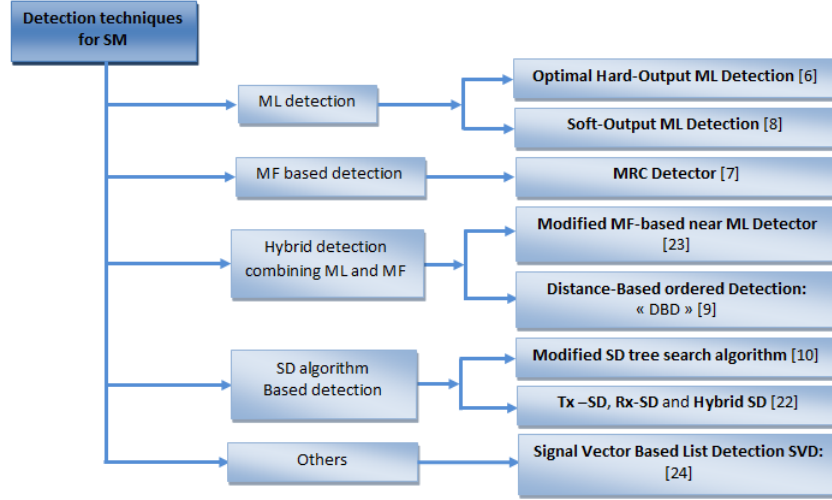


Fig. 4: Overview of SM detectors.

where $\mathbf{g}_{jq} = \mathbf{h}_j \mathbf{x}_{j,q}$, $1 < j < N_t$, $1 < q < M$, M being the modulation order and $p_y(\mathbf{y}|\mathbf{x}_{j,q}, \mathbf{H}) = \pi^{-N_r} \exp(-\|\mathbf{y} - \mathbf{H}\mathbf{x}_{j,q}\|_F^2)$ is the *PDF* of $\mathbf{y}$, conditioned on $\mathbf{x}_{j,q}$ and $\mathbf{H}$.

C. Signal Vector-Based Detector (SVD)

SVD principle is based on the observation that the received vector $\mathbf{h}_j \mathbf{x}_{j,q}$ and $\mathbf{h}_j$ have the same direction, as described in [21]. The direction is found by estimating the angle θ_j^c between $\mathbf{h}_j$ and $\mathbf{y}$, as follows:

$$\theta_j^c = \arccos \frac{\|\langle \mathbf{h}_j^c(k), \mathbf{y}^c(k) \rangle\|_F}{\|\mathbf{h}_j^c(k)\|_F \|\mathbf{y}^c(k)\|_F} \quad (12)$$

Then, the TX-antenna index can be calculated using the following equation:

$$\hat{j}_{SVD}^c = \arg \min_j \theta_j^c \quad j = \{1, \dots, N_t\} \quad (13)$$

Finally, the emitted symbol can be estimated as:

$$\hat{q}_{SVD}^c = \arg \min_q \|\mathbf{y}^c(k) - \mathbf{h}_{\hat{j}_{SVD}^c}^c(k) x_q\|_F^2 \quad q = \{1, \dots, M\} \quad (14)$$

D. Distance-Based ordered Detection (DBD)

The DBD algorithm calculates the distances between the estimated symbols and their demodulation constellations [21]. First, for DBD principle, the received vector $\mathbf{y}$ is multiplied by the pseudo-inverse of $\mathbf{h}$ as:

$$\mathbf{z}^c(k) = (\mathbf{H}^\dagger)^c(k) \mathbf{y}^c(k) \quad j = \{1, 2, \dots, N_t\} \quad c = \{1, 2, \dots, N\} \quad (15)$$

Then the estimated symbol $\hat{x}_q$ is obtained by the demodulator as described in the following expression.

$$\hat{x}_q^c = Q(\mathbf{z}^c(k)) \quad (16)$$

Let $d_{j,q}$ denotes the distance between $\hat{x}_q^c$ and all possibly values of the transmitted sequences x_q by applying this following equation:

$$d_{j,q} = \|\mathbf{h}_j^c(k)\| \|\hat{x}_q^c - x_q\| \quad (17)$$

Hence the TX-index $\hat{j}$ and the corresponding modulated symbol x_q are estimated by:

$$[\hat{j}^c, \hat{q}^c] = \min_{j,q} (d_{j,q}) \quad (18)$$

VI. PROPOSED DETECTOR FOR SM-OFDM SCHEME

Two processing steps are involved in this detector, resulting from the merging of two approaches in the estimation of

the TX-antenna index, the selected rotated constellation, and the emitted symbol. It is performed by using MRC technique and cosine rule principle.

1— *Antenna index estimation*: First, the antenna index should be computed. Hence, MRC is used as expressed in the following equation:

$$\tilde{z}_j^c(k) = \frac{(\mathbf{h}_j^c(k))^H}{\|\mathbf{h}_j^c(k)\|_F^2} \mathbf{y}^c(k) \quad \text{for } c = \{1, 2, \dots, N\} \quad (19)$$

where $\mathbf{h}_j^c(k)$ refers to the j^{th} channel vector, corresponding to the c^{th} subcarrier, and $\mathbf{y}^c(k)$ is the received vector on each c^{th} subcarrier. Therefore, the TX-antenna index $\hat{j}^c$ will be calculated by:

$$\hat{j}^c = \arg \max_j |\tilde{\mathbf{z}}^c(k)| \quad (20)$$

Based on “19”, the vector $\mathbf{z}^c(k)$ could be presented as:

$$\mathbf{z}^c(k) = [z_1^c(k), z_2^c(k), \dots, z_{N_t}^c(k)]^T \quad (21)$$

2— *Constellation and symbol indices estimation*: At the receiver side, based on the assumption that a perfect channel knowledge is available, and that one transmit antenna being activated at the c^{th} subcarrier and is transmitting the q^{th} symbol mapped to the p^{th} constellation, the transmitted vector $\tilde{\mathbf{z}}_j^c(k)$ can be defined as:

$$\tilde{\mathbf{z}}_j^c(k) = \chi_q^{p,c}(k) + \mathbf{n}(k) \quad (22)$$

In a noiseless environment, $\tilde{\mathbf{z}}_j^c(k)$ could be presented as:

$$\tilde{\mathbf{z}}_j^c(k) = [\text{Re}(\tilde{z}_j^c(k)), \text{Im}(\tilde{z}_j^c(k))]^T \quad (23)$$

$$= [\text{Re}(\chi_q^{p,c}(k)), \text{Im}(\chi_q^{p,c}(k))]^T \quad (24)$$

$$= \mathbf{M}_p[\text{Re}(s_0^c(k)), \text{Im}(s_0^c(k))]^T \quad (25)$$

The angle between $\tilde{\mathbf{z}}_j^c(k)$ and the emitted one, $\chi_q^{p,c}(k)$, should be zero, in the absence of noise. In adverse, these vectors fluctuated around each other in a noisy environment.

We propose the cosine-based rule, and is based on the observation that the emitted vector $\tilde{\mathbf{z}}_j^c(k)$ share the same direction with the rotated symbol vector $\chi_q^{p,c}(k)$; therefore the cross-angle will be zero. Subsequently, the vector $\chi_q^{p,c}(k)$ having the smallest angle value $\varphi_q^{p,c}$ corresponds to the expected one, and the angle can be computing using this equation:

$$\varphi_q^{p,c} = \arccos \frac{\langle \tilde{\mathbf{z}}_j^c(k), \chi_q^{p,c}(k) \rangle}{|\tilde{\mathbf{z}}_j^c(k)| |\chi_q^{p,c}(k)|} \quad (26)$$

Afterwards, the estimated p^{th} constellation and the q^{th} emitted symbol index are calculated by looking for the minimal value of the equation in “26” as described in the following equation:

$$[\hat{p}, \hat{q}] = \arg \min_{p,q} (\varphi_q^{p,c}) \quad (27)$$

VII. NAKAGAMI-M FADING CHANNEL MODEL - CHANNEL IMPAIRMENTS

The generalized Nakagami- m distribution is adopted herein to encompass as fading scenarios as possible, and the channel envelope in such case is given by the known formula:

$$f_{|\omega_l|}(x) = \frac{2}{\Gamma(m_l)} \left(\frac{m_l}{\Omega_l}\right)^{m_l} x^{2m_l-1} \exp\left(-\frac{m_l}{\Omega_l} x^2\right) \quad l = 1, 2, \dots, L \quad (28)$$

where $\Gamma(\cdot)$ is the gamma function defined as $\Gamma(m) = \int_0^\infty x^{m-1} e^{-x} dx$. The parameter $m \geq \frac{1}{2}$ indicates the fading severity and $\Omega_l = \varepsilon\{|\omega_l|^2\}$ refers to the mean-square value of the l^{th} path signal where ω_l represents the multipath gain. The spatial fading correlation is taken into account, by separating the parts arising from the receive and the transmit sides, thereby adopting the so-called Kronecker model. Because of the adopted SM modulation scheme, one transmit antenna is concerned at each time instant. Therefore the transmit correlation is not of interest, which yields the following channel expression:

$$\mathbf{H} = \mathbf{R}_r^{1/2} \mathbf{H} \quad (29)$$

where $\mathbf{H}_w$ is the matrix containing the $N_r \times N_t$ channel vectors $\mathbf{h}_{\rho\nu}$ introduced earlier. The superscript $(\cdot)^{\frac{1}{2}}$ stands for the Hermitian square root of a matrix and the receive correlation matrix $\mathbf{R}_r \in R^{N_r \times N_r}$ has the following Toeplitz structure:

$$\mathbf{R}_r = \begin{bmatrix} 1 & \alpha & \dots & \alpha^{N_r-1} \\ \alpha & 1 & \dots & \alpha^{N_r-2} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha^{N_r-1} & \alpha^{N_r-2} & \dots & 1 \end{bmatrix} \quad (30)$$

where $\alpha = \exp(-\beta)$, with β standing for the correlation decay coefficient set to $\frac{2\pi}{\lambda} d \sin \phi$, with λ being the wavelength, d the inter-element spacing and ϕ describing the angle of arrival (AoA) offset. The channel coefficients at the receive side are said to be perfectly correlated when $0 < \alpha < 1$. Moreover, in practice, obtaining perfect channel estimates is hardly feasible, therefore the availability of only imperfect channel state information (CSI) should be considered. Thus, by taking into account both spatial correlation and imperfect CSI at the receiver side in Nakagami fading, the channel estimate could be obtained as follows:

$$\hat{\mathbf{H}} = \rho \mathbf{R}_r^{1/2} \mathbf{H} + (1 - \rho) \lambda^{N_r \times N_t} \quad (31)$$

where λ is a normally distributed random variable with zero mean and unit variance. The coefficient $0 < \rho < 1$ is a factor that determines the similitude of the estimated CSI to the actual one, thus leading to perfect CSI estimation scenario when $\rho = 1$.

VIII. SIMULATION RESULTS

We consider the following parameters: Rayleigh channel model with $L = 7$ taps configuration is retained, the $FFT = 128$ is used, and 4 antennas are considered at the transmitter and the receiver sides, respectively. 2PSK modulation is retained. The receive antennas are assumed separated sufficiently such that to eliminate spatial correlation. BER performance is illustrated in Fig. 5 when considering MIMO channel model. As expected, the performance of SM operated by ZF, MMSE, DBD, OD, SVD, and MRC-cosine-based rule detectors, ZF and MMSE are suboptimal detections schemes, which performances are inferior to OD, DBD, SVD and MRC-cosine-based rule detectors, mainly because antenna indices are not accurately estimated. The channel matrices in this case are not orthogonal and a linear detection, such as MMSE algorithm, which has a limitation to find antenna indices, as compared to OD and SVD and DBD detectors.

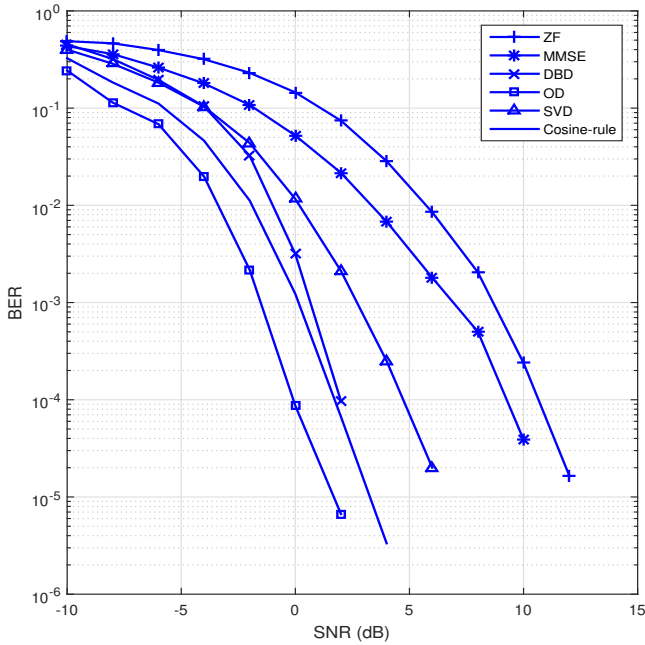


Fig. 5: Performance of SM-OFDM detectors.

Fig. 6 compares the achieved BER performance for different values of m , and different correlation and CSI conditions, when adopting the three presented detectors. Rayleigh channel scenario, obtained when $m = 1$, is compared against purely Nakagami channel, simulated with $m = 10$. Furthermore, uncorrelated ideally estimated channels are herein taken as a reference and are obtained by setting the values of ρ and α to one and zero, respectively. From this figure it can be seen that, in ideal channel conditions, Nakagami channel is more favorable to communication than Rayleigh channel, whatever is the used detector. This was expected because of the high scattering level characterizing propagation in Rayleigh channel (uncorrelated perfectly known channel). Performance gain in Nakagami channel is negligible when opting for a given detector over the others, unlike in Rayleigh channel where OD exhibits the best performance and MMSE is the least performing, which consolidates the observations reported

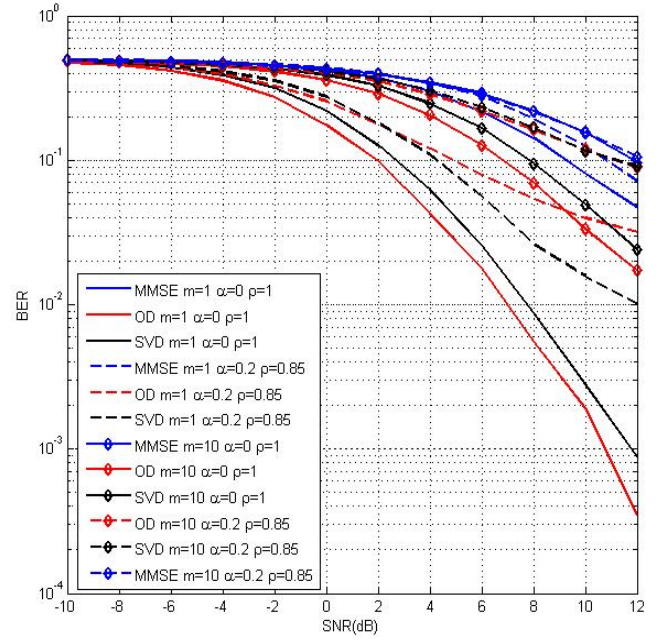


Fig. 6: Comparison of the achieved BER performances of three different SM-OFDM architectures in Nakagami channel with different CSIs and correlations.

in different references [22]. This is mainly due to the lack of accuracy in estimating the values of antenna indices in MMSE. The attained data reliability is quite interesting since low BER values are provided at reasonable SNR levels. However, as shown in this figure, increasing slightly the receive spatial correlation and CSI imperfection levels, has a drastic degradation effect on the data reliability, more particularly in purely Nakagami channel.

IX. CONCLUSION

This paper has presented the recent research progress on SM-OFDM detectors. Moreover, the operating principles of SM and SM-OFDM are provided. The major contribution is the proposed MRC-cosine-based rule detector. Its provided robustness and improved performance have been confirmed. The performance of SM-OFDM scheme has been investigated with different detectors over Nakagami- m fading channel, in the presence of channel impairments. It was found that, in ideal correlation and CSI conditions, and in the presence of Rayleigh fading. SM-OFDM with OD detector outperforms MMSE and SVD alternatives. Nakagami channel offers a more favorable propagation scenario, and the attained BER performance is quite viable. Moreover, SM-OFDM was shown to be quite sensitive to the increase of spatial correlation and CSI imprecisions, regardless of the used detector and the considered channel.

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Numerical simulation of natural convection heat transfer in a cavity with finned surface under linear temperature profile

Mohamed Tarek ATTOUCHI, Salah LARBI, and Sofiane KHELLADI

Abstract– This paper is dedicated to a numerical study of natural convection through a cavity with finned surface where its base temperature is variable with linear profile. An approach based on modelling of internal fluid flow around a finned surface in laminar and at steady state conditions is used. Thermal and hydrodynamic aspects of the fluid flow were analyzed through the numerical resolution of equations of fluid dynamics. For this purpose, we have developed a computer code (in Fortran 90) based on the finite volume method. The studied model consists of a rectangular cavity where vertical walls are thermally insulated, the horizontal ones are maintained at different temperatures: cold and constant temperature on the upper wall and hot temperature with linear profile on the lower wall. The Rayleigh number used is in the range of 103 to 106 and the Prandtl number is fixed at 0.71. The plotted results are related to temperature distribution, streamlines, velocity fields as well as the mean Nusselt number.

Keywords– Natural Convection, Enclosures, Laminar Flow, Fins, Numerical Simulation, Finite Volume Method.

NOMENCLATURE

a_P, a_E, a_W, a_N and a_S	Coefficients associated with variable ϕ
C_P	Specific heat at constant pressure, [J/kg.°C]
d	Spacing between fins, [m]
g	Acceleration due to gravity; [m/s ²]
G_k	Term of generation of the turbulent kinetic energy due to
ENP	Ecole Nationale Polytechnique.
ENTR	Entropy (Shannon entropy).
DWT	Discrete Wavelet Transform.
HV	High Voltage.
STD	Standard Deviation.
THD	Total Harmonic Distortion.

the gradient of the mean velocity with: $G_k = -\sqrt{\frac{1}{PrRa}} \frac{\mu_t}{\sigma_T} g\beta \frac{\partial T}{\partial y}$

Gr	Grashof number
H	Cavity height, [m]
K	Thermal conductivity, [W/m.K]
k	Turbulent kinetic energy
L	Cavity length, [m]; l : fin width, [m]
Nu_L	Local Nusselt number, Nu_M : Average Nusselt number
P	Pressure [Pa]
Pr	Prandtl number; $Pr = \frac{\nu}{\alpha}$
T	Local fluid temperature, [°C]
α	Thermal diffusivity, [m ² /s]
β	Coefficient of thermal expansion
$\sigma_k; \sigma_T$	Turbulent Prandtl number for kinetic and turbulent energy respectively
ϕ	General dependent variable (e.g. temperature, speed, ect.)
ε	Energy dissipation rate
ρ	Volumic mass, [Kg/m ³]
ν	Fluid kinematic viscosity [m ² /s]

μ	Fluid dynamic viscosity [kg/(m.s)]
μ_t	Turbulent viscosity
ψ	Streamlines [m ² /s]
c	Cold
h	Hot

I. INTRODUCTION

Heat transfer by natural convection, in enclosures with finned surfaces and under differential heating conditions, is of primary importance in many engineering applications. It is encountered in various fields of industrial applications, such as the cooling devices of electronic instruments, the operation of safety of nuclear reactors, the solar collectors, the boilers, the fire fighting, the energy storage systems, the radioactive waste storage, the energy efficiency of buildings. Many researchers have conducted the studies related to the various aspects of this problem theoretically and experimentally. In the following, we present a bibliographical study allowing us to make a synthesis of research work based essentially on cavities with finned surfaces and subjected to different boundary conditions.

W. S. Fu and W. J. Shieh [1] carried out a theoretical study of natural convection in a square enclosure partitioned by a single adiabatic fin attached to the ceiling wall. The Rayleigh numbers considered are 104 and 105. The results showed that the heat transfer coefficients were influenced by the height and the location of the fin.

M. H. Novak and E. S. Nowak [2] used the CAV computer code for a 2D numerical analysis of natural convective laminar heat transfer and fluid flow distribution in rectangular and square thin-windowed cavities with or without inner fins. Recommendations on window design were made. The numerical analysis carried out by I. Dagtekin and H.F. Oztop [3], was based on the study of heat transfer by natural convection and the flow of the fluid in an enclosure equipped with two heated fins. The right and lower sidewalls were perfectly insulated, while the left and upper ones were kept at the same uniform temperature. The fins were placed at the bottom of the enclosure with higher temperatures than that of the uninsulated walls. The effects of the position and the height of the fins on the heat transfer and the flow field as well as the average Nusselt number were studied.

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N. Yucel and A. H. Ozdem [4] numerically studied fluid flow and heat transfer in partially divided square enclosures, the horizontal end walls of the enclosure were chosen adiabatic and the left and right side walls were maintained at uniform, but different temperatures and were subjected to appropriate boundary conditions and a Rayleigh number range of 103 to 5.105. It has been observed that, the average Nusselt number increases with increasing Rayleigh number and decreases with increasing number of partitions, however, the decrease in average Nusselt number is apparent at low Rayleigh numbers. Note that the increase in the height of the partition induces a decrease in the average Nusselt number.

E. Bilgen [5] carried out a numerical study on differentially heated square cavities. The horizontal walls are adiabatic while the vertical ones are isothermal. A thin fin is attached to the active wall. It was found that the Nusselt number increases with increasing Rayleigh number and decreases with increasing fin length. However, there is an optimum fin position, often at or near the center of the cavity, to minimize heat transfer by natural convection.

A. F. Fahaid [06] has numerically studied laminar natural convection in vertical enclosures fitted with conductive fins attached to the hot vertical wall. The side vertical walls were kept at constant but different temperatures, while the horizontal wall and the bottom wall were kept adiabatically. It has been observed that the rate of heat transfer through an enclosure is largely affected and therefore it can be controlled by the number of fins attached to the vertical sidewall of the enclosure, as well as the fin lengths.

A. Ben-Nakhi and A.J. Chamkha [7] have numerically examined natural convection in steady and laminar regimes in a square enclosure with a thin inclined fin of arbitrary length and perfectly conducting. The fin is attached to the hot vertical left wall, while the other three cold walls are thick, of the same thickness, and of arbitrary thermal conductivity. The results showed the effects of the angle of inclination of the thin fin on the length and the thermal conductivity of the surfaces concerned.

A. Al Amiri and K. Khanafer [8] presented a study, which deals with the buoyancy induced by the heat transfer in a partially divided square enclosure. The Rayleigh number is taken in the range: $104 \leq Ra \leq 107$. The results revealed that all parameters related to the geometric dimensions of the heating element significantly affect the flow field, isotherms and heat transfer. The study revealed that increasing the heat exchange surface of the heating element, increasing its height, width and location improved heat transfer.

A.F. Costa [9] has numerically studied natural convection in a square, partitioned, air-filled enclosure. Two fins of finite thickness are placed in the enclosure, whose position, length and thermal conductivity vary both as a function of the Rayleigh number and as a function of the boundary conditions. The overall thermal performance of the enclosure was analyzed through the Nusselt number.

F. Selimefendigil and H. F. Öztop [10] conducted a numerical study on the heat transfer from a square cavity in the presence of an adiabatic and tilted thin fin. The upper and lower walls of the cavity were maintained at a constant temperature, while the vertical ones are assumed adiabatic. The average Nusselt number was determined as a function of the height of the fin, its angle of inclination and the Richardson number.

C. Benseghir and S. Rahal [11] carried out a numerical study of

transient laminar natural convection in a square cavity differentially heated and filled with air. Thin rectangular fins made with a high conductivity material were placed on the hot wall of the cavity. A single value of Rayleigh $Ra=105$ was considered. This study highlights the existence of an optimal length of the fins allowing the maximization of the natural convection on the cold wall.

S. G. Martyusheva, M. A. Sheremeta [12] performed a two-dimensional numerical analysis of the combined heat transfer (transient natural convection, thermal radiation and conductive surface). The enclosure concerned is square and filled with air. The walls, solid and conductive, have a finite thickness and a local heat source with conditions of heat exchange by convection with the surrounding environment.

It was found that, the average Nusselt number increases with the Rayleigh number and the thermal conductivity ratio. However, it decreases with surface emissivity and the ratio of wall thickness to cavity spacing. It is important to note that the above investigations consider constant hot and cold temperatures in the study of natural convection in finned enclosures. The novelty of our work lies in (i) the consideration of a linear temperature profile at the level of the fins, (ii) the adoption of our own boundary conditions, (iii) the presentation of the velocity fields not plotted in the research aroused. Indeed, the main objective of the present study is to analyze the natural convection in laminar and permanent regimes in a square enclosure equipped with fins heated in a non-uniform way according to a linear temperature profile. A careful analysis of the effect of the heating elements on the heat transfer inside the cavity will be carried out by imposing well-defined boundary conditions, for various parameters, namely the Rayleigh number, the Prandtl number, the height and fin width and number of heating elements.

II. PHYSICAL MODEL

The physical model considered is shown in Figure 1. It is a cavity of length L and height H .

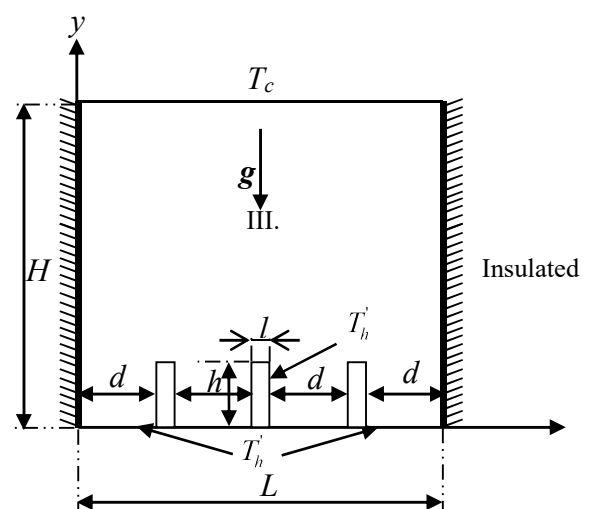


Fig. 1: Schematic representation of the physical model used.

The vertical walls are thermally insulated. As for the horizontal walls of the enclosure, they are maintained at different temperatures T_c and T_h' .

Identical fins, of thickness l and height h , are placed equidistantly on the lower wall of the cavity and maintained at

a temperature T'_h such that $h/H = 1/5$ and its width $l/L = 1/20$. The heat transfer in the cavity is done by natural convection and the Prandtl number is 0.71. The vector g represents the acceleration due to gravity.

III. MATHEMATICAL MODEL

A dimensionless form of the equations reduces the number of independent parameters in the equations and makes the solutions more general for a given set of parameters. It helps to save machine time by increasing the convergence speed of the solution.

The dimensionless equations are derived in such a way that only the Prandtl and Rayleigh numbers of the fluid are dimensionless parameters.

A. Governing equations

The mathematical model used is based on the classical equations of conservation balances (mass, momentum and energy), is given by equations (1), (2), (3) and (4).

- For the mass balance equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \quad (1)$$

- For the momentum balance equation along the x direction:

$$\begin{aligned} \rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} \\ = -\frac{\partial P}{\partial x} + \sqrt{\frac{Pr}{Ra}} \frac{\partial}{\partial x} \left[(\mu + \mu_t) \left(2 \frac{\partial u}{\partial x} \right) \right] \\ + \sqrt{\frac{Pr}{Ra}} \frac{\partial}{\partial y} \left[(\mu + \mu_t) \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right] \end{aligned} \quad (2)$$

- For the momentum balance equation along the y direction:

$$\begin{aligned} \rho \frac{\partial v}{\partial t} + \rho u \frac{\partial v}{\partial x} + \rho v \frac{\partial v}{\partial y} = -\frac{\partial P}{\partial y} + \sqrt{\frac{Pr}{Ra}} \frac{\partial}{\partial y} \left[(\mu + \mu_t) \left(2 \frac{\partial v}{\partial y} \right) \right] + \\ \sqrt{\frac{Pr}{Ra}} \frac{\partial}{\partial x} \left[(\mu + \mu_t) \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right] + (T - T_{ref}) \end{aligned} \quad (3)$$

- For the energy balance equation:

$$\begin{aligned} \rho \frac{\partial T}{\partial t} + \rho u \frac{\partial T}{\partial x} + \rho v \frac{\partial T}{\partial y} = \frac{1}{\sqrt{PrRa}} \frac{\partial}{\partial x} \left[\left(\mu + \frac{\mu_t Pr}{\sigma_T} \right) \frac{\partial T}{\partial x} \right] + \\ \frac{1}{\sqrt{PrRa}} \frac{\partial}{\partial y} \left[\left(\mu + \frac{\mu_t Pr}{\sigma_T} \right) \frac{\partial T}{\partial y} \right] \end{aligned} \quad (4)$$

The governing equations given by the relations (1) to (4) with their corresponding boundary conditions are given in a dimensionless form in order to generalize the study. It should also be noted that the terms of turbulence and those dependent on time are not taken into account in this study. We define the expressions of the speed as a function of the current function as:

$$v = -\frac{\partial \psi}{\partial x}; u = \frac{\partial \psi}{\partial y}$$

Consider u^* and v^* , T^* , P^* , x^* and y^* , the dimensionless forms of the components of velocity vectors, temperature, pressure and space variables:

$$\begin{aligned} u^* = \frac{u}{u_{ref}} = \frac{u}{\sqrt{g\beta H \Delta T}}; v^* = \frac{v}{v_{ref}} = \frac{v}{\sqrt{g\beta H \Delta T}}; T^* = \frac{T - T_c}{T_h - T_c}; P^* = \frac{P}{P_{ref}}; x^* = \frac{x}{L}; y^* = \frac{y}{H} \\ \psi^* = \frac{\psi}{\alpha}; T_{ref} = \frac{T_h + T_c}{2}; \Delta T = T_h - T_c \\ u_{ref} = v_{ref} = \sqrt{g\beta H \Delta T}; P_{ref} = \rho u_{ref}^2 \end{aligned}$$

The index *ref* represents the reference value for all the variables.

T_c : is the cold temperature; T_h : hot temperature.

The temperatures T_h and T_c become equal to 1 and 0 on a dimensionless scalar.

The local Nusselt number at the hot wall is given by:

$$Nu_L = -\frac{L}{\Delta T} \frac{\partial T}{\partial y} \Big|_{x^*=0}$$

While the average Nusselt number is represented by:

$$Nu_M = -\frac{L}{\Delta T} \int \left(\frac{\partial T}{\partial y} \right)_{x^*=0} dx$$

B. Numerical Scheme and Solving Method

The final form of the general two-dimensional discretization equation is written:

$$a_P \phi_P = a_W \phi_W + a_E \phi_E + a_S \phi_S + a_N \phi_N + b \quad (5)$$

with

$$\begin{aligned} a_E &= D_e A(|P_e|) + \text{Max}[-F_e, 0] \\ a_W &= D_w A(|P_w|) + \text{Max}[F_w, 0] \\ a_N &= D_n A(|P_n|) + \text{Max}[-F_n, 0] \\ a_S &= D_s A(|P_s|) + \text{Max}[F_s, 0] \end{aligned}$$

with

$$A(|P_i|) = \text{Max}[0, (1 - 0.1|P|)^5]$$

According to the power law used in this calculation,

$$\begin{aligned} b &= S_C \Delta x \Delta y + a_P^0 \phi_P^0 + M \\ a_P &= (a_W + a_E + a_N + a_S) + \Delta F + (a_P^0 - S_P \Delta x \Delta y) \\ \Delta F &= F_e - F_w + F_n - F_s \\ a_P^0 &= \frac{\rho_P^0 \Delta x \Delta y}{\Delta t} \end{aligned}$$

F : The convection force or mass flow through the interfaces of the control volumes;

D : Diffusion conductance and $P = \frac{F}{D}$ (Number of *Peclet*), with

$$\begin{aligned} F_e &= (\rho u)_e \Delta y; F_w = (\rho u)_w \Delta y \\ F_n &= (\rho v)_n \Delta x; F_s = (\rho v)_s \Delta x \\ D_e &= \frac{\Gamma_e \Delta y}{(\delta x)_e} = \frac{(\mu + \mu_t)_e}{(\delta x)_e}; D_w = \frac{\Gamma_w \Delta y}{(\delta x)_w} = \frac{(\mu + \mu_t)_w}{(\delta x)_w} \end{aligned} \quad (6)$$

$$D_n = \frac{\Gamma_n \Delta x}{(\delta y)_n} = \frac{(\mu + \mu_t)_n}{(\delta y)_n}; D_s = \frac{\Gamma_s \Delta x}{(\delta y)_s} = \frac{(\mu + \mu_t)_s}{(\delta y)_s} \quad (7)$$

$$P_e = \frac{F_e}{D_e}; P_w = \frac{F_w}{D_w}; P_n = \frac{F_n}{D_n}; P_s = \frac{F_s}{D_s} \quad (8)$$

M : Modification to momentum equations

$$M = \left[\sqrt{\frac{Pr}{Ra}} \frac{\partial}{\partial x} \left[(\mu + \mu_t) \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \right] \right]$$

($M = 0$ for incompressible fluids where the density does not change with time).

IV. NUMERICAL RESOLUTION

Governing equations, from (1) to (4), were used in their dimensionless form in the calculation with the aim of reducing the number of independent parameters in the equations and making the solutions more general for a data set of parameters in a concern for gain in machine time by increasing the speed of convergence of the solution. The characteristic dimensionless numbers are those of Prandtl and Rayleigh. The numerical modeling of the problem was established through the resolution of fluid dynamics equations, using the finite volume method with adequate boundary conditions. The power law scheme is chosen and used in this calculation, the Rayleigh numbers also used are in the range $10^3 \leq Ra \leq 10^6$, while the Prandtl number is taken equal to $Pr=0.71$. The mesh grids used in the calculation are of different sizes depending on the geometry of the cavity, ranging from 102×102 up to 302×302 . They are regular in the temperature calculation and are staggered for the speed calculation. A convergence test of an order taken in an interval:

$10^{-2} \leq \varepsilon \leq 10^{-4}$ based on the residuals of T , u and v , is applied in this calculation as follows:

$$\phi^{(n)} - \phi^{(n-1)} = \varepsilon$$

V. BOUNDARY CONDITIONS

The boundary conditions used for solving the governing equations, (1) to (4), are related to the non-slip condition for the velocity and the Dirichlet or Newman condition for the temperature. The fins are heated at non-uniform temperature, T'_h given by

$$T'_h = T_h - (T_h - T_c) \frac{x}{L}$$

For temperature, the dimensionless boundary conditions are:
Bottom wall:

$-T^*(x^*, 0) = 1 - x^*$ for the variable hot temperature at the level of the fins and on the rest of the wall.

Top wall:

$-T^*(x^*, 1) = 0$ is a constant cold temperature.

$-\frac{\partial T^*}{\partial x^*}(0, y^*) = 0$ on the left wall.

$-\frac{\partial T^*}{\partial x^*}(1, y^*) = 0$ on the right wall.

* For velocities and stream functions:

$-u^*(0, y^*) = u^*(1, y^*) = 0$; $u^*(x^*, 0) = u^*(x^*, 1) = 0$

and $v^*(0, y^*) = v^*(1, y^*) = 0$; $v^*(x^*, 0) = v^*(x^*, 1) = 0$

$-\psi^*(0, y^*) = \psi^*(1, y^*) = 0$; $\psi^*(x^*, 0) = \psi^*(x^*, 1) = 0$

$-u^* = v^* = \psi^* = 0$ at the fins.

- A regular mesh (figure 2-a) is used for the calculation of the pressure fields, and of the temperature.

- A staggered mesh whose nodes are located halfway from those of the main mesh, as shown in figure 2-(b,c) to calculate the horizontal u and vertical v components of the velocity. Each component of the velocity is staggered according to its direction.

Mesh generating functions

In the problem presented above, a sine function is used to produce the mesh in the x and y directions for the single cavities. This function can be expressed mathematically as

$$\frac{xu(i)}{H} = \frac{i-2}{imax} - \frac{1}{2\pi} \sin\left(2\pi \frac{i}{imax}\right) \quad i = imin, imax$$

$$\frac{yv(j)}{H} = \frac{j-2}{jmax} - \frac{1}{2\pi} \sin\left(2\pi \frac{j}{jmax}\right) \quad j = jmin, jmax$$

with $imin=jmin=2$, $imax=NI-2$, and $jmax=NJ-2$

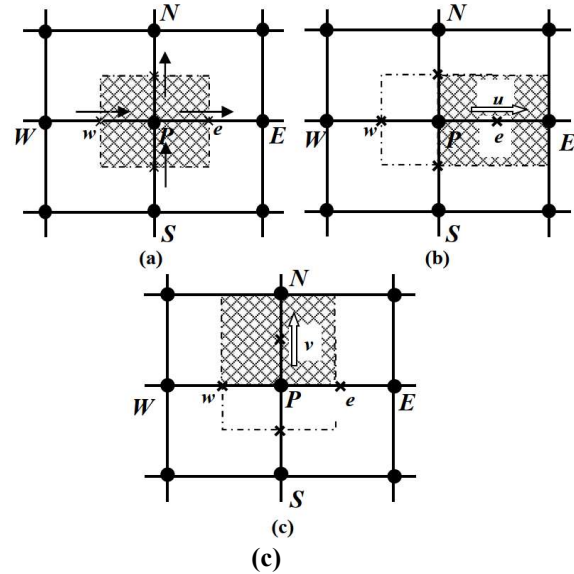


Fig. 2: Storing used parameters
(a) Storage of pressure and temperature P, T
(b) Storage of horizontal velocity u
(c) Storage of vertical velocity v

The sine function generates a non-uniform mesh that is carefully spaced near the wall and sparsely far away from the wall.

For the cavities equipped with fins, it is preferred to use a uniform mesh defined as follows:

$$\frac{xu(i)}{H} = \frac{i-2}{imax}; \quad i = imin, imax$$

$$\frac{yv(j)}{L} = \frac{j-2}{jmax}; \quad j = jmin, jmax$$

VI. CODE VALIDATION

The developed computer code is validated by the Benchmark solutions relating to the problem of natural convection in a differentially heated square cavity under different Rayleigh numbers [22].

For the validation of the numerical code without the presence of fins case, the results of references [14, 15] correspond to a closed square cavity whose left and right vertical walls are differentially heated, while the horizontal walls are adiabatic.

The validation of the computer code in the case of finned surface is done [22]. The results of reference [8] correspond to a closed square cavity. The bottom wall is characterized by the presence of an obstacle, while the left and right vertical walls are maintained at a constant cold temperature T_c , while the walls horizontal are maintained adiabatic, the isothermal obstacle is maintained at a constant temperature T_h and is placed at a well determined distance from the right and left wall. Table 1 gives the results comparison between our results for maximum, minimum and average Nusselt number with those of

the literature review for a cavity without fins under the same operating conditions.

VII. RESULTS PRESENTATION

The cavity model used in the computer code where the dimension les height and width of the fin equal respectively $h/H=1/5$, $l/L=1/20$, the distance between fins is:

$d/L=((L-N \times l)/(N+1))/L$ for N fins.

We take: $Pr=0.71$, T_h' variable and $10^3 \leq Ra \leq 10^6$

*Case of a cavity without fins

With linear temperature profile: $T_h' = T_h - (T_h - T_c) \frac{x}{L}$

The results are plotted in the form of isotherms, current functions and iso-velocities u e v respectively.

The contours of isotherms, iso-currents and iso-velocities u and v , as well as the local Nusselt number as function of the Rayleigh number of the top wall and the bottom wall are plotted respectively for the Rayleigh values: $10^3, 10^4, 10^5$ and 10^6 :

Figure 3 illustrates the isotherms, the streamlines and the iso-velocity contours (u and v) for $Pr=0.71$ and $Ra=10^3$ respectively and without fins.

- Linear heating of the bottom wall without fins

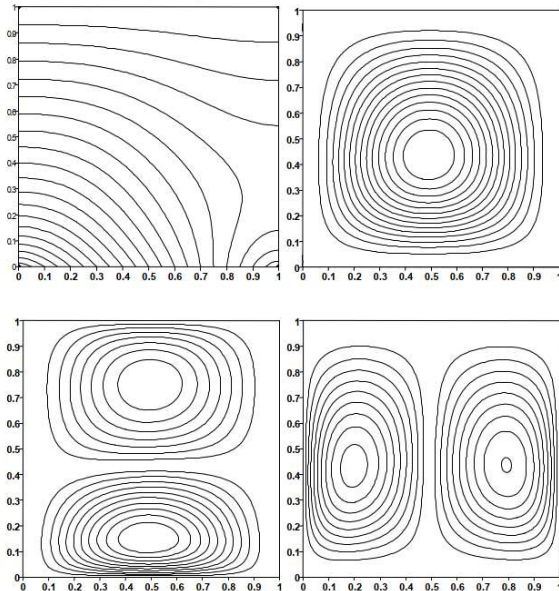


Fig. 3: Results for a finless cavity at $Ra=10^3$
Linear heating of the bottom wall without fins

Figures (4-a and 4-b) show the local Nusselt number for top and bottom wall for $Pr=0.71$ and $Ra=10^3$ respectively and without fins.

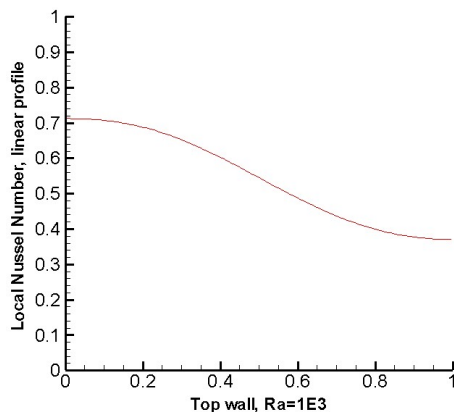


Fig. 4-a: Local Nusselt number
Top wall, for a finless cavity at $Ra=10^3$

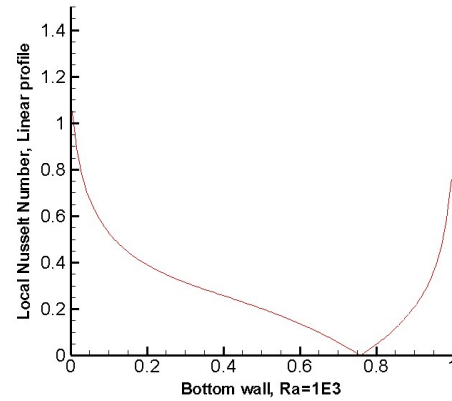


Fig. 4-b: Local Nusselt number
bottom wall, for a finless cavity at $Ra=10^3$

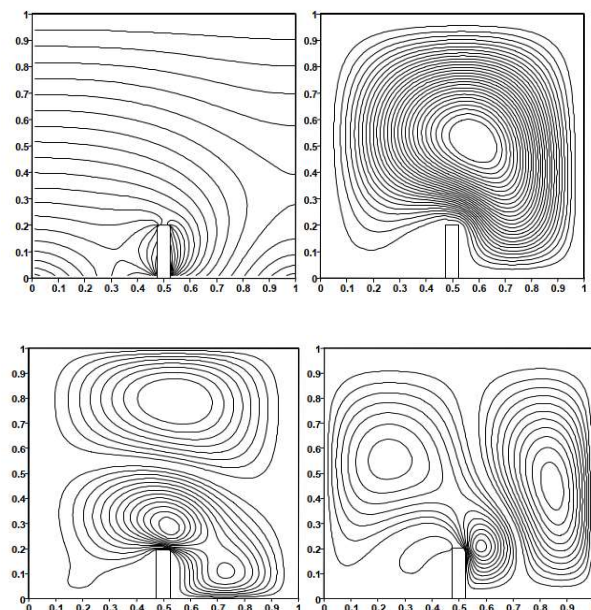
Table 1. Results comparison between the present study and those of the literature review

Ra	Nus	[16]	[17]	[18]	[19]	[20]	[21] FEM-DSC	Present study
10^3	Max	1.50	-	-	1.47	1.5062	1.501-1.444	1.506
	Min	0.692	-	-	0.623	0.6913	0.691-0.665	0.697
	Moy	1.12	-	1.117	1.074	-	1.117-1.073	1.1008
10^4	Max	3.53	3.5	-	3.47	3.5305	3.576-3.441	3.535
	Min	0.586	-	-	0.497	0.5850	0.577-0.528	0.605
	Moy	2.243	-	2.243	2.084	-	2.254-2.155	2.225
10^5	Max	7.71	7.71	-	7.71	7.7084	7.945-7.662	7.724
	Min	0.729	-	-	0.614	0.7282	0.698-0.678	0.784
	Moy	4.52	-	4.521	4.3	-	4.598-4.352	4.486
10^6	Max	17.92	17	-	17.46	17.5308	17.86-17.39	17.4912
	Min	0.989	-	-	0.716	0.9845	0.9132-0.903	1.185
	Moy	8.8	-	8.806	8.743	-	8.976-8.632	8.7268

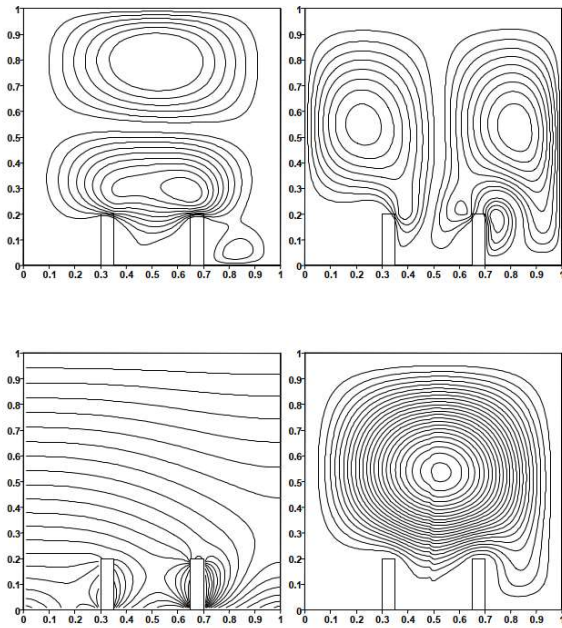
** Case of a cavity with fins

Figure 5 illustrates the isotherms, the streamlines and the iso-velocity (u and v) contours for different fin numbers ($N=1, 2, 3, 5, 7$) with $h/H=0.20$, $w/W=0.05$ and for: $Pr=0.71$ and $Ra=10^3$.

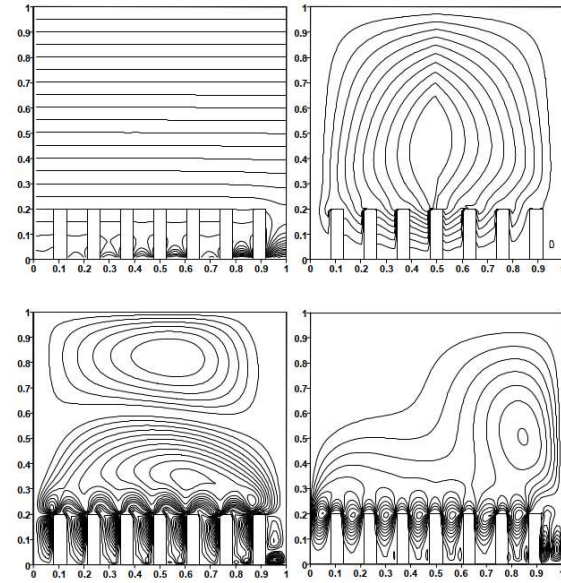
-Case of a one fin



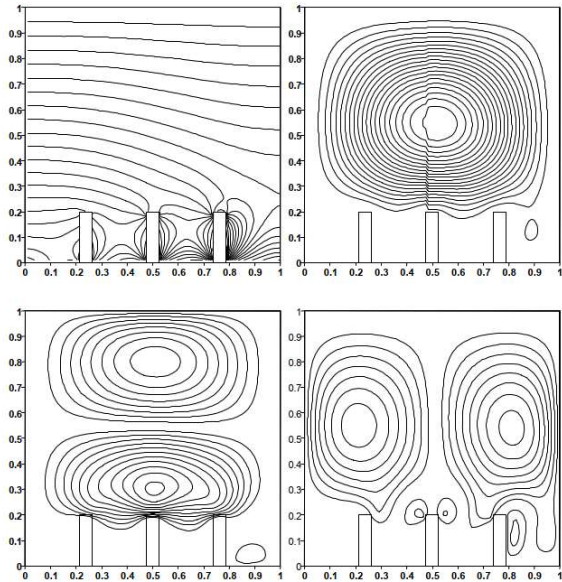
-Case of two fins



-Case of seven fins

Fig. 5: Results for a finned cavity at $Ra=10^3$

-Case of three fins



-Case of five fins

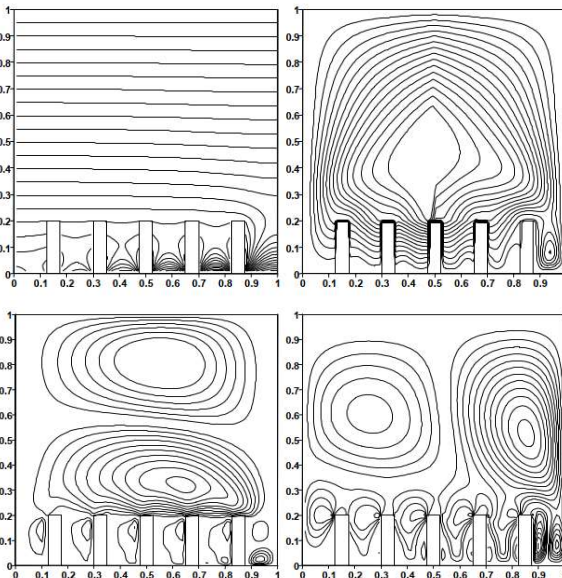
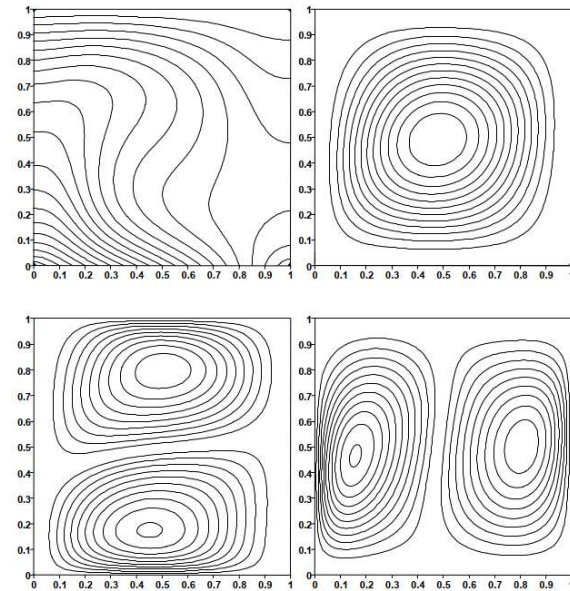
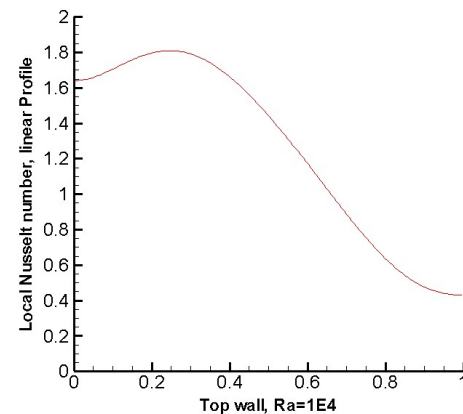


Figure 6 illustrates the isotherms, the streamlines and the iso-velocity contours (u and v) for $Pr=0.71$ and $Ra=10^4$ respectively without fins.

Fig. 6: Results for a finless cavity at $Ra=10^4$

Figures (7-a and 7-b) show the local Nusselt number for top and bottom wall for $Pr=0.71$ and $Ra=10^4$ respectively without fins.

Fig. 7-a: Local Nusselt number
Top wall, for a finless cavity at $Ra=10^4$

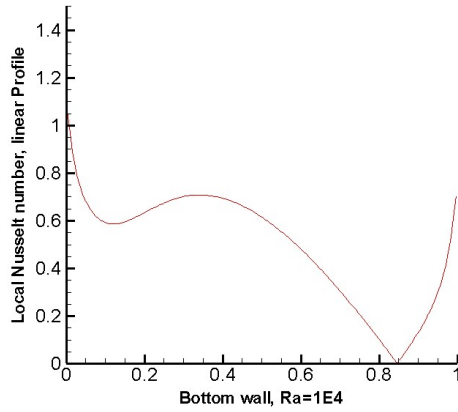
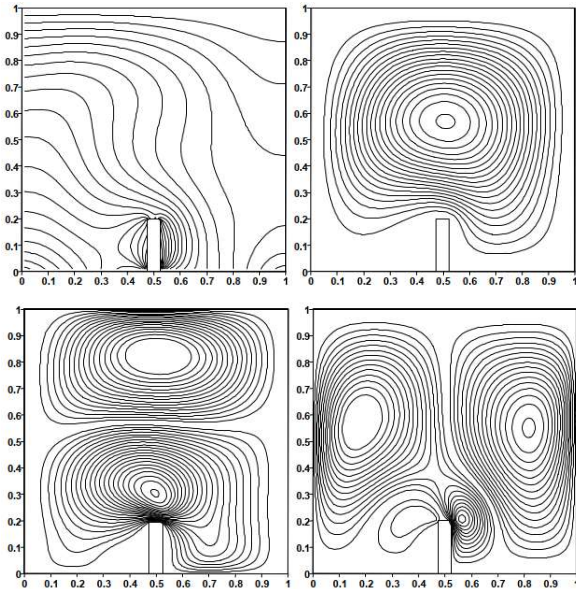


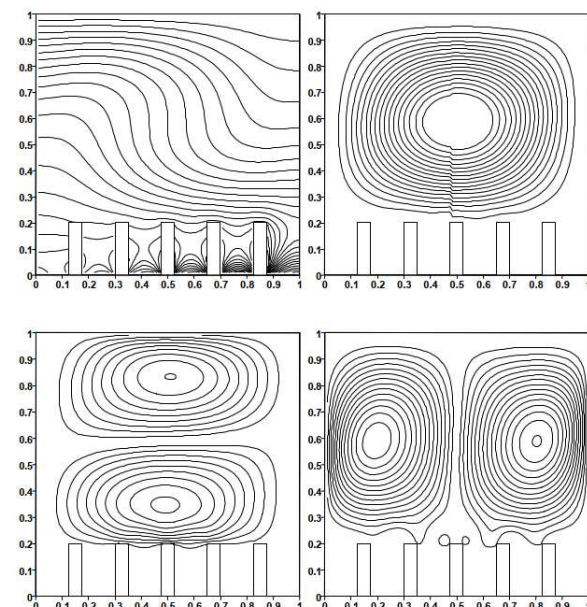
Fig. 7-b: Local Nusselt number bottom wall, for a finless cavity at $Ra=10^4$

Figure 8 illustrates the isotherms, the streamlines and the iso-velocity (u and v) contours for different fin numbers ($N=1, 2, 3, 5, 7$) with $h/H=0.20$, $w/W=0.05$ and for: $Pr=0.71$ and $Ra=10^4$.

-Case of a one fin



-Case of two fins



-Case of seven fins

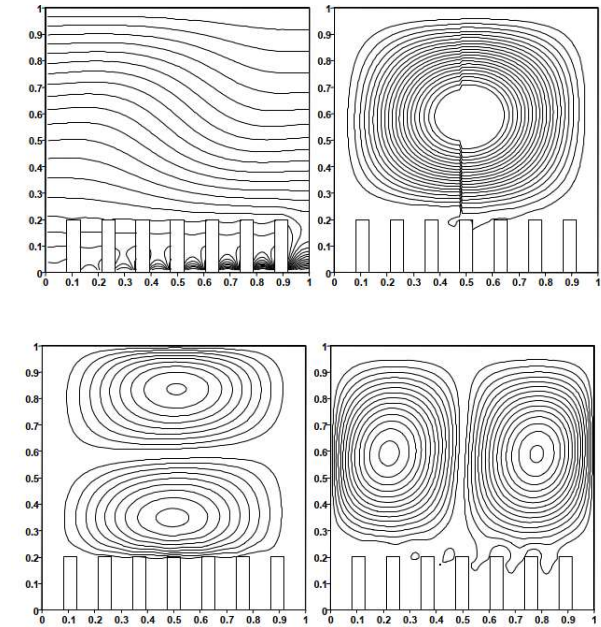


Fig. 8: Results for a finned cavity at $Ra=10^4$

Figure 9 shows the isotherms, the streamlines and the iso-velocity contours (u and v) for $Pr=0.71$ and $Ra=10^5$ respectively without fins.

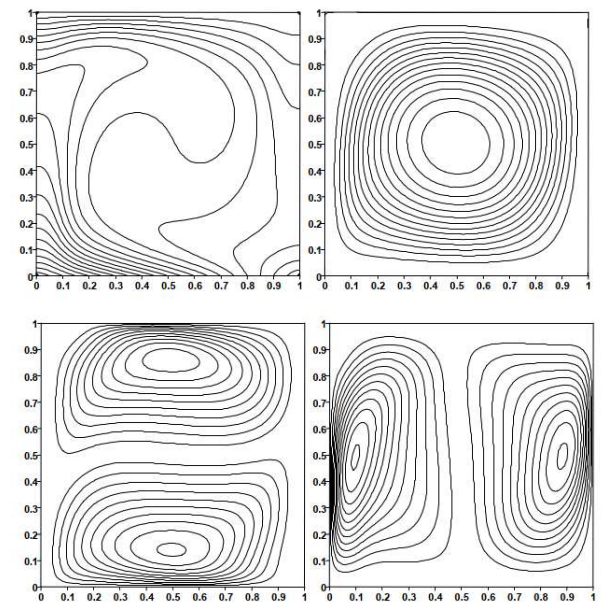


Fig. 9: Results for a finless cavity at $Ra=10^5$

Figures (10-a and 10-b) illustrate the local Nusselt number for top and bottom wall for $Pr=0.71$ and $Ra=10^5$, respectively without fins.

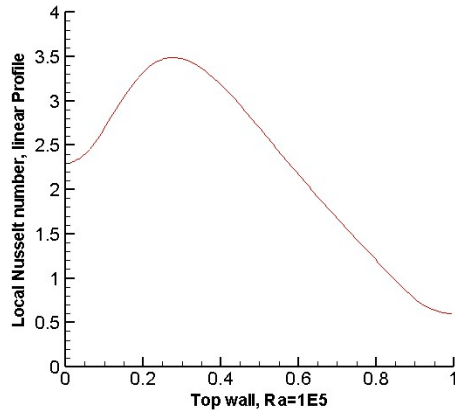


Fig. 10-a: Local Nusselt number
Top wall, for a finless cavity at $Ra=10^5$

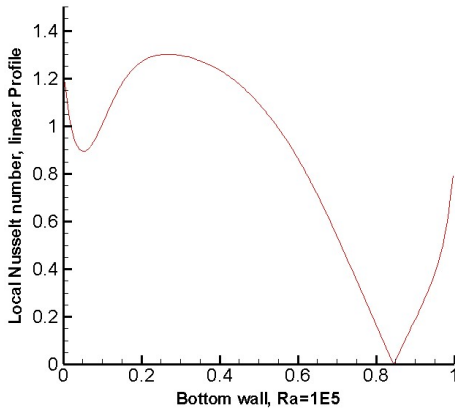
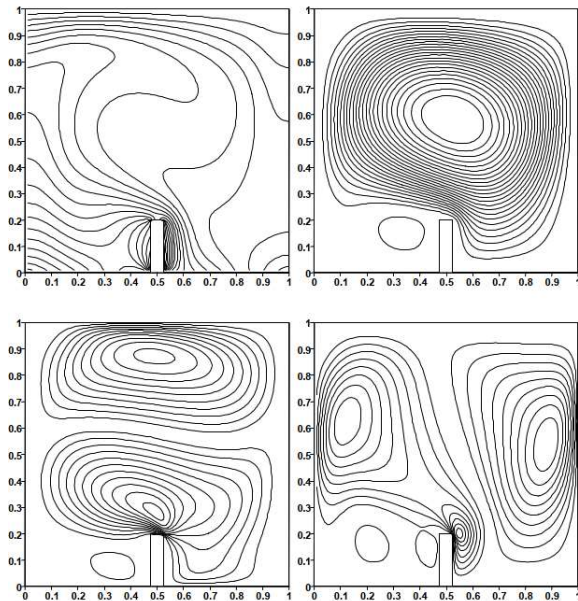


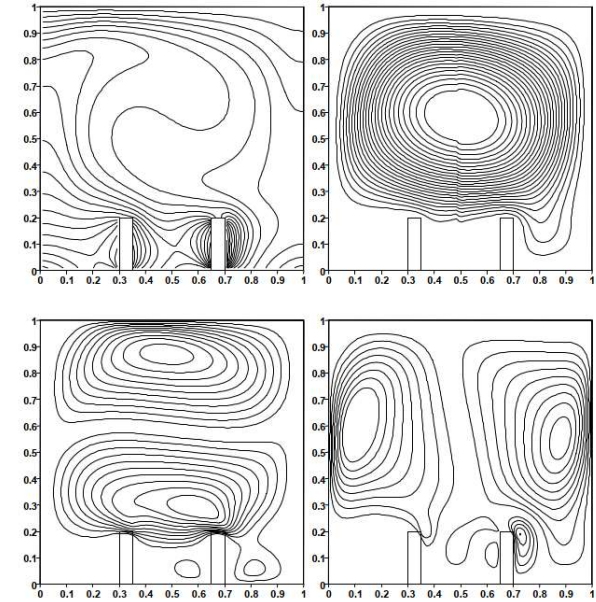
Fig. 10-b: Local Nusselt number
bottom wall, for a finless cavity at $Ra=10^5$

Figure 11 shows the isotherms, the streamlines and the iso-velocity (u and v) contours for different fin numbers ($N=1, 2, 3, 5, 7$) with $h/H=0.20$, $w/W=0.05$ and for: $Pr=0.71$ and $Ra=10^5$.

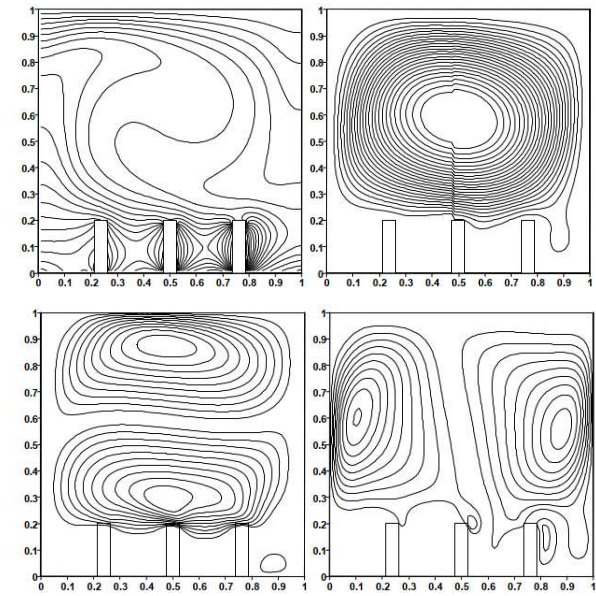
-Case of one fin



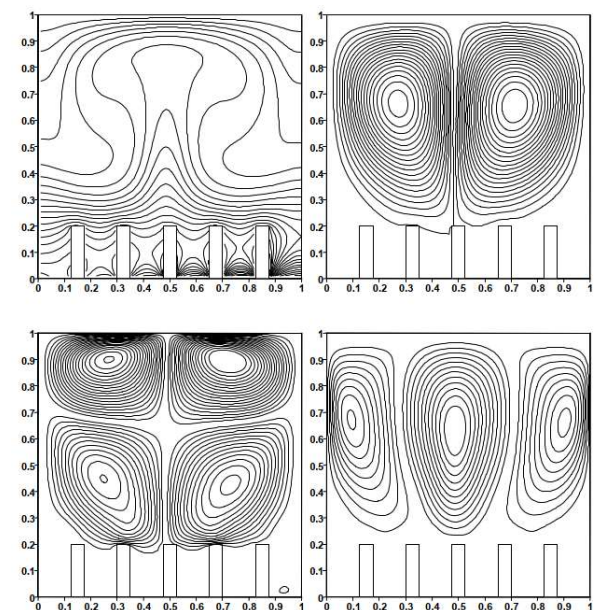
-Case of two fins



-Case of three fins



-Case of five fins



-Case of seven fins

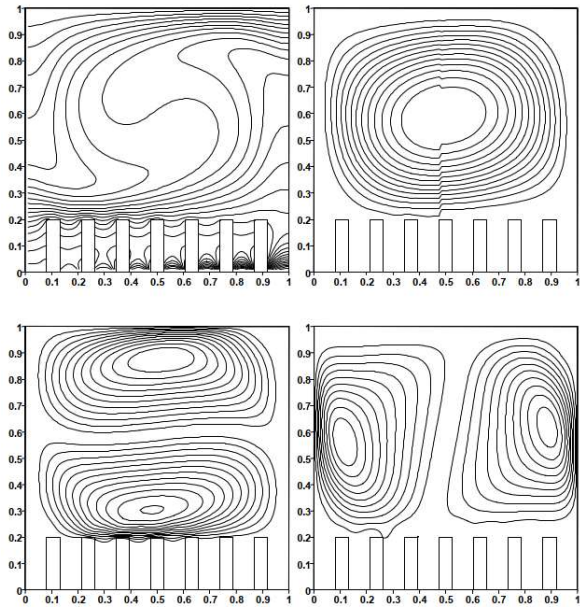


Fig. 11: Results for a finned cavity at $Ra=10^5$

Figure 12 illustrate the isotherms, the streamlines and the iso-velocity contours (u and v) for $Pr=0.71$ and $Ra=10^6$ respectively without fins.

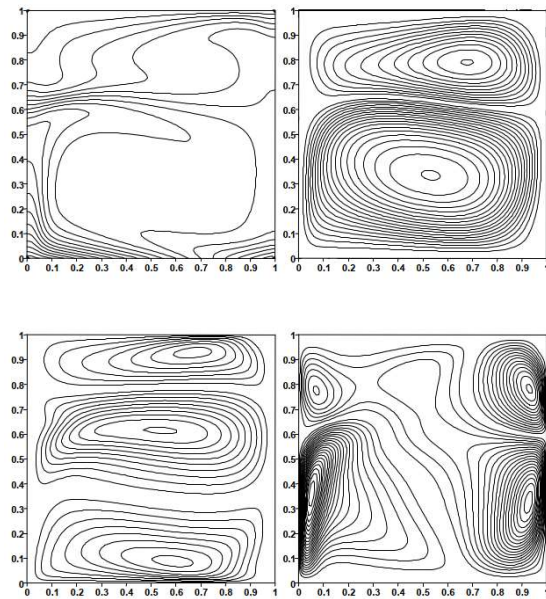


Fig. 12: Results for a finless cavity at $Ra=10^6$

Figures (13-a and 13- b) show the evolution of the local Nusselt number for top and bottom wall for $Pr=0.71$ and $Ra=10^6$ respectively without fins

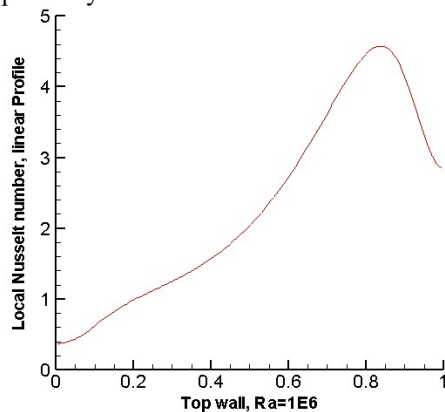


Fig.13-a: Local Nusselt number
Top wall, for a finless cavity at $Ra=10^6$

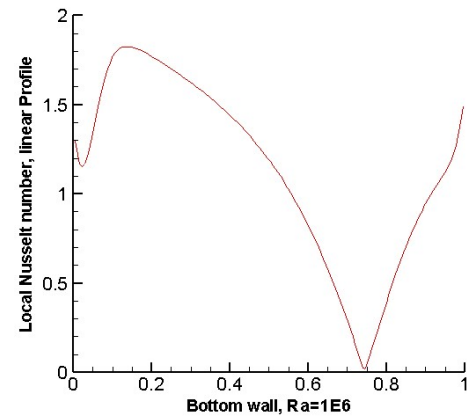
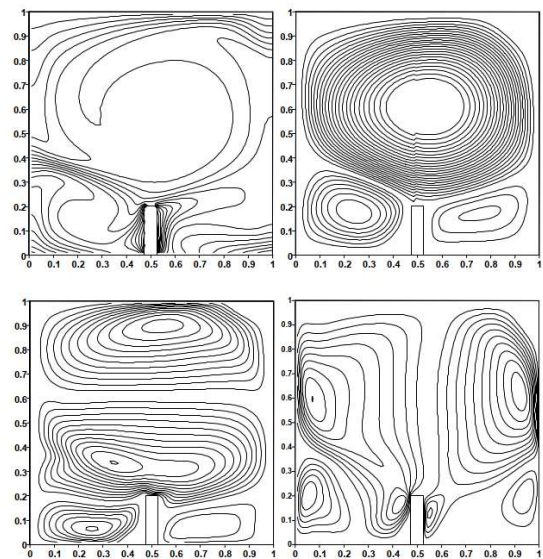


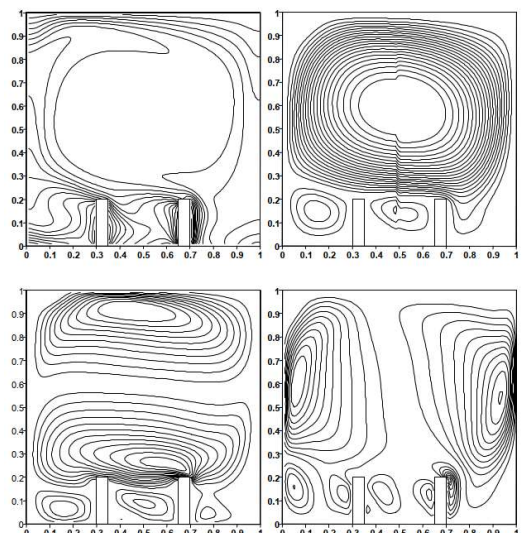
Fig.13-b: Local Nusselt number
bottom wall, for a finless cavity at $Ra=10^6$

Figure 14 illustrates the isotherms, the streamlines and the iso-velocity (u and v) contours for different fin numbers ($N=1, 2, 3, 5, 7$) with $h/H=0.20$, $w/W=0.05$ and for: $Pr=0.71$ and $Ra=10^6$

-Case of one fin

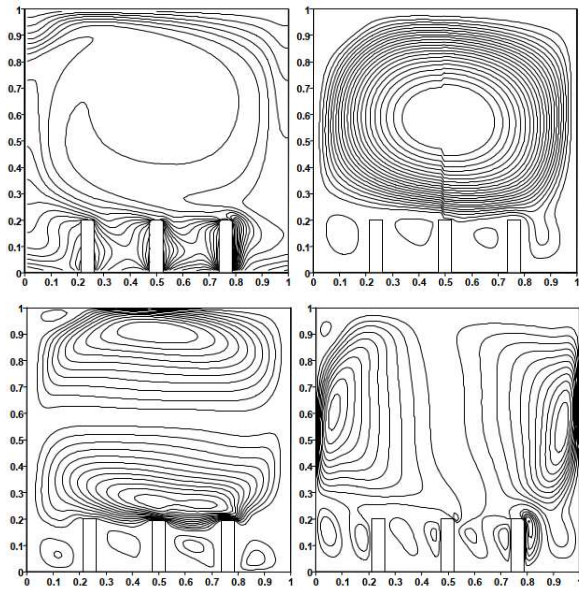


-Case of two fins

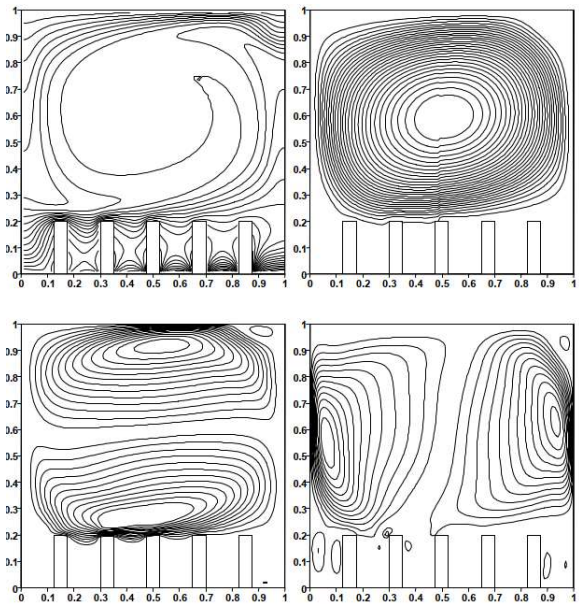


VIII. DISCUSSIONS

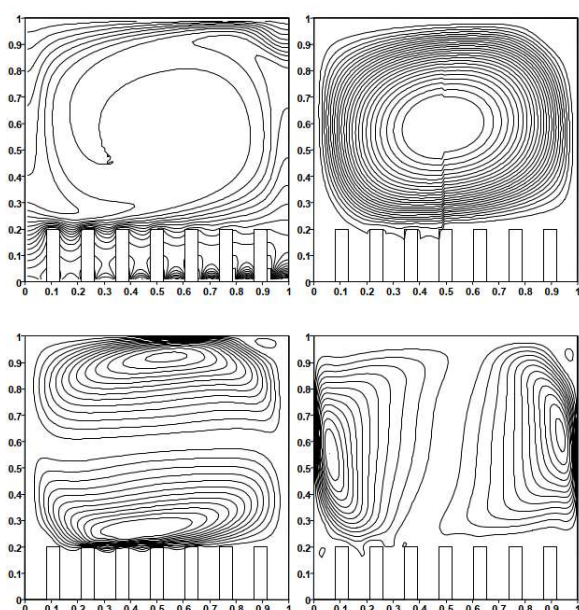
-Case of three fins



-Case of five fins



-Case of seven fins

Fig. 14: Results for a finned cavity at $Ra=10^6$ ***Linear heating of the bottom wall without fins**

The contours of the isotherms, of the current functions, as well as the iso-velocities for $Ra = 10^3$ - 10^6 and $Pr = 0.71$ are represented in figures (3), (6), (9) and (12). When the bottom wall is heated in a non-uniform and linear manner, the non-uniform heating suppresses singularities at the edges of the bottom wall and exhibits an even temperature distribution throughout the cavity.

For $Ra = 10^3$, the thermal boundary layer develops only over 60% of the cavity (figures 3). It is observed in the vicinity of $Ra=10^4$, it can be noted that the temperature at the bottom wall is not uniform and a temperature maximum occurs (Figure 6), at the center, therefore, higher rates of heat transfer occur in the center.

At $Ra = 10^5$, the circulation model is qualitatively similar to the uniform heating case (Figure 9).

Due to non-uniform bottom heating, the heating rate near the wall is generally lower, which induces less buoyancy resulting in a lower thermal gradient throughout the domain. The uniformity of the temperature distribution and the temperature gradient are even less observed on the regime of the central part inside the upper half of the domain. The effect of lower buoyancy also leads to a large area of temperature stratification at the vertical line of symmetry.

From $Ra = 10^6$, the mechanism of convection becomes more pronounced and consequently the central vortex moves upwards. Figure (12) also shows that the isotherms are horizontal and vertical inside enclosure; the boundary layers become very thin, this can be attributed to a current inside the cavity, which also causes a reduction in temperature gradients in the center of the cavity at high convection.

The contours of the iso-velocities reveal several important characteristics, in the contours of the iso-velocities- u , for the values of $Ra=10^3, 10^4, 10^5$ and 10^6 , we notice four horizontal vortices, which form, two below the two others. For the contours of the iso-velocity- v , if we take the values of $Ra=10^3, 10^4$ and 10^5 , three dominant vertical circulations one of each on the left middle and right zones of the cavity almost completely swept. The shapes can be simulated for the three values quoted, a slight deformation is observed at the extremities of the vortices with the increase in the number of Rayleigh $Ra=10^6$.

Figures (4), (7), (10) and (13), present the variation of the local Nusselt number compared to the cold wall of height and the bottom hot wall for different Rayleigh numbers, the variation is more seen for hot walls which explains the higher temperature effect.

**** Linear heating of the bottom wall with fins**

The contours of isotherms, current functions and iso-velocities are represented in figures (5), (8), (11) and (14) for $Ra = 10^3$ - 10^6 and $Pr = 0.71$ when the bottom wall is heated in a non-uniform linear manner, the non-uniform heating suppresses singularities at the edges of the bottom wall and presents regular temperature distribution throughout the cavity.

For $Ra = 10^3$ and $Pr = 0.7$ (Figure 5), the thermal boundary layer develops more than 70% of the cavity, which is lower compared to the case of uniform heating, it is observed in the around $Ra=10^4$ (Figure 8). It can be noted that the temperature at the bottom wall is not uniform and a temperature maximum occurs at the center. Therefore, higher rates of heat transfer occur at the center.

At $Ra = 10^5$ (Figure 11), due to non-uniform bottom heating, the heating rate near the wall is generally lower, which induces less buoyancy resulting in a lower thermal gradient throughout the domain.

The uniformity of the temperature distribution and the temperature gradient are even less observed on the regime of the central part inside the upper half of the domain. The effect of lower buoyancy also leads to a large zone of temperature stratification at the vertical line of symmetry.

From $Ra = 10^6$ (Figure 14), the mechanism of convection becomes more pronounced and therefore the central vortex moves upwards. We also notice that the isotherms are horizontal and vertical inside the enclosure, the boundary layers become very thin, this can be attributed to a current inside the cavity which also causes a reduction in temperature gradients in the center of the high convection cavity.

The contours of the iso-velocity reveal several important characteristics, in the contours of the iso-velocity- u , for the values of $Ra=10^3$, 10^4 and 10^5 , we notice four main horizontal vortices form above the fins, two other secondary ones form at the bottom of the cavity above the hot wall. For the value of $Ra=10^6$, two main vortices move away upwards towards the adiabatic wall while in the rest of the cavity several secondary vortices form around the fin.

For the contours of the iso-velocity- v , if we take the values of $Ra=10^3$, 10^4 and 10^5 three dominant vertical circulations one of each on the left middle and right zones of the cavity almost completely swept. The gaits can be simulated for the three values quoted, a slight deformation is observed at the extremities of the vortices with the increase in the number of Rayleigh $Ra=10^6$.

It should be noted that the presence of the fins has only slightly influenced the temperature, streamlines or iso-velocities, the change is only observed in the vicinity of the fins.

We present below the average Nusselt number variation for top and bottom walls, as function of Rayleigh number for different number of fins.

Figure 15 shows the evolution of the average Nusselt number with respect to the Rayleigh number for different numbers of fins with $h/H = 0.20$ and $w/W = 0.05$ on the hot and cold horizontal walls respectively.

As shown in the figure, the Nusselt number increases with increasing Rayleigh number for a number of fins ranging from 0 to 7 fins. The maximum values of the Nusselt number are reached for the case with three fins near the hot side of the cavity, on the other hand, the maximum values of the Nusselt number are reached for the case with seven fins near the cold side of the cavity.

Figure 16 illustrates the variation of the average Nusselt number function of the number of fins. Notice that the maximum value of the average Nusselt number is reached for the case with three fins near the hot side of the cavity at $Ra=10^6$, however it is reached at seven fins near the cold wall.

Figure 17 shows the evolution of the average Nusselt number in according with the height of the fin for different numbers of fins, as shown in the figure. Note that the average Nusselt number is reached for the case of two and three fins near the hot side of the cavity at $h/H=3/5$, contrary to the case of the cold wall, it is reached at two, tree and seven fins with $h/H=3/5$. The average Nusselt number variation for the top and bottom wall is presented below, as function of the number of fins for different Rayleigh numbers.

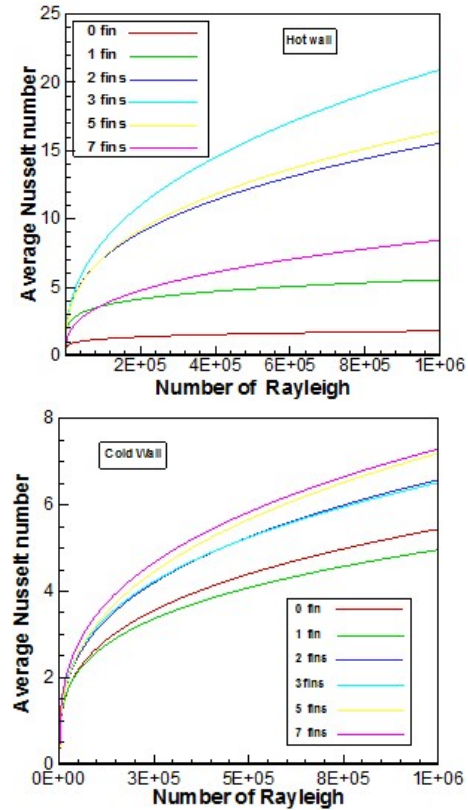


Fig. 15: Average Nusselt number according to Rayleigh number

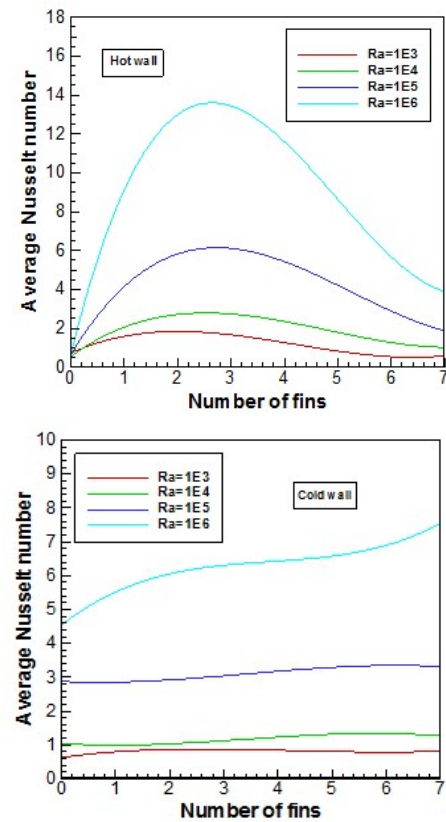


Fig.16: Average Nusselt number according to the number of fins

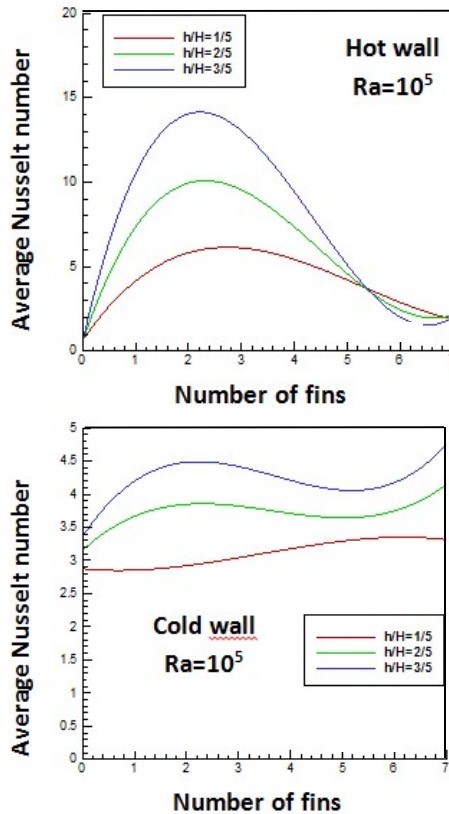


Fig. 17: Average Nusselt number at $Ra=10^5$ in according with the height of the fin

IX. CONCLUSION

A numerical study has been performed to investigate natural convection in a square cavity for laminar flow case. The cavity is made up of a finite number of fins on the bottom wall whose purpose is to improve the overall rate of heat transfer. The fins are heated using a non-uniform linear temperature profile; the vertical walls are adiabatic while the horizontal walls are subjected to cold temperatures at the top walls and hot and non-uniform temperatures at the bottom. The finite volume method is used for solving the governing equations. This method allows to obtain more realistic solutions in terms of isotherms, current functions as well as iso-velocities for a wide range of Rayleigh number and a Prandtl number fixed at 0.71.

The heat transfer rate is lower at the edges by the non-uniform heating, while it exhibits higher heat transfer rates at the center of the bottom wall. Boundary layer formation has been shown to occur, thermal boundary layer has been observed to grow about 60% for non-uniform heating. Non-uniform heating can be used in small regimes followed by uniform heating. This combination can be suitable to achieve improved heat transfer effects. The results obtained are also very useful for a better understanding of heat transfer by natural convection which takes place in partitioned enclosures, and towards the best and cheapest way to achieve thermal performance of these heat transfer elements.

Finally, we would like to stress the idea that the modeling of a heat transfer or a fluid flow system must be guided by the objectives and the constraints of the system.

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